Centre for Climate Change Economics and Policy

Grantham Research Institute on Climate Change and the Environment

Declining discount rates and the Fisher Effect: Inflated past, discounted future?

Mark C. Freeman, Ben Groom, Ekaterini Panopoulou and Theologos Pantelidis

April 2013

Centre for Climate Change Economics and Policy
Working Paper No. 129

Grantham Research Institute on Climate Change and the Environment
Working Paper No. 109

The Centre for Climate Change Economics and Policy (CCCEP) was established by the University of Leeds and the London School of Economics and Political Science in 2008 to advance public and private action on climate change through innovative, rigorous research. The Centre is funded by the UK Economic and Social Research Council and has five inter-linked research programmes:

1. Developing climate science and economics
2. Climate change governance for a new global deal
3. Adaptation to climate change and human development
4. Governments, markets and climate change mitigation
5. The Munich Re Programme - Evaluating the economics of climate risks and opportunities in the insurance sector

More information about the Centre for Climate Change Economics and Policy can be found at: http://www.cccep.ac.uk.

The Grantham Research Institute on Climate Change and the Environment was established by the London School of Economics and Political Science in 2008 to bring together international expertise on economics, finance, geography, the environment, international development and political economy to create a world-leading centre for policy-relevant research and training in climate change and the environment. The Institute is funded by the Grantham Foundation for the Protection of the Environment and the Global Green Growth Institute, and has five research programmes:

1. Global response strategies
2. Green growth
3. Practical aspects of climate policy
4. Adaptation and development
5. Resource security

More information about the Grantham Research Institute on Climate Change and the Environment can be found at: http://www.lse.ac.uk/grantham.

Declining discount rates and the Fisher Effect: Inflated past, discounted future?

Mark C. Freeman, Ben Groom, Ekaterini Panopoulou and Theologos Pantelidis*

May 8, 2013

Abstract

Uncertain, yet persistent, real rates of return to capital underpin one argument for using a declining schedule of social discount rates. Yet persistency is only present in approximately the first three-quarters of the time-series of US Treasury bond yields used by Newell and Pizer [37] to estimate the term structure for the US Environmental Protection Agency. This coincides with the period in which the series reflects nominal, rather than real, interest rates. To overcome this disconnect the 'Fisher Effect' is estimated using a cointegrated model of inflation and nominal interest rate data. The real interest rate series is then simulated and the certainty equivalent discount rate calculated without the need for extensive data transformations, such as smoothing out negative real interest rates. An arguably more credible schedule of declining discount rates is then estimated. International guidelines on Cost-Benefit Analysis should be updated to reflect this methodological advance.

JEL: Q48, C13, C53, E43

Keywords: Declining Discount Rates, Fisher Effect, Real and Nominal Interest Rates, Social Cost of Carbon.

1 Introduction

Despite some puzzles along the way, the burgeoning theoretical literature on discounting distant time horizons points more or less unanimously towards the use of a declining term structure of social discount rates (DDRs) [17, 18, 49, 52, 54].[1] This conclusion is robust to an individual's position in relation to the normative-positivist debate, which characterised the heated aftermath of the Stern Review, provided that the primals of the discounting problem are assumed to exhibit persistence over time [3, 12, 17].[2]

Consensus in an area of theory as potentially fraught as social discounting is a rare thing. Perhaps for this reason the literature on DDRs has been highly influential. The UK, French and Norwegian governments now recommend DDRs for intergenerational Cost-Benefit Analysis (CBA) [27, 31, 34]. The literature on DDRs also motivates the US Environmental Protection Agency's (USEPA) recommendation that a lower discount rate with a flat term structure should be applied to intergenerational projects for sensitivity analysis [50]. The US Interagency Working Group on the Social Cost of Carbon recommends similar practices [28]. Furthermore, DDRs are currently being considered by the USEPA and the Office of Management of Budgets (OMB) after a recent consultation of experts.[3]

*Mark Freeman, School of Business and Economics, Loughborough University, United Kingdom. E-mail: M.C.Freeman@Lboro.ac.uk. Ben Groom, Department of Geography and Environment, London School of Economics, United Kingdom. E-mail: B.Groom@lse.ac.uk. Ekaterini Panopoulou, Department of Statistics and Insurance Science, University of Piraeus, Greece. E-mail: Apano@Unipi.gr. Theologos Pantelidis, Department of Economics, University of Macedonia, Greece. E-mail: Pantelidis@Uom.gr. Author for correspondence: Ben Groom.

[1] This is for risk-free discounting of certainty-equivalent future costs and benefits. See [19] for a discussion about project and systematic risk in discounting.

[2] Strictly speaking, there are also additional conditions required on the nature of the inter-temporal social welfare function (e.g. [17]).

[3] For the out come of the RFF expert panel meeting see Arrow et al. (2012).

Yet in the process of policy debate, it has become clear that there is no clear consensus on how to operationalise a schedule of DDRs for use in CBA. One need only look at the heterogeneous and occasionally ad hoc motivations for the current policies as evidence for this (e.g. [27]). This lack of consensus turns out to be important since the outcomes of intergenerational valuations are sensitive to the empirical choice of discount rates. Indeed, the range of policy prescriptions arising from different empirical approaches to estimating the DDR schedule is comparable to that emanating from the distinct positions taken in the thorny normative-positive debate (e.g. [12, 23, 40]).

In this paper we explore the empirical sensitivities associated with the declining certainty equivalent discount rate proposed by Martin Weitzman [52] when uncertainty is characterised using historical interest rate data. Newell and Pizer [36, 37] (henceforth N&P) showed that US bond yields have exhibited sufficient persistence in the past two centuries for the empirical schedule of DDRs to exhibit a rapid decline, raising the US$ (2000) social cost of carbon from $5.7/tC to between $6.5/tC and $10.4/tC in the process [37]. This latter result was shown to be highly sensitive to the time-series model used to characterise interest rate uncertainty. Subsequent work by Groom, Koundouri, Panopoulou and Pantelidis [23] (henceforth GKPP) showed that once a wider range of time-series models is considered, and a process of model selection undertaken, there are good theoretical and empirical reasons to prefer models which allow for more flexible characterisation of uncertainty in the interest rate data generating process. Their preferred model and associated schedule of DDRs raised the social cost of carbon yet further to $14.4/tC. More global approaches to estimating the DDR using international bond yields have been undertaken by [20] and [26].

The idea that uncertainty about future interest rates leads to DDRs has strongly influenced the current UK, US and Norwegian governments' guidance on long-term discounting ([27, p98.] [28, p 24.] [34, Ch 5, p79.] [50, Ch 6, p23.]).[4] It also has prominence in the current consultation taking place in the US (e.g. [3]) and is expected to serve as an input into the 2013 'refresh' of the UK Treasury Green Book. Taken together, the prominence of this approach and the sensitivity of policy decisions to the empirical methods employed motivates further investigation into the robustness or otherwise of the results in the literature.

To this end, in this paper we focus on the time-series of bond yields used by N&P. This data set has been particularly important as it was also used by GKPP in the other main US-focussed study in this area. N&P use annual market yields for long-term government bonds for the period 1798 to 1999. Starting in 1950, nominal interest rates are converted to real ones by subtracting a ten-year moving average of the expected inflation rate of the CPI as measured by the Livingston Survey of professional economists. This ex-ante measure of inflation does not exist prior to 1950, and so expected inflation is assumed to equal zero for the first three-quarters of the series. [20] also assume that nominal and real interest rates are equivalent before 1950 in their international study. This paper focusses on testing the robustness of empirical schedules of the DDR to this assumption.

It has been well documented that, prior to 1950, the United States went through periods of both highly positive and highly negative inflation (see, for example, [13] and [5]). A priori, therefore, it seems highly likely that the time-series of real and nominal interest rates would have significantly differed during the first three-quarters of N&P's sample period; this is a conjecture that we confirm later in the paper. This is potentially of great importance since the persistence which underpins the decline in the DDRs is more prominent in the nominal interest rate series used from 1798 to 1950. The remaining real interest rate series up to 1999 is arguably mean reverting. Furthermore, the volatility of real and nominal interest rates is typically very different. These observations suggest that the shapes of the term structure reported in earlier studies may potentially be non-robust to different assumptions about the inflation process.

[4] For instance, the Norwegian Guidelines conclude: "Beyond 40 years, it is reasonable to assume that one will be unable to secure a long-term rate in the market, and the discount rate should accordingly be determined on the basis of a declining certainty-equivalent rate as the interest rate risk is supposed to increase with the time horizon. A rate of 3 percent is recommended for the years from 40 to 75 years into the future. A discount rate of 2 percent is recommended for subsequent years."

Indeed, if real interest rates have been mean-reverting through the entire sample period, then the resulting schedule of social discount rates will be effectively flat.

