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NON-IDENTICAL QUADRUPLETS: FOUR NEW ESTIMATES OF THE ELASTICITY OF MARGINAL UTILITY FOR THE UK[1]

Ben Groom[2] **and David Maddison**[3]

Abstract

This paper reviews the empirical evidence on the value of the elasticity of marginal utility for the United Kingdom. This parameter is a key determinant of the social discount rate and also informs equity weighting in applied Cost Benefit Analysis. Four different empirical methodologies are investigated: the equal-sacrifice income tax approach, the Euler-equation approach, the Frisch additive-preferences approach and the subjective-wellbeing approach. New estimates are presented using contemporaneous and historical data. Combining the estimates using meta-analytical techniques yields a best-guess estimate of 1.5 for the elasticity of marginal utility. Critically the confidence intervals for this estimate exclude unity, which is HM Treasury's current official estimate of the elasticity of marginal utility and the value used in the Stern Review (2007). The paper illustrates the implications for the UK term structure of discount rates. We use extensions to the Ramsey rule reflecting uncertainty and auto-correlated consumption growth rates. Other things equal, our estimate of 1.5 for the elasticity of marginal utility would lead to a social discount rate of 4.5 percent for the short term. From this starting point, estimates of the term structure for the UK point to a long-run rate of 3.75%. This is a much higher and flatter term structure than currently recommended in the Treasury Green Book. Using over 150 years of growth data, however, implies a term structure that starts at 3.6% and declines to 2.4% in the long-run. All these results raise conceptual and empirical questions about the current UK guidelines on social discounting.

Keywords: Elasticity of Marginal Utility, Social Time Preference Rate, Cost Benefit Analysis, Inequality Aversion, Prudence

JEL classification: D60, D61, H24, R13

1. Introduction

In this paper we are concerned with estimating the elasticity of marginal utility (η) for use in calculating the Social Discount Rate (SDR) via the 'Ramsey' rule.[4] Put simply, with inter-temporal welfare represented by the sum of time-separable discounted utility, the Ramsey rule provides the test rate of return for a public project. A project with a rate of return above that specified by the Ramsey rule will increase welfare.

The Ramsey rule has been highly influential as an organisational framework for intertemporal decision-making and appears to be at the heart of discounting policy in many countries e.g. HMT (2003), Lebegue (2005), ADB (2007) and MNOF (2012). It is typically stated as:

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[2] Department of Geography and Environment, London School of Economics. Corresponding author: b.groom@lse.ac.uk
[3] Department of Economics, University of Birmingham.
[4] We recognise that the elasticity of marginal utility can also inform equity weighting more generally in CBA, as described in the HM Treasury guidelines (HMT, 2003).

(1) $$r = \delta + \eta g$$

where r is the social rate of return to capital, δ is the pure rate of time preference and g is the average consumption growth rate and η is the elasticity of marginal utility. The RHS of the equation, $\delta + \eta g$, is referred to as the Social Rate of Time Preference (SRTP). It measures the welfare preserving rate of compensation for deferring consumption. In an optimal growth context, a social planner whose objective is to maximise inter-temporal social welfare would chose a consumption path that equates the social rate of return to capital to the SRTP. The equality in (1) would also hold in a frictionless decentralised economy in which agents' preferences can be summarised by the representative agent. In these special circumstances r and SRTP would coincide with i, the market rate of interest. In such cases the SRTP, r and i are all candidates for the Social Discount Rate (SDR) i.e. the discount rate to be used in public policy appraisal.

As a consequence of this theory we observe all three conceptions of the SDR being used in policy contexts. There are however compelling reasons to prefer the SRTP for this purpose, as is the case in the U.K. Where markets are distorted and projects are funded by deferring consumption, e.g. via income taxes, the SRTP is a more appropriate rate since it reflects the minimum rate of compensation that public projects should provide for the rescheduling of consumption that investment entails.[5] Comparison against this consumption side measure will ensure that inter-temporal welfare is not reduced by public investment. This logic is even more compelling when projects with long-term consequences are considered. Here, there are typically no financial instruments from which can draw inference on the welfare increasing inter-temporal trade-off. Even if there were, it is not clear that market rates of return necessarily contain the correct 'ethical material' for such long-term decisions (e.g. Dietz et al., 2008).

In the Ramsey context, the task of calculating the SRTP is essentially one of choosing values for the three parameters: δ, η and g. Internationally however, views differ as to the value of these parameters. The French Government for example, argues that values of 0, 2 and 2 are respectively defensible, leading to a (medium term) SDR of 4 percent (Lebegue, 2005). In the United Kingdom HM Treasury's guidelines on CBA (contained in the so-called 'Green Book') uses values of 1.5, 1 and 2 respectively for a (medium term) SDR of 3.5 percent (HMT, 2003).[6]

Clearly, it is important to be as precise as possible in determining the parameters δ, η and g since the SDR (and potentially therefore the outcome of any CBA whose costs and benefits fall in different time periods) can be extremely sensitive to the values that are chosen. This is particularly true for longer term projects such as nuclear power or the mitigation of climate change (Stern et al, 2006).[7] In fact however, disagreement on the correct value of δ has

[5] Strictly speaking the opportunity cost of raising public funds should also be recognised in CBA. We ignore this issue for the remainder of the paper.
[6] An exception was made for forestry which was entitled to use a discount rate of 3 percent.
[7] Recent literature on social discounting argues that the term structure of discount rates should be declining with the time horizon considered (e.g. Arrow et al, 2013a; Arrow et al, 2013b). Declining discount rates can also be found in the Green Book (HMT, 2003). The French and Norwegian governments also use a declining term structure for risk free projects (Lebegue, 2005; MNOF 2013), while the US government has been influenced strongly by the theoretical literature on Declining Discount Rates (IAWG, 2010; USEPA, 2010).

rumbled on since Ramsey (1928) whilst, albeit for quite different reasons, opinions also differ sharply on the correct value of η.[8]

One source of disagreement over the value of η arises because it has several quite different interpretations in the Ramsey context. It can be interpreted as a measure of intra-temporal inequality aversion, inter-temporal inequality aversion and risk aversion. Placing the same numerical value on each of these concepts can be understood as the normative position in the discounted Utilitarian framework that aversion to differences in the level of consumption is identical irrespective of whether it arises between individuals at a single point in time, across different time periods or across different states of the world. The many interpretations of η have given rise equally numerous methodologies to estimate it. Whilst on the face of it this is helpful, it also gives rise to disagreement over which method and numerical value is appropriate for insertion into the SRTP.

One chief reason for disagreement arises because the normative position is not borne out in empirical estimates. That is, it has been observed that individuals do not treat these different concepts in the same way. For instance, in their stated preference study of preferences for risk aversion, inter-temporal substitution and inequality aversion, Atkinson et al. (2009), find that responses differed significantly in each case. For this reason, they suggest that the concepts are "siblings, not triplets", meaning related, but not identical numerically.

Furthermore, those who take a positive view of social discounting point out that the Ramsey model does a poor job of describing the world as we see it, giving rise to various empirical-theoretical puzzles (e.g. Mehra and Prescott, 2003). Disentangling inter-temporal substitution and risk-aversion goes some way to solving the equity premium puzzle, for instance (Epstein and Zin, 1989). On the normative side, others argue that *intra*-temporal inequality aversion ought to be explicitly accounted for in the discount rate (Emmerling, 2012).[9] Finally, perhaps the most fundamental source of disagreement on the estimation of the elasticity of marginal utility stems from whether a positive or a normative perspective is taken on the matter of social discounting.[10]

Unsurprisingly, a wide range of estimates for η exist in the literature. Based on a variety of methodologies such as the equal absolute sacrifice approach and estimates of the elasticity of inter-temporal substitution, Stern (1977) advocates a value of around 2 with a possible range of 1 to 10. Based on a review of revealed preference literature, Pearce and Ulph (1995) suggest a value of between 0.7 to 1.5 and a best-guess estimate of 0.83 based on the panel data estimates of the elasticity inter-temporal substitution by Blundell et al (1994). Cowell and Gardiner (1999) point to a range of 0.5 to 4.0 based on similar range of methodologies. Evans and Sezer (2005) apply the equal absolute sacrifice approach to the countries of the European Union and find that values fall between 1.3 and 1.6. More recently, in response to the Stern Review, Gollier (2007) points to values of around 4 based on experimental evidence on risk aversion, while Dasgupta (2008) prefers a value of 2 on the basis of introspection on inequality aversion.