To investigate the sensitivity of current policy recommendations to assumptions about inflation we propose a method which removes the disconnect between nominal and real interest rates that occurs in 1950. In this paper, the real interest rate series is determined by empirically characterising the theoretical relationship between nominal interest rates and inflation known as the 'Fisher Effect' [10]. Modelling real interest rates in this way then results in a term structure of certainty equivalent discount rates that are inflation-adjusted for the whole sample period.

The techniques that we use have other methodological advantages over those originally employed by N&P and GKPP. Historically there have been repeated periods (including the time of writing) of negative real interest rates, yet N&P remove this possibility by transforming the data to a three year moving average. Furthermore, a logarithmic model is used which then removes the possibility of a negative certainty equivalent discount rate.[5] Neither of these adjustments are necessary within this paper.

What is perhaps surprising, but heartening, about our results is that previous schedules of the DDR appear to be largely robust to more rigorous treatment of inflation. The schedules that we describe generally decline more sharply at long horizons than either N&P or GKPP. However, at the short end, social discount rates are higher than those described by GKPP. As a consequence, the estimated Social Cost of Carbon lies between the estimates of N&P and GKPP, but closer to the latter than the former.

2 A Theory of Declining Discount Rates

When using market bond yields to inform the discount rate, policy makers are taking a positivist approach to social discounting. A project with a consumption certainty equivalent future benefit V_t at future time t and zero at all other times is then, from a valuation perspective, economically equivalent to a zero-coupon default risk-free bond with maturity t. The appropriate positivist valuation approaches can therefore be taken directly from the asset pricing literature.

A well-known result from financial economics (see, for example, [2, Equation 16]) is that the present value of the project under consideration at some earlier time h is given by:

$$P_h = E_h\left(V_t \exp\left(-\sum_{\tau=h}^{t-1} r_\tau\right)\right) \tag{1}$$

where r_τ is defined as the logarithmic expected single-period return for holding a claim on V_t over the interval $[\tau, \tau+1]$: $\exp(r_\tau) = E_\tau[P_{\tau+1}/P_\tau]$. The derivation of equation 1 emerges simply from repeated iteration of the single-period Net Present Value equation.

Define the variable $r(t)$ by $P_0 = E_0[V_t]\exp(-tr(t))$. If V_t is non-stochastic, or at least uncorrelated with $\sum_{\tau=h}^{t-1} r_\tau$ (something that [53] calls a 'pragmatic-decomposition' assumption), then:

$$r(t) = -\frac{1}{t}\ln\left(E_0\left(\exp\left(-t\overline{r}_t\right)\right)\right) \tag{2}$$

where $\overline{r}_t = t^{-1}\sum_{\tau=0}^{t-1} r_\tau$ is the average value of r_τ over the horizon of the project. Following Weitzman [52] we call $r(t)$ the *certainty equivalent discount rate*, and the corresponding *certainty-equivalent forward rate,* $\widetilde{r}_t$, for discounting between adjacent periods at time t is:

$$\widetilde{r}_t = \frac{E(P_t)}{E(P_{t+1})} - 1 \tag{3}$$

This is commonly referred to as the Expected Net Present Value (ENPV) approach. Crucially, exponential functions are convex and so, by Jensen's inequality, $r(t) < E_0(\overline{r}_t)$. The magnitude

[5] N&P argue that short-term fluctuations are not strictly relevant to the time horizons that are the focus of their paper. Furthermore, negative real rates do not appear in their data, the argument being that these are transitory phenomena [36, p 10].

of this inequality is driven by two parameters; the value of t and the uncertainty over $\overline{r}_t$. That the inequality gets greater with larger t causes the term structure of social discount rates to decline with the horizon of the project. That the inequality also gets greater with more uncertainty over $\overline{r}_t$ means that understanding the volatility of average future costs of capital is the critical empirical task facing those who wish to operationalise the ENPV approach.

When parameterising equation 2, N&P and others estimate the statistical properties of $\overline{r}_t$ from a historic time-series of yields on a long-term bond. However, it is not immediately obvious that single period expected returns on long bonds, with horizons of a few decades, and a many-century $t-$period default risk-free fixed income security should be the same. In general, empirical estimates of the Treasury yield curve are upward sloping, suggesting that $\overline{r}_t$ is likely to be higher than an average long-term bond yield. However, the literature on social discount rates generally ignores these yield curve issues by assuming that the liquidity premium on bonds of all horizons is zero. We retain this assumption here, the motivation for which is two-fold.

Within environmental economics, it has been common to justify the ENPV approach through the original thought experiment of [52]. He assumes that future interest rates are currently unknown but that, in one instant, all uncertainty will be removed. The true value of r_0 will be revealed and $r_\tau = r_0$ with certainty for all future τ. In this case, the ENPV approach with r_τ proxied by a short-term risk-free rate has been justified through the literature on the so-called 'Weitzman-Gollier puzzle'. This starts with [15] and thus far culminates with [21] and [49] via [25], [7] and [11]. In fixed income pricing, the use of the ENPV equation in the absence of liquidity premia is given by [8] in continuous time and [14] in discrete time. Here equation 2 is referred to as the Local Expectations Hypothesis. In this case, rather than all uncertainty being removed in one instant, a less restrictive 'local certainty' equivalent is required. By having consumption at time $\tau + 1$ fully known at time τ, all assets have a zero consumption beta and therefore all risk and liquidity premia are also zero. Consistent with the 'Weitzman-Gollier' puzzle literature (excluding [11], who uses a pure exchange economy), logarithmic utility for the social planner is a critical condition to justify local certainty of consumption.

The social planner's current uncertainty over the far-horizon average future Treasury long-bond yield will depend on two things; the volatility of r_τ itself and the persistency of shocks to this series. Even if interest rates are highly volatile, provided that these shocks are transitory then the long-term average of r_τ will be relatively stable, leading to a slowly declining schedule of social discount rates. However, if shocks are persistent, then these will remain important into the distant future. N&P use their data to estimate an AR(3) model and compare this to a fully persistent Random Walk specification. The uncertainty in the discount rate is then simulated using multiple forecasts. In both cases persistence is found to be sufficient to cause a rapidly declining term structure. N&P could not distinguish between the two models on statistical grounds. For this reason, the USEPA guidelines on discounting take an average between these two model to inform their lower 2.5% rate for intergenerational projects (e.g. [50]).

Subsequent work showed that the empirical schedule of DDRs based on N&P's data is not robust to different empirical models, making model selection crucial for informing policy (e.g. [23]). In particular, in the US and UK cases, rigorous model selection leads to a preference for models that can deal with more flexible and complex characterisations of the mean and variance of the interest rate process (e.g. [22]).

While N&P and GKPP have concentrated on the robustness of the N&P approach to model selection, [20] and [26] concentrated instead on the choice of data. Their contributions lie in internationalising the debate. If the social planner is interested in the global social cost of carbon, then this cannot be estimated using US bond yields alone. The analysis in this paper is also primarily focussed on data-related issues, although we also make methodological improvements to previous studies. Rather than internationalising the data, we remain within a US context but handle inflation more rigorously than either N&P or GKPP. As discussed above, the empirical term structure of the certainty equivalent discount rates is extremely sensitive to the estimated persistence and volatility present in the data. Indeed, closer scrutiny of the time-series used in the previous literature exposes several transformations of the data to which the empirical schedule of DDRs is almost certainly going to be sensitive, and we explore these issues below.

3 Data on Interest Rates

N&P base their analysis on nominal long-bond Treasury yields for the period 1798 to 1999. Starting in 1950, the Livingston Survey of professional economists is used to construct a measure of expected inflation, which is then used to create real interest rates. No adjustment to nominal yields is made before 1950. The interest rates are then converted to their continuously compounded equivalents and estimations are made using a three-year moving average of this series. Finally, N&P used logarithms of the series which preclude negative rates and makes interest rate volatility more sensitive to the level of interest rates. A trend correction is also required [36].

N&P have an extremely thorough description of their methods and the treatment of their data, as well as a convincing justification for the steps taken (see also [36]). Nevertheless, there are certain features of their 200 year series and the transformations undertaken which are worthy of further investigation given the sensitivity of the schedule of DDRs to different empirical treatments. Appendix A shows some descriptive statistics and statistical tests on the N&P series, while Appendix B compares the N&P series to series on nominal and real interest rates sourced from Global Financial Data (GFD) for the duration over which there is overlap with the N&P data: 1820-1999. Both serve to motivate our closer scrutiny of the N&P data and our subsequent alternative methodological approach.

First, in Appendix A, Figures A1-A4 show the result of a rolling estimates of the AR(3) model of interest rates that forms the central model of N&P, together with the associated p-value of the Augmented Dickey Fuller (ADF) test for a unit root.[6] Figures A1 and A2 use unsmoothed data, while Figures A3 and A4 use the three year moving average data used by N&P. Figure A1 uses a 50 year window for the rolling estimation and shows that there are only two periods when the null hypothesis of a unit root is rejected. The first is the set of 50 year windows with starting points between 1810 and 1830. The second is the set of 50 year windows with starting points from 1945 until 1950. The latter set of windows are made up predominatly of the real data series. The pattern becomes more clear in Figure A2 in which a 100 year window is used for the rolling estimation. By this measure, it becomes clear that persistence is a pre-1950 phenomenon associated with the nominal but not the real interest rate data.

Figure A3 and A4 show the results of a similar exercise for the smoothed data used by N&P, for 50 and 100 year windows respectively. Qualitatively speaking, Figures A3 and A4 show that the extent of persistence again declines towards the more recent windows of data containing a greater proportion of the post-1949 real interest rate series. More importantly, when the data is smoothed there is no 50 or 100 year window in which the null hypothesis of a unit root can be rejected.[7] A comparison of Figures A1 and A2 with Figures A3 and A4 shows that, whatever the theoretical logic, smoothing the data inevitably increases persistence in the series.

Additional evidence for the existence of a unit root in the nominal interest rate data, but not the real interest rate data, can be found in Table A1. Here an ADF test is undertaken on the pre-1950 nominal data and the post-1949 real data, unsmoothed and smoothed. The unit root hypothesis is rejected for the smoothed real (post-1949) data. This underpins the rejection of the null when the entire smoothed series is tested.

Finally, Appendix B illustrates how the unsmoothed N&P series compares with the GFD data on real and nominal interest rates since 1820. The first thing to notice is that the N&P series has smoothed away three periods during which real interest rates were negative: the early 1900s, the late 1930s to early 1940s and the late 1960s to early 1970s. Second, the GFD real interest rate data is much more volatile than the N&P data, particularly pre-1950 when the N&P data is nominal. Table B1 indicates that the correlation of the N&P data with the GFD data is much weaker pre-1950 for both real and nominal GFD series. Furthermore, the N&P

[6] The ADF test contains the lagged difference terms appropriate for the AR(3) model.

[7] The rolling ADF test is undertaken without a trend component, although similar results arise when the trend is included.

series is more strongly correlated with the nominal GFD data than the real. Lastly, Table B2 shows that the autocorrelation coefficients for each data source: N&P, GFD nominal and GFD real, are quite different.

Much of this analysis is merely descriptive of course. However, from a theoretical and empirical perspective it seems clear that some of the assumptions underpinning the series used by N&P and GKPP are not completely satisfactory. Smoothing, the removal of negative real interest rates and, in particular, the disconnect between nominal and real interest rates before 1950 appear to be driving some of the time-series properties of the data that are important from the perspective of DDRs. There may also be some conceptual problems with the use of the Livingston Survey of Professionals data on the CPI since the interest rate data is for a long-bond, while the survey is typically concerned with one-year inflation estimates. It is also worth noting that a fairly dim view of the Livingston survey is taken in some quarters.[8] In the following section we propose an alternative empirical and theoretical approach for estimating the DDR schedule which addresses these problems.

4 A Bivariate Model for Calculating the Declining Discount Rates

The key problem highlighted in the previous section is the disconnect between nominal and real interest rates in the N&P data. The reasons for this approach were reasonable in the sense that data on expectations of inflation were not available prior to 1950 when the Livingston survey of economists started collecting such data. We propose a solution to this problem which allows expected inflation to be modelled using data on observed inflation, and hence real interest rates to be derived using data on nominal interest rates and inflation. As we explain, this generates a real interest rate series for 200 years.

4.1 A model of nominal interest rates and inflation: The 'Fisher effect'

We are interested in estimating the long-run behaviour of ex ante real interest rates using data on nominal interest rates and inflation. This relationship is often analysed in the context of the 'Fisher' relationship [10]. Specifically, if we let $y_t(m)$ denote the $m-$period nominal interest rate at time t, $x_t^e(m)$ to be the expected rate of inflation from time t to $t+m$, and $r_t^e(m)$ stand for the ex-ante $m-$period real interest rate, we can express the 'Fisher effect' as follows:[9]

$$y_t(m) = x_t^e(m) + r_t^e(m) \tag{4}$$

The additional assumption of rational expectations (see, e.g. [33]) allows us to link realised inflation to expected inflation, $x_t(m) = x_t^e(m) + \upsilon_t$, where υ_t is a white noise process, orthogonal to $x_t^e(m)$. Finally, if we further assume that the real interest rate is a white noise process with a mean value r, we end up with the following equation:

$$y_t(m) = r + \theta x_t(m) + u_{1t} \tag{5}$$

In the literature, there are alternative theories about the magnitude of the θ parameter in the above equation. The traditional Fisher hypothesis suggests that $\theta = 1$. However, there are different approaches that suggest a θ that is either greater than unity (e.g. [9]) or less than unity (e.g. [35]). On an empirical basis, the findings are also mixed. Mishkin was one of the first researchers who pinpointed the problem of spurious regression when examining the relationship between nominal interest rates and inflation due to the non-stationarity of the series [33]. Therefore, he suggested that cointegration techniques are necessary to investigate the Fisher effect. However, even when someone applies appropriate cointegration methods, the

[8] It has been described as being "poorly designed throughout most of its history, having been intended more for journalistic than scientific purposes.." [48, p.127]

[9] This is an approximate Fisher model. The exact relationship being: $(1 + y_t(m)) = (1 + x_t^e(m))\,(1 + r_t^e(m))$. The approximation works well when $x_t(m)r_t^e(m)$ is small.

small sample properties of the cointegrating estimators play an important role introducing a significant level of uncertainty in the estimated value of the θ parameter. In our empirical analysis, we choose not to impose any restrictions on the value of θ and organize our simulation in a framework that takes into account the uncertainty surrounding the value of θ.[10]

Next, we describe a Data Generating Process (DGP) for the relationship between interest rates and inflation rates. The DGP, put forward by Phillips [44, 45], develops a general framework for the dynamics of the variables under scrutiny and it is often used in the literature to examine the finite sample properties of cointegrating estimators (see, [41, 47]).

4.2 The triangular data generating process

We consider the triangular DGP for the I(1) vector $\mathbf{z}_t = [y_t,\, x_t]^\top$ given in equation 5, and:[11]

$$\Delta x_t = u_{2t} \tag{6}$$

The cointegrating error, u_{1t}, and the error that drives the regressor, u_{2t}, compose an I(0) process, $\mathbf{u}_t = [u_{1t}, u_{2t}]^\top$, described by the following VAR(1) model:

$$\mathbf{u}_t = A\mathbf{u}_{t-1} + \mathbf{e}_t \tag{7}$$

where A is a 2×2 parameter matrix and $\mathbf{e}_t$ is a white noise process. More specifically, $\mathbf{u}_t$ is given by:

$$\begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix} = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix} \begin{pmatrix} u_{1t-1} \\ u_{2t-1} \end{pmatrix} + \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix} \tag{8}$$

and

$$\begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix} \sim NIID \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \Sigma_e = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{pmatrix} \right] \tag{9}$$

Note that this DGP suggests that the cointegration parameter θ is time-invariant. We test and provide evidence that supports this assumption in the empirical part of our study.

4.3 Implications of the triangular model

Before proceeding to more detailed empirical estimations, N&P present a simple AR(1) representation of their model to show the role that persistency, volatility and maturity play in determining the DDR schedule. Here, we undertake a similar task for our DGP under the assumption that $\theta = 1$. In this case, the real interest rate $r_t = y_t - x_t = r + u_{1t}$. If r is currently unknown, but is believed to be distributed according to $N\left(\overline{r}, \sigma_r^2\right)$, then, from equation 1,

$$\begin{aligned} P_0 &= E\left(\exp(-rt)\right) E\left(\exp\left(-\sum_{\tau=1}^{t} u_{1\tau}\right)\right) \\ &= \exp(-\overline{r}t + 0.5t\sigma_r^2) E\left(\exp\left(-\sum_{\tau=1}^{t} u_{1\tau}\right)\right) \end{aligned} \tag{10}$$

The structure of the summation term term in (10) depends on the value of the parameters in A and Σ_e. If $a_{12} = 0$, our DGP for the real interest rate becomes a simple mean-reverting process. This coincides with that of N&P and thus it is persistence, measured by a_{11}, and uncertainty, measured by σ_r^2 and σ_{11}, that determine the speed of decline in the social discount rate. On the other hand, when $a_{12} \neq 0$, the dynamics become more complicated.

[10] Our focus on the cointegrating relationship between nominal interest rates and inflation means that we are not interested in modelling the real interest rate directly, and hence we do not follow the procedures associated with previous models of the certainty equivalent discount rate found in GKPP.

[11] For expository purposes, we drop m.