[8] The impartial consequentialist tradition famously takes the normative view that all generations should be equal treated equally with regard to their utility: $\delta = 0$. Axiomatic treatments such as Koopmans (1960) and agent relative ethics (e.g. Arrow 1999) suggest $\delta > 0$, typically without specifying a number.

[9] The Ramsey Rule is essentially at the centre of the empirical paradox known as the equity premium puzzle. There has been some success in resolving this puzzle using more flexible preferences such as the recursive Epstein-Zin-Weil preference structure, which disentangle the elasticity of intertemporal substitution from risk aversion.

[10] This lay at the heart of the debate following the Stern Review (e.g. Dasgupta, 2008; Nordhaus, 2007; Stern 2007).

The purpose of our paper is to survey and update the empirical evidence on the value of η for the United Kingdom. We identify four methodologies that have in the past been used to estimate η some of which have been influential in shaping United Kingdom policy on discounting. These are the equal-sacrifice approach, the Euler-equation approach, the wants-independent approach and the subjective wellbeing approach.[11] Our paper then empirically implements each of these methodologies in turn improving on previous research into the value of η in several important ways. First, we analyse historical data on income tax schedules stretching back to 1948 whilst addressing a number of technical issues thrown up by earlier research. Second, we estimate the Euler equation using data from 1970 to 2011 checking carefully for the existence of parameter stability and endogeneity (the bugbears of previous analyses). Third, in undertaking the 'wants-independent' approach, we test the underlying assumption of the additive preferences necessary for obtaining estimates of η from household expenditure data (a requirement that previous researchers have overlooked).

Taking estimates from the four techniques we then combine them using meta-analytical procedures to obtain a single 'best' estimate and formally test (and indeed reject) the hypothesis that η is equal to unity. Next, we calculate the value of the SRTP given our estimate of η using the Green Book's estimates of δ and g. Finally, we illustrate the consequences of adopting our preferred estimate of η when the Ramsey rule is extended to include uncertainty in the rate of growth of consumption. In this case there is a 'prudence' effect and, where consumption growth rates exhibit persistence, a discount rate which declines over time (e.g. Gollier, 2012).

Our paper focusses mainly on revealed preference and emphasises the empirical testing and precision of estimates. In so doing our meta-analysis fails to reject the hypothesis of parameter homogeneity. Hence, despite being conceptually very different, inequality aversion, inter-temporal substitution, wants independence and subjective well-being estimates can be treated as not statistically different from one another in the UK. In other words they are more like non-identical quadruplets than simply siblings.

The remainder of the paper is organised as follows. In the following section we briefly review the Ramsey rule and its recent extensions. Sections three to six discuss each of the different techniques for determining the value of η in turn. Section seven contains a meta-analysis and section eight calculates the implied SRTP using the Ramsey Rule and its variants. The final section concludes.

2. The Ramsey Rule and Extensions

In this section we derive the Ramsey rule and outline some extensions to it that allow for various forms of uncertainty in the rate of growth of consumption. The Ramsey rule is based on a time-separable discounted utility function. More specifically, the inter-temporal welfare function W is given by the discounted sum of utilities (U) over all time periods from t=0 onwards:

(2) $$W = \sum_{t=0}^{t=\infty} \beta^t U(C_t)$$

[11] Space does not allow a review the stated preference approach to estimating the elasticity of marginal utility generated by controlled experiments examining aversion to inequality or risk aversion. Neither do we estimate the pure rate of time preference or undertake a review of the extensive literature on its normative and positive arguments (e.g. Arrow, 1999; Frederickson et al., 2002).

Where C is consumption and:

(3) $$\beta^t = (1+\delta)^{-t}$$

And in the case of a constant elasticity of marginal utility:

(4) $$U(C_t) = \frac{C_t^{1-\eta} - 1}{1-\eta}$$

The SRTP in this case represents the welfare preserving rate of compensation that the representative agent would require in order to defer consumption. This represents the test rate against which marginal risk-free projects should be compared in CBA.[12]

When the growth rate of consumption is uncertain the Ramsey rule can be extended to reflect this uncertainty and, in the simplest case in which the growth rate is *i.i.d.* normal with mean μ and variance σ^2, the Ramsey Rule becomes:[13]

(5) $$SRTP = \delta + \eta\bar{g} - 0.5\eta(\eta+1)\sigma^2$$

Where $\bar{g}$ is the expected annualised consumption growth rate and σ^2 is the variance of the growth rate of consumption.[14] The final term on the RHS of this equation, $0.5\eta(\eta+1)\sigma^2$, is known as the 'prudence' effect, and reflects a precautionary motive for saving. If $\eta > 0$ this reduces the SRTP.