We next calculate the expected value of $\exp(-\sum_{\tau=1}^{t} u_{1\tau})$ based on the following infinite Moving Average (MA) representation of u_{1t}

$$u_{1t} = \sum_{i=0}^{\infty} \Phi_i \mathbf{e}_{t-i} \tag{11}$$

where $\Phi_0 = I_2$ is a 2×2 identity matrix, and $\Phi_i = A^i$, $i = 1, 2, \ldots$. Given the Cholesky decomposition of $\Sigma_e = BB^\top$, we obtain the following representation

$$u_{1t} = \sum_{i=0}^{\infty} \Theta_i \mathbf{w}_{t-i} \tag{12}$$

where $\Theta_i = \Phi_i B$ and $\mathbf{w}_t = B^{-1}\mathbf{e}_t \sim IIDN(\mathbf{0}, I_2)$ [32].

In an attempt to avoid unnecessary complications, let us assume that $a_{21} = 0$. In this case, the eigenvalues of A, denoted as λ_1 and λ_2, are equal to a_{11} and a_{22} respectively. Then, given that the generation mechanism starts at time $t = 1$, we end up with the following result:

$$E[\exp(-\sum_{\tau=1}^{t} u_{1\tau})] = \exp\{0.5(1 + R_1 + R_2)\}, \text{ where} \tag{13}$$

$$R_1 = \sum_{\tau=1}^{t-1}[1 + (\frac{\sqrt{\sigma_{11}}\lambda_1}{1-\lambda_1} + \frac{a_{12}\sigma_{12}\lambda_1}{\sqrt{\sigma_{11}}(\lambda_1-\lambda_2)(1-\lambda_1)})(1-\lambda_1^\tau) - \frac{a_{12}\sigma_{12}\lambda_2}{\sqrt{\sigma_{11}}(\lambda_1-\lambda_2)(1-\lambda_2)}(1-\lambda_2^\tau)]^2$$

$$R_2 = \sum_{\tau=1}^{t-1}[\frac{\sqrt{\sigma_{22} - \frac{\sigma_{12}^2}{\sigma_{11}}}a_{12}\lambda_1}{(\lambda_1-\lambda_2)(1-\lambda_1)}(1-\lambda_1^\tau) - \frac{\sqrt{\sigma_{22} - \frac{\sigma_{12}^2}{\sigma_{11}}}a_{12}\lambda_2}{(\lambda_1-\lambda_2)(1-\lambda_2)}(1-\lambda_2^\tau)]^2$$

Substituting equation 13 into equation 10, we obtain an expression for the expected value of the discount factor and then the instantaneous discount rate at time t in the future is calculated based on the continuous-time equivalent of the certainty equivalent forward rate in equation 3. This expression is algebraically lengthy, and not reported for brevity, but allows us to conclude that, similarly to the case of the AR(1) model of N&P, it is persistence, measured by λ_1 and λ_2, and uncertainty, measured by σ_r^2 and the elements of Σ_e, that determine the speed of decline in the discount rates. As expected, the discount rate is a decreasing function of t. Figure 1 illustrates the path of the DDR for two different levels of persistence which is controlled by the value of λ_1 while keeping the remaining parameters fixed. It is clear that the discount rate declines faster reaching much lower levels as λ_1 increases. The same picture arises from Figure 2 that plots the DDRs for values of λ_1 in the interval $[0.6, 0.95]$, while keeping the remaining parameters of the process fixed.

5 Empirical Results and Simulation

We now turn to empirical estimates of the cointegrating relationship between inflation and nominal interest rates. A number of models are deployed to allow for flexible estimation of the Fisher parameter, θ, and to check for the robustness of the certainty equivalent discount rate to different specifications of the cointegrating relationship. Expected inflation is proxied by the 10-year average realised inflation rate as calculated from the CPI deflator (CPI data to 2009). This matches the inflation horizon with the bond horizon. The real interest rate series is not modelled directly, but is derived from predictions from the cointegration estimators. Our results confirm the widely held view that interest rates and inflation rates are I(1) processes and cointegrated.[12] As a result, the first condition for the Fisher hypothesis, i.e. the condition that $y_t(m)$ and $x_t(m)$ are cointegrated processes is satisfied. Next, we outline the alternative cointegration estimators we employ.

[12] Detailed tables of unit root tests and cointegration tests are available from the authors upon request.

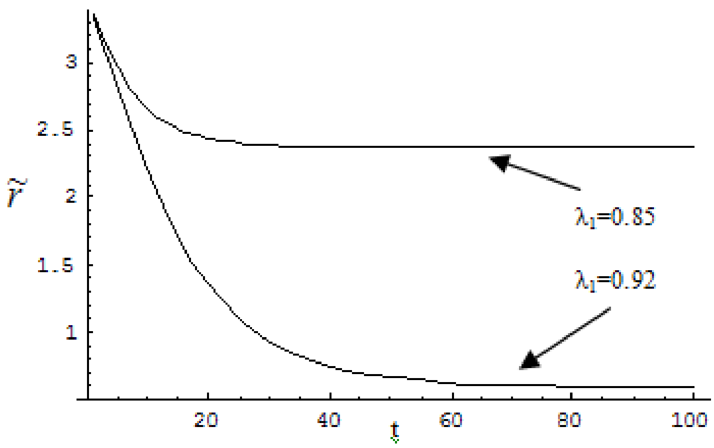

Figure 1: Certainty-Equivalent Discount Rate

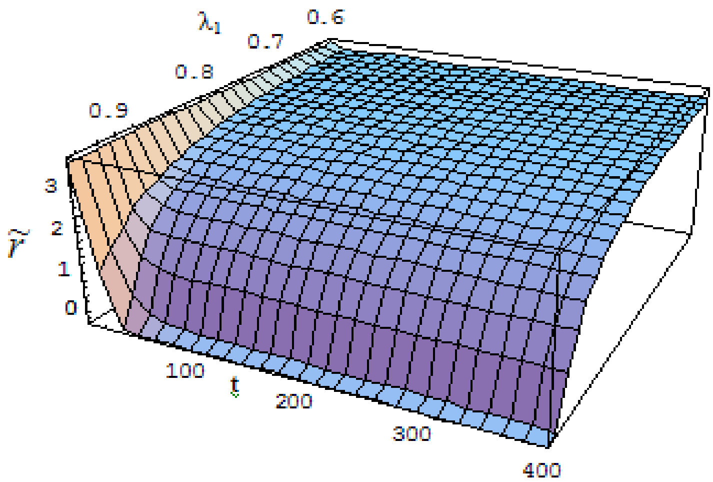

Figure 2: Certainty-Equivalent Discount Rate

5.1 Estimation of the cointegrating parameters

We consider both parametric and semi-parametric cointegration estimators, the majority of which are asymptotically efficient provided that the conditions of the Functional Central Limit Theorem (FCLT) are satisfied. Next, we provide a brief description of these estimators:

Dynamic OLS (DOLS(p,t)): This estimator has been suggested by several papers [45, 46, 47]. It provides a direct way to estimate the cointegrating relationship and asymptotically leads to valid test statistics. It utilises the static equation 5, augmented by lags and leads of the first difference of the regressor, i.e.:

$$y_t = \theta x_t + \sum_{i=1}^{p-1} \gamma_i \Delta x_{t-i} + \sum_{j=1}^{t-1} d_j \Delta x_{t+j} + v_t \tag{14}$$

Existence of serial correlation of v_t does not raise any serious problems in the estimation of θ and can be dealt with by consistently estimating the long-run variance of v_t as proposed by Newey and West [38].

Fully Modified Least Squares (FMLS): The FMLS estimation method, proposed by Phillips and Hansen [44], employs semi-parametric corrections for the long run correlation and endogeneity effects, which fully modify the OLS estimator and its attendant standard error. This estimator is based on consistent estimation of the long-run covariance matrices, which requires the selection of a kernel and the determination of the bandwidth. We employ the Quadratic Spectral kernel and select the bandwidth parameter by applying the Newey-West procedure [39]. Moreover, we consider the "prewhitened" version of FMLS which filters the error vector $\widehat{\mathbf{u}}_t$ prior to estimating the long-run covariance matrices.[13]

Canonical Cointegrating Regression (CCR): Park's Canonical Cointegrating Regression (CCR) is closely related to FMLS, but instead employs stationary transformations of the data to obtain least squares estimates and remove the long run dependence between the cointegrating error and the error that drive the regressors [42]. As in FMLS, consistent estimates of the long-run covariance matrices are required. To this end, we consider the "prewhitened" version of CCR and employ the Quadratic Spectral kernel with the bandwidth selected by the Newey-West procedure.

Johansen's Maximum Likelihood (JOH): This is the well-known system-based maximum likelihood estimator of θ, suggested by Johansen [29, 30]. The order of the JOH estimator corresponds to the lag-order of the Vector Autoregressive model on which this estimator is based. An important difference between this estimator and the other cointegration estimators considered in this study is that it has been developed and proved to be asymptotically optimal in the context of a Gaussian Vector Autoregression which accommodates a rather narrow class of DGPs.

Augmented Autoregressive Distributed Lag (AADL(q,r,s)): This estimator is based on the following AADL(q,r,s) model [43]:

$$y_t = \theta x_t + \sum_{i=1}^{q-1} a_i \Delta x_{t-i} + \sum_{j=1}^{r-1} b_j \Delta y_{t-j} + \sum_{h=1}^{s-1} a_h \Delta x_{t+h} + \epsilon_t$$

The parameter of interest is equal to the long-run multiplier of y_t with respect to x_t. A direct estimate of the parameter of interest θ along with its standard error may be obtained by

[13] [41] perform Monte Carlo simulations for a variety of DGPs and show that significant gains can emerge when the "pre-whitened version" of the FMLS estimator is employed.

transforming the AADL model into the Bewley form (see [6, 51, 4]). Estimates of the coefficients and their standard errors can be obtained by using the Instrumental Variables (IV) estimator, with the original matrix of regressors being the instrumental variables [51].

5.2 Stability and estimates of the cointegration vector

Before proceeding to the estimation of the cointegrating regression (5), we first test its stability over the two centuries of data that we employ. Specifically, we employ three tests, namely the L_c, *MeanF* and *SupF* tests, each with the null hypothesis that the cointegrating vector is time-invariant [24]. Each can be derived as Langrange multiplier (LM) tests in correctly specified likelihood problems, which differ in their alternative hypotheses. Specifically, the null hypothesis in each case is that the cointegrating vector is constant, while the alternative is that parameters either follow a martingale process (L_c, *MeanF*) or exhibit a single structural break at unknown time t (*SupF*).[14] Each test tends to have power in similar directions and can detect whether the proposed model captures a stable relationship. The asymptotic distribution of the test statistics is non-standard and depends on the nature of trends in the cointegrating regression. [24] provides both tabulated critical values and function p-values that map the observed test statistic into the appropriate value in the range of $p \in [0, 1]$ and more specifically into the range of interest: $p \in [0, 0.20]$. Table 1 presents the stability tests for the parameters in the cointegrating regression. Test statistics are calculated on the basis of a fully modified estimation with the covariance parameters estimated using the Quadratic Spectral kernel and prewhitened residuals with a VAR(1) model. The bandwidth is selected by means of the Andrews (1991) procedure [1].[15] P-values are calculated by the function p-value methodology (see [24]). A p-value of 0.20 suggests significance at > 0.20 level.

Table 1. Parameter stability tests

Test	L_c	(p-val)	*MeanF*	(p-val)	*SupF*	(p-val)
United States	0.151	(0.20)	1.099	(0.20)	2.130	(0.20)

Overall, our findings suggest that the cointegrating relationship between the US inflation and the nominal interest rate is stable. To this end, we proceed with the estimation of the parameters in the Fisher equation. Specifically, we employ the five estimators described in Section 5.1 along with the Akaike Information Criterion (AIC) to choose the lag and lead specification for DOLS and AADL as well as the lag specification for JOH. AIC is also used to determine the optimal lag specification for the estimation of the long-run covariance matrix in the context of FMLS and CCR. Table 2 presents the estimated values of r and θ, together with the standard errors of the estimates for all the estimators under consideration.

Table 2. Cointegrating Regression Parameter Estimates

	DOLS		FMLS		CCR		JOH		AADL	
	Estimate	*s.e.*	*Estimate*	*s.e.*	*Estimate*	*s.e.*	*Estimate*	*s.e.*	*Estimate*	*s.e.*
r	3.652	0.421	3.995	0.889	4.301	0.805	0.087	2.062	3.173	1.336
θ	0.541	0.221	0.434	0.260	0.287	0.187	2.259	0.608	0.815	0.485

Our findings suggest that estimates are quite heterogeneous across estimators. Specifically, estimates of θ range from as low as 0.287 (CCR) to 2.259 (JOH) associated with a high level of uncertainty as depicted in the standard errors. Similar findings pertain to the r estimate with estimates ranging from 0.087 (JOH) to 4.301 (CCR).

[14] The tests are built in the context of fully modified estimation of the cointegrated regression. To save space, we do not give details on the formulation of the tests. The interested reader is referred to [24].

[15] Alternative specifications with respect to the choice of kernel, bandwidth and prewhitening yielded qualitatively similar results. We thank Prof. Hansen for making the codes available at http://www.ssc.wisc.edu/~bhansen/progs/progs.htm.

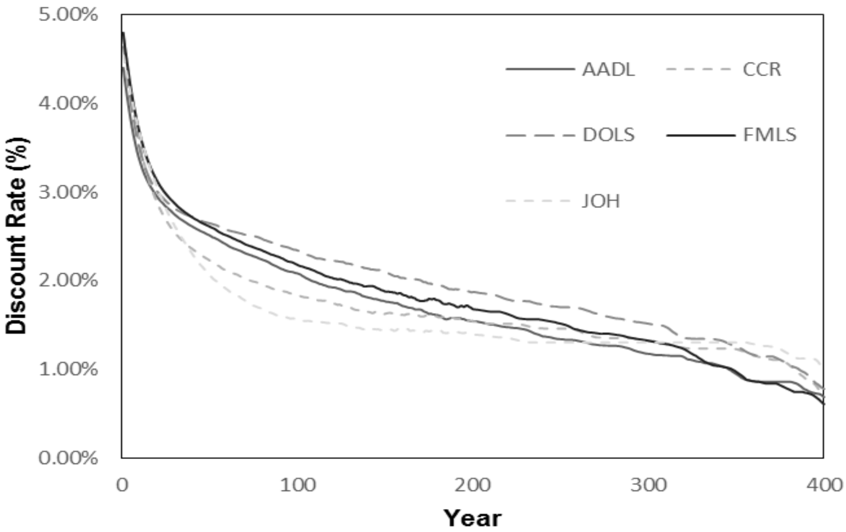

Figure 3: Term Structures of the Social Discount Rate- Cointegration Estimators

5.3 Calculation of certainty-equivalent discount rates

To characterise the uncertainty of future real interest rates, we first simulate multiple future paths of real interest rates and then calculate the certainty equivalent rate following the simulation approach proposed by N&P adjusted for the DGP proposed above. The estimates (and the corresponding standard errors) of r and θ given in Table 2 are employed to estimate the residual series u_{1t} and u_{2t}. Once the residual series are obtained, we fit a VAR(1) model and get estimates for the elements of the A and Σ_e matrices. The variance-covariance matrix Σ_A of the estimated $vecA$ is also obtained. 300,000 future paths (of 400 years length) are simulated for the nominal interest rate and the inflation rate taking into account: i) the stochastic dynamics of the DGP; ii) the uncertainty surrounding the estimated parameters; and, (iii) the in-sample properties of the US real interest rate. Appendix C provides a detailed account of the steps taken in the simulation.

Figure 3 shows the term structure of the social discount rate resulting from the simulations and calculation of the certainty equivalent discount rate for each of the five cointegration estimators.

Strikingly, the term structures arising from our proposed methods appear quite similar irrespective of the choice of the estimator. For each estimator the resulting term structure at $t = 0$ is set equal to 4.4 percent and falls below 3 percent after 25 years.[16] The fastest decline appears when we employ JOH and CCR which reach 2 percent after 54 and 73 years respectively. The respective values for AADL, DOLS and FMLS are 108, 170 and 128 years respectively. Finally, the discount rate approaches zero in the very long-run, ranging from 0.42 percent to 0.67 percent after 400 periods for FMLS and JOH, respectively. Ultimately, there is sufficient persistence in the cointegrated series to cause a significant decline in the term structure over a policy relevant time horizon.

For comparative purposes in Figure 4 we plot the term structure from the AADL model with the preferred term structures of the previous empirical work in this area, alongside the UK Treasury Green Book term structure. The AADL model is chosen since each of the empirical models is theoretically equivalent, but the AADL model is widely regarded to have better empirical qualities.

These results are an important robustness check on previous work and indicate that if a government is to take this positive approach to social discounting long-term time horizons, care is needed not only in model selection, as discussed in GKPP [22, 23], but first and foremost

[16] The starting point for the term structure is in each case the value of the last data point in the series: 1999.