This extension of the Ramsey rule further illustrates, again in the iso-elastic case, the critical role played by the parameter η. When uncertainty is introduced η governs the magnitude of the wealth effect associated with the growth of consumption and the response to uncertainty in the growth of consumption reflected in the prudence effect. Gollier (2012) has analysed the value of the discount rate when the prudence effect is taken into account. For a given value of η countries with a high variation in consumption growth rates, typically developing countries, have a large downwards adjustment for prudence.

The outcome of CBA is typically sensitive to the level of the discount rate. This is particularly so for projects with long-term and intergenerational consequences. For this reason there is now much interest at the policy level in the long-run term structure of the

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[12] Consider two time periods, period 0 and period *t*, so that the SWF can be written as: $W_0 = u(c_0) + \beta^t u(c_t)$. Now suppose that project *j* is available that costs one unit of consumption today, and yields $(1+r_j)$ units at time *t*. The change in welfare associated with project *i* is approximately: $\Delta W_0 = -u'(c_0) + \beta^t u'(c_t)(1+r_i)$. The SDR is the welfare preserving rate of return, that is, the return, *r**, that sets the welfare change equal to zero. Rearranging when $\Delta W_0 = 0$ and using a first order Taylor Series expansion of the felicity function yields: $r^* = \delta - (u''(c_0)/u'(c_0))c_0 g$, where $g = (c_t - c_0)/c_0 t$ is the annualised growth in consumption between time period zero and *t*.

[13] That is: $c_t = c_0 \exp(xt)$, where $x \sim N(\mu, \sigma_c^2)$.

[14] When *x* is normal the growth rate of expected consumption is given by: $\bar{g} = E\exp(\tilde{x}) = \mu + 0.5\sigma^2$

discount rate. Many theoretical arguments have been presented for a term structure which declines with the time horizon. Most require uncertainty and persistence in either growth or the interest rate itself. For instance, if growth of consumption is subject to persistent innovations, this can provide a justification for declining discount rates. In the simple case where the rate of growth of consumption follows a mean reverting process it is straightforward to show that short-run and long-run SRTP become respectively (Gollier, 2012):[15]

(6)
$$r_t = \begin{cases} \rho + \eta\mu - 0.5\eta^2\left[\sigma_x^2 + \sigma_y^2\right] & \text{for } t = 1 \\ \rho + \eta\mu - 0.5\eta^2\left[\sigma_x^2 + \dfrac{\sigma_y^2}{(1-\phi)^2}\right] & \text{for } t = \infty \end{cases}$$

where ϕ measures the degree of persistence in growth. In this context, the value of η has additional significance since, other things equal, it determines the shape of the term structure in the intervening period.[16]

This concludes our brief summary of the theory of discounting. The influence of η clearly goes well beyond application of the standard Ramsey rule since it appears in the extensions under uncertainty. The following sections turn to the task of estimating η for the UK. To this end we explore four separate conceptions of η and use empirical methods which rely on revealed preference to generate updated estimates.

3. Estimating η from income tax schedules: The equal absolute sacrifice approach

Basic Approach and Previous Studies

In this section we use 'socially-revealed' preferences to infer the value of η. More specifically, we analyse information on the progressivity of the income tax schedule to infer the value of η under the assumption of equal sacrifice. This approach also requires that the utility function takes a known (almost invariably iso-elastic) form. The justification for the assumption of equal sacrifice may be traced back to Mill (1848) who states: "Equality of taxation, as a maxim of politics, means equality of sacrifice". In this exercise η can also be interpreted as the Government's or society's inequality aversion parameter.