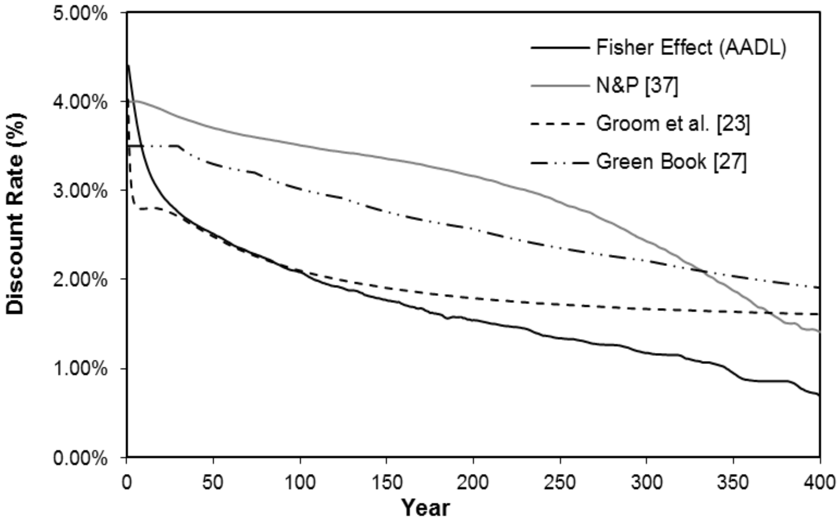

Figure 4: Empirical Term Structures for the Social Discount Rate

in the treatment of the interest rate data. It is clear that the term structure emerging from modelling the Fisher Effect are broadly consistent with, but clearly distinct from, those that make more arbitrary assumptions concerning the data. The policy implications of this finding are likely to be important for intergenerational projects. We now make this claim explicit by evaluating a typical intergenerational question: the Social Cost of Carbon.

6 Application: The Social Cost of Carbon (SCC)

Following N&P and GKPP we used the Social Cost of Carbon to illustrate the policy implications of the different discounting approaches. The marginal damages of an additional tonne of carbon are estimated using the DICE model (See Figure **??**1 in Appendix D). The SCC is the present value of this profile of carbon damages which remain positive for a 400 year horizon at least. Table 3 shows the implications of the alternative discounting approaches for the SCC in dollars per tonne of carbon. The N&P approach provides the lowest SCC and the other approaches are compared in percentage terms to that. The current approach is at the top end of the estimates, being 102% higher than N&P and 12% smaller than GKPP [23].

Table 3. The Social Cost of Carbon ($US 2000)

Discounting Approach	SCC	% difference to N&P (MR)
Flat 4%	5.7	-11%
N&P [37] (Mean Reverting)	6.4	0%
N&P [37] (Random Walk)	10.4	62%
Green Book [27]	12.5	95%
Fisher Effect (AADL)	**12.9**	**102%**
GKPP [23]	14.4	125%
USEPA (Flat 2.5%) [50]	16.6	159%

7 Conclusion

The empirical estimation of Weitzman's [52] declining term structure of the Social Discount Rate using historical interest rate data undertaken by Newell and Pizer [37] (N&P) and Groom, Koundouri, Panopoulou and Pantelidis [23] (GKPP) has directly influenced governmental guidance in the US, and indirectly influenced policy in a number of other countries [27, 34, 28, 50]. Yet the US interest rate data series used by N&P and GKPP reflects nominal interest rates pre-1950 and real interest rates thereafter. Furthermore negative real interest rates are removed and smoothing takes place. A cursory analysis of this series and comparisons to historic real interest rates shows that the time-series properties of the nominal and real data series differ markedly. These properties, such as persistence, volatility, and the lower bound are important determinants of the term structure of certainty equivalent discount rates.

By modelling the relationship between inflation and nominal interest rates in the US we are able to proceed in calculating the certainty equivalent on real interest rate data alone, without the need for the removal of negative real interest rates or smoothing. In essence we use an empirically testable theoretical structure to remove the disconnect between nominal and real interest rates present in the N&P data. The conclusions are qualitatively similar to N&P in that a declining term structure emerges. Yet the decline is closer to that of GKPP [23] which is more rapid. This results in a social cost of carbon which, at $12.9/tC, is over 100% higher than N&P's mean reverting value.

These results are an important robustness check on previous work and indicate that if a government is to take this positive approach to the social discounting of long-term time horizons, care is needed not only in model selection, but first and foremost in the treatment of the interest rate data. The ongoing discussions on discounting in the US and the upcoming 'refresh' of the UK Treasury Green book should take heed.

References

[1] Andrews D.W.K. (1991), Heteroskedasticity and autocorrelation consistent covariance matrix estimation. Econometrica, 59, 817-858.

[2] Ang, A., and J. Liu, (2004), How to discount cashflows with time-varying expected returns, Journal of Finance, 59, 2745–2783.

[3] Arrow, K. J., Maureen L. Cropper, Christian Gollier, Ben Groom, Geoffrey M. Heal, Richard G. Newell, William D. Nordhaus, Robert S. Pindyck, William A. Pizer, Paul R. Portney, Thomas Sterner, Richard S. J. Tol and Martin L. Weitzman (Forthcoming), 'How Should Benefits and Costs be Discounted in an Intergenerational Context?' Review of Environmental Economics Policy.

[4] Banerjee, A., Dolado, J.J., Galbraith, J.W. and D.F. Hendry (1993), Cointegration, Error Correction and the Econometric Analysis of Non-Stationary Data, Oxford, Oxford University Press.

[5] Barsky, R.B. and J.B. De Long (1991), Forecasting pre-World War I inflation: The Fisher Effect and the Gold Standard. The Quarterly Journal of Economics, 106(3), 815-836.

[6] Bewley, R.A (1979), The direct estimation of the equilibrium response in a linear model, *Economics Letters,* 3, 357-61.Mishkin, F.S.(1992), Is the Fisher effect for real? A reexamination of the relationship between inflation and interest rates. *Journal of Monetary Economics*, 30, 195-215.

[7] Buchholz, W., and J. Schumacher, (2008), Discounting the long distant future: a simple explanation for the Weitzman–Gollier puzzle. Working paper: University of Regensburg.

[8] Cox,J. C., Ingersoll, J. E. & Ross, S. A. (1981), 'A re-examination of traditional hypotheses about the term structure of interest rates', *Journal of Finance,* 36, 769–799.

[9] Darby, M.R. (1975), The financial and tax effects of monetary policy on interest rates, *Economic Inquiry,* 13, 266-269.

[10] Fisher, I. (1930), *The theory of interest*, Macmillan.

[11] Freeman, M.C. (2010), Yes, the far-distant future should be discounted at the lowest possible rate: A resolution of the Weitzman-Gollier puzzle, Economics: The Open-Access, Open-Assessment E-Journal, 2010-13, pp.1-21.

[12] Freeman, M.C. and B. Groom (2010), Positively gamma discounting. Working Paper: Loughborough University.

[13] Friedman, M. and A.J. Schwartz (1963), A Monetary History of the United States. Princeton University Press, Princeton.

[14] Gilles, C. and S.F. LeRoy (1986), A note on the local expectations hypothesis: A discrete-time exposition, Journal of Finance, 41, 975–979.

[15] Gollier, C. (2004), Maximizing the expected net future value as an alternative strategy to gamma discounting. Finance Research Letters, 1, 85–89.

[16] Gollier, C. (2007), The consumption-based determinants of the term structure of discount rates. Mathematics and Financial Economics, 1(2), 81–102.

[17] Gollier, C. (2008), Discounting with fat-tailed economic growth, J Risk Uncertain (2008) 37:171–186. DOI 10.1007/s11166-008-9050-0

[18] Gollier, C. (2009), Should we discount the far-distant future at its lowest possible rate?., Economics - The Open-Access, Open-Assessment E-Journal 3(2009.25), 1.14.

[19] Gollier, C. (2012), Evaluation of long-dated investments under uncertain growth trend, volatility and catastrophes. CESifo Working Paper: Industrial Organisation, No. 4052. http://hdl.handle.net/10419/69569

[20] Gollier, C., P. Koundouri & T. Pantelidis (2008), .Declining discount rates: Economic justifications and implications for long-run policy., Economic Policy 23(56), 757.795.

[21] Gollier, C. and M. Weitzman. (2010), How Should the Distant Future be Discounted when Discount Rates are Uncertain? Economics Letters, 107(3): 350-353.

[22] Groom, B., P. Koundouri, E. Panopoulou, & T. Pantelidis (2004). Discounting the Distant Future: How much does model selection affect the certainty equivalent rate? Discussion Paper 04-02, Department of Economic, University College London.

[23] Groom, B., P. Koundouri, E. Panopoulou, & T. Pantelidis (2007). Discounting the Distant Future: How much does model selection affect the certainty equivalent rate? Journal of Applied Econometrics, 22, 641–656.

[24] Hansen, B. E., (1992), Tests for Parameter Instability in Regressions with I(1) Processes, Journal of Business & Economic Statistics, American Statistical Association, vol. 10(3), pages 321-35, July.

[25] Hepburn, C. and B. Groom (2007), Gamma Discounting and Expected Net Future Value. Journal of Environmental Economics and Management, 53(1), p99-109.

[26] Hepburn, C., P. Koundouri, E. Panopoulou and T. Pantelidis (2008), Social discounting under uncertainty: A cross-country comparison, Journal of Environmental Economics and Management, 57,140–150.