Algebraically, the principle of equal (absolute) sacrifice implies that for all income levels Y the following equation must hold:

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[15] Where consumption evolves according to the following system:

$$c_{t+1} = c_t \exp(x_t)$$
$$x_t = \mu + y_t + \varepsilon_{xt}$$
$$y_t = \phi y_{t-1} + \varepsilon_{yt}$$

The error terms are assumed to be mean zero and *i.i.d.* normal (Gollier, 2012). The term structure of the SRTP is then given by:

$$SRTP = \delta + \eta\bar{g} - 0.5\eta^2\left[\sigma_x^{\ 2} + \frac{\sigma_y^{\ 2}}{(1-\phi)^2}\right] + \left[\eta y_{-1}\phi\frac{1-\phi^t}{t(1-\phi)} - 0.5\eta^2\frac{\sigma_y^{\ 2}}{(1-\phi)^2}\left(\frac{\phi^{2t}-1}{(\phi^2-1)} - 2\phi\frac{\phi^t-1}{t(\phi-1)}\right)\right]$$

where y is an observed state space variable. Equation (6) assumes $y_{-1} = 0$, hence growth is at trend.

[16] Other justifications for a declining discount rate in which η plays an important role include parameter uncertainty in the mean and variance of growth, the expectation of catastrophic states in the future (see Gollier, 2012).

(7) $$U(Y) - U(Y - T(Y)) = k$$

Where k is a constant, Y is gross income, U is utility and $T(Y)$ is the total tax liability. Assuming an isolelastic utility function: $U(Y) = (Y^{1-\eta} - 1)/(1-\eta)$, substitution yields:

(8) $$\frac{Y^{1-\eta} - 1}{1-\eta} - \frac{[(Y - T(Y))^{1-\eta} - 1]}{1-\eta} = k$$

Differentiating this expression with respect to Y and solving for η yields:

(9) $$\eta = \frac{\ln\left(1 - \frac{\partial T(Y)}{\partial Y}\right)}{\ln\left(1 - \frac{T(Y)}{Y}\right)}$$

Where $T(Y)/Y$ is the average tax rate (ATR) and $\partial T(Y)/\partial Y$ is the marginal tax rate (MTR).

Cowell and Gardiner (1999) argue that there is good reason to take seriously estimates derived from tax schedules: decisions on taxation have to be defended before an electorate and the values implicit in them ought therefore to be applicable in other areas where distributional considerations are important such as discounting or the determination of welfare weights. At the same time however, there are concerns about whether a progressive income tax structure consistent with equal sacrifice would adversely impact work incentives (Spackman, 2004). Furthermore, satisfactory tests of the equality of sacrifice assumption are impossible since they are necessarily based on a particular utility function. Furthermore, strictly speaking equal sacrifice combined with a smooth utility function would imply an MTR that continually varies. For this reason alone an income tax structure characterised by a limited number of tax thresholds cannot fit perfectly the equal sacrifice model.

Previous studies have used income tax schedules to estimate η in many different countries as well as at different income levels.[17] Our focus however, is on evidence for the United Kingdom. Stern (1977) calculates η using data for the tax year 1973-4 when there were nine different tax rates and the top rate was 75 percent. The calculations are based on the income tax liabilities of a married couple with two children. Using a regression approach Stern emerges with an estimate for η of 1.97.[18] Cowell and Gardiner (1999) present estimates of η

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[17] Evans (2005) for instance, provides evidence for 20 OECD countries. One striking observation about these estimates is that they all lie in the range 1-2, with the smallest estimate for Ireland (η = 1) and the largest being for Austria (η = 1.79).

[18] Invoking the assumption of equal sacrifice two alternative methods of obtaining estimates of η are described in the literature. These will henceforth be referred to as the 'direct' method and the 'regression' method. The direct method is simply to evaluate η for a given income (almost invariably average income). The only advantage of the direct method is its simplicity. The disadvantage is that resulting value of η may not be representative of the population of income tax payers. The regression method involves analysing tax rates from a sample of income tax payers. More specifically an estimate of η is obtained from the following regression:

(9a) $$Log(1 - MTR_i) = \eta \times Log(1 - ATR_i) + \varepsilon_i$$

for the years 1998-9 and 1999-2000. Although they provide scant detail their results clearly illustrate the effect of excluding National Insurance Contributions (NICs) without which the estimated values of η are much higher