[27] HMT (2003), Guidelines on Cost Benefit Analysis. HM Treasury, UK.

[28] IAWG (2010), Appendix 15a: Social Cost of Carbon for Regulatory Impact Analysis under Executive Order 12866. US Interagency Working Group on the Social Cost of Carbon of the United States of America. http://www1.eere.energy. gov/buildings/appliance_standards/commercial/pdfs/sem_finalrule_appendix15a.pdf

[29] Johansen, S. (1988), Statistical analysis of cointegrating vectors, *Journal of Economic Dynamics and Control*, 12, 231-254.

[30] Johansen, S. (1991), Estimation and hypothesis testing of cointegration vectors in Gaussian vector autoregressive models, *Econometrica*, 59, 1551-1580.

[31] Lebegue, D. (2005), Revision du taux d'actualisation des investissemnet publics. Rapport du groupe d'experts, Commisariat Generale de Plan. http://catalogue.polytechnique.fr/site.php?id=324&fileid=2389

[32] Lutkepohl, H. (1993), Introduction to Multiple Time Series Analysis, Springer-Verlag.

[33] Mishkin, F.S.(1992), Is the Fisher effect for real? A reexamination of the relationship between inflation and interest rates. *Journal of Monetary Economics*, 30, 195-215.

[34] MNOF (2012). Cost Benefit Analysis, Official Norwegian Reports NOU 2012: 16. Ministry of Finance, Norway.

[35] Mundell, R. (1963), Inflation and Real Interest, *Journal of Political Economy* (71), pp. 280-283.

[36] Newell R., W. Pizer (2001), Discounting the benefits of climate change mitigation: How much do uncertain rates increase valuations? Discussion Paper 00-45, Resources for the Future, Washington DC.

[37] Newell R., W. Pizer (2003), Discounting the benefits of climate change mitigation: How much do uncertain rates increase valuations? *Journal of Environmental Economics and Management* **46 (1)**: 52-71.

[38] Newey, W.K. and K.D. West (1987), A simple positive, semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix, *Econometrica*, 55, 703-708.

[39] Newey, W.K. and K.D. West (1994), Automatic lag selection in covariance matrix estimation, *Review of Economic Studies,* 61, 4, 631-653.

[40] Nordhaus, W. D. (2007), .A review of the Stern Review on the economics of climate change., Journal of Economic Literature 45, 686.702.

[41] Panopoulou E. and N. Pittis (2004), A comparison of autoregressive distributed lag and dynamic OLS cointegration estimators in the case of a serially correlated cointegration error, *The Econometrics Journal,* 7(2), 585-617.

[42] Park, J.Y. (1992). Canonical cointegrating regressions, *Econometrica*, 60, 119-143.

[43] Pesaran H.M. and Y. Shin (1999), An Autoregressive Distributed Lag Modelling Approach to Cointegration Analysis, *Econometrics and Economic Theory in the 20th*

[44] Phillips, P.C.B. and E.J. Hansen (1990), Statistical inference in instrumental regressions with I(1) processes, *Review of Economic Studies*, 57, 99-125.

[45] Phillips, P.C.B. and M. Loretan (1991), Estimating long-run economic equilibria, *Review of Economic Studies*, 58, 407-436.

[46] Saikkonen, P. (1991), Asymptotically efficient estimation of the cointegrating regressions, *Econometric Theory*, 7,1, 1-27.

[47] Stock, J.H. and M.W. Watson (1993), A simple estimator of cointegrating vectors in higher-order integrated systems, *Econometrica*, 61, 783-820.

[48] Thomas, L.B. (1999), Survey Measures of Expected U.S. Ináation, Journal of Economic Perspectives, 13, p.127.

[49] Traeger, C. (2012), What's the Rate? Disentangling the Weitzman and the Gollier Effect, CUDARE Working Paper 1121, University of California, Berkeley.

[50] USEPA (2010), Guidelines for Preparing Economic Analyses. United States Environmental Protection Agency, Washington DC. EPA 240-R-10-001. http://yosemite.epa.gov/ee/epa/eed.nsf/pages/Guidelines.pdf.

[51] Wickens, M.R. and T.S. Breusch (1988), Dynamic specification, the long run and theestimation of transformed regression models, Economic Journal, 98, (Conference 1988), 189-205.

[52] Weitzman, M.L. (1998), Why the far distant future should be discounted at its lowest possible rate. *Journal of Environmental Economics and Management* **36**: 201-208.

[53] Weitzman, M.L. (2001), Gamma discounting. *American Economic Review* **91**: 261-271.

[54] Weitzman, M.L. (2007), Subjective expectations and the asset-return puzzle. American Economic Review, 97, 1102–1130.

A Autocorrelation and Unit Root Tests of the Newell and Pizer Series, [36, 37]

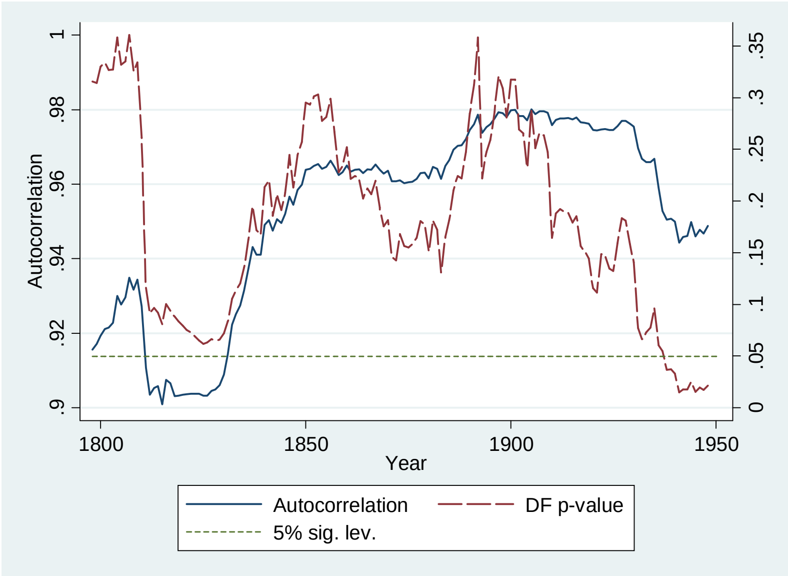

Figure A1. Rolling Estimation of Autocorrelation Coefficient (AR(3)) and Augmented Dickey-Fuller Test p-value (50 year window, unsmoothed data)

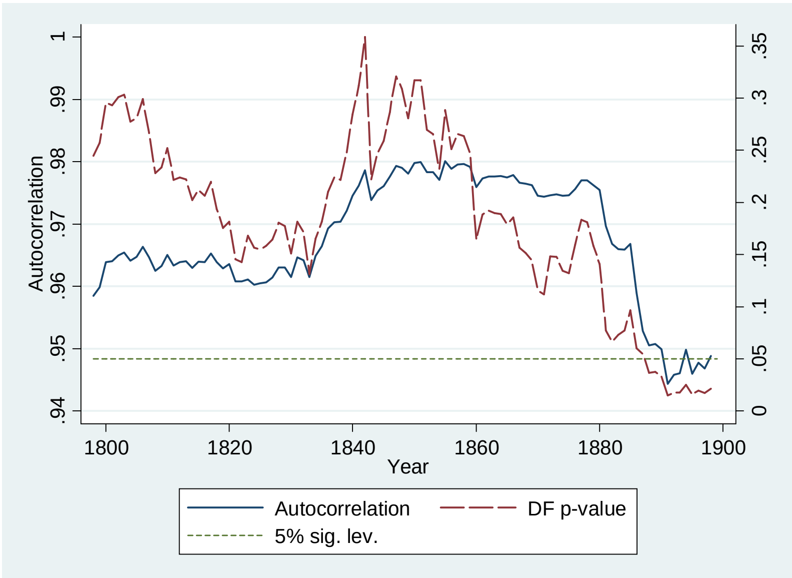

Figure A2. Rolling Estimation of Autocorrelation Coefficient (AR(3)) and Augmented Dickey-Fuller Test p-value (100 year window, unsmoothed data)

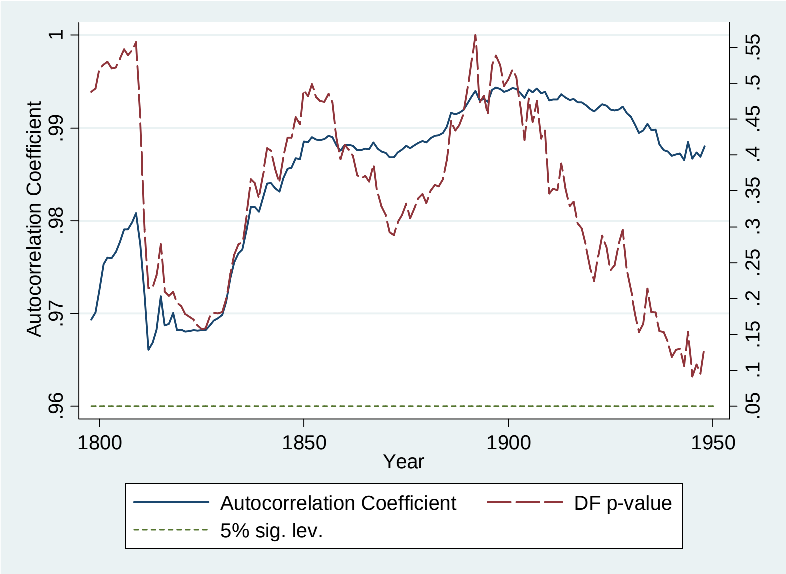

Figure A3. Rolling Estimation of Autocorrelation Coefficient (AR(3)) and Augmented Dickey-Fuller Test p-value (50 year window, smoothed data)

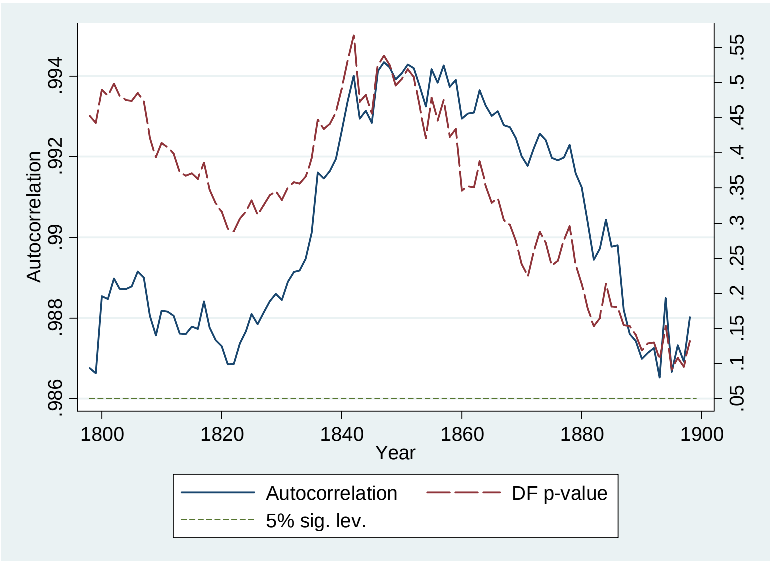

Figure A4. Rolling Estimation of Autocorrelation Coefficient (AR(3)) and Augmented Dickey-Fuller Test p-value (100 year window, smoothed data)

Table A1. Augmented Dickey Fuller Tests (AR(3))

	Smoothed (3yr M.A.)			Unsmoothed		
Test	All	Pre 1950	Post 1949	All	Pre 1950	Post 1949
ADF	-2.46	-1.51	-2.43	-3.29**	-1.68	-3.15**
ADF (trend)	-3.15*	-2.80	-2.24	-3.97**	-3.24	-3.10*

Significance levels: *** = 1%, ** = 5% and * = 10%

B Interest Rate Data 1820 - 1999

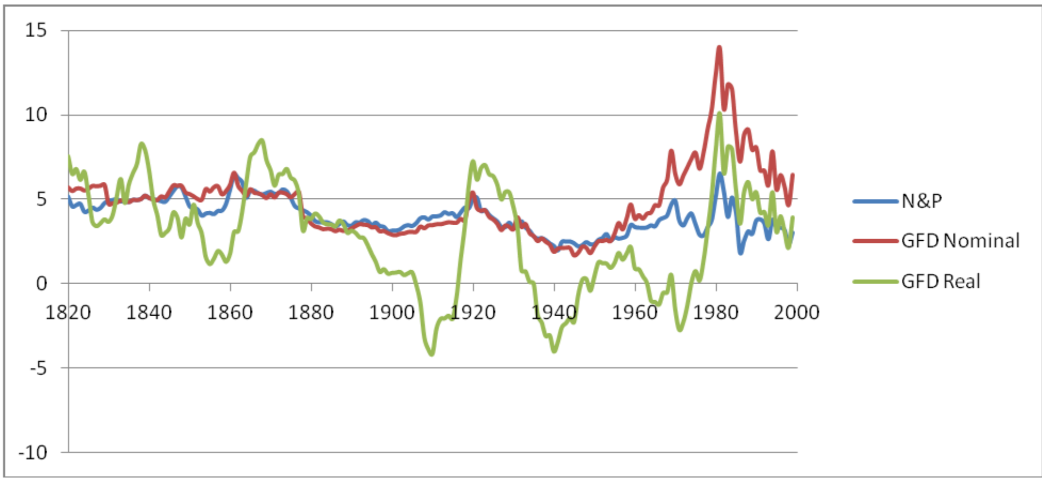

Figure B1. Real and nominal interest rates: 10 year bonds and 10 year inflation expectations

Table B1. Correlations (10 year Inflation)		
Years	N&P, GFD real	N&P, GFD nominal
1820 -1949	0.665	0.905
1950 -1999	0.355	0.677

Table B2. Auto-correlation (10 year inflation)			
Order	N&P	GFD Nominal	GFD real
1	0.905	0.946	0.947
5	0.601	0.797	0.609
10	0.574	0.616	0.138

C Simulation

The following steps are taken to simulate possible future paths of real interest rates and calculate the certainty equivalent discount rate:

1. We generate random values for $\mathbf{e}_t = [e_{1t}, e_{2t}]^\top$ from the bivariate Normal distribution $N(\mathbf{0}, \widehat{\Sigma_e})$ based on the estimated variance-covariance matrix $\widehat{\Sigma_e}$.
2. We obtain random values for the elements of A from the multivariate Normal distribution $N(vec\widehat{A}, \widehat{\Sigma_A})$ and generate random values for $\mathbf{u}_t = [u_{1t}, u_{2t}]^\top$ from equation 7.
3. We generate random values for r and θ from $N(\widehat{r}, se(\widehat{r}))$ and $N(\widehat{\theta}, se(\widehat{\theta}))$ respectively.
4. We use equations 5-6 to generate a random path for both the nominal interest rate, y_t, and the inflation rate, x_t. In this way, we calculate a future path for the real interest rate, $y_t - x_t$.
5. We check whether the estimated real interest rate fluctuates between the minimum and maximum values of the observed real interest rate of our sample for the US. If this condition is not satisfied, the simulated sample is discarded. Specifically, the min/max filter discards the entire simulated series if it exceeds 10% or is less than -4.15%, yet without direct restrictions on the underlying series of cointegrated nominal interest rates and inflation. This approach is undertaken in order to purge the simulation of explosive processes and is typical in many simulation exercises.[17]
6. Steps 1-4 are repeated as many times as needed to generate 300,000 simulated samples.
7. Finally, we calculate the *certainty-equivalent* discount factor and the *certainty equivalent* forward rate based on equations 1 and 3 respectively.

D Simulated DICE Damages

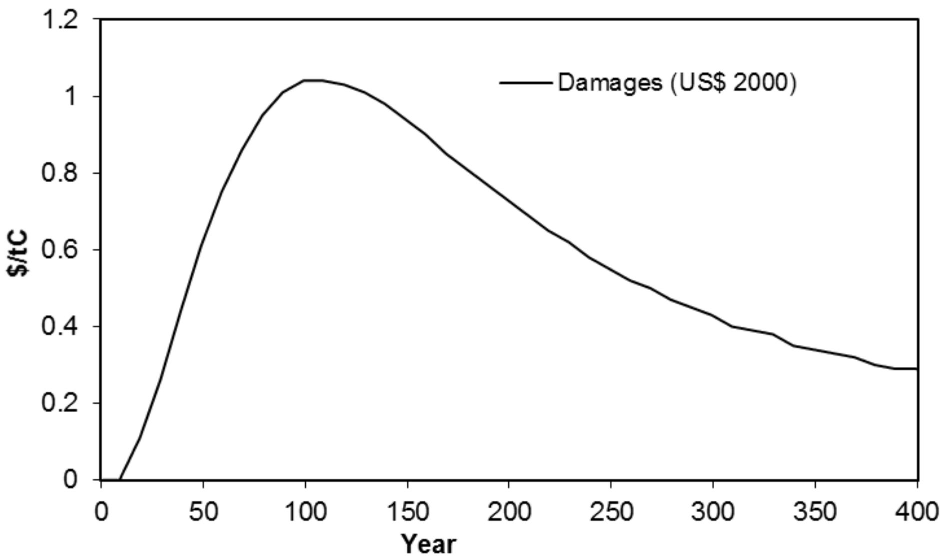

Figure D1. Marginal Carbon Damages (US$ 2000)

[17] N&P do something similar by discarding all simulated paths when the randomly drawn parameters lead to explosive processes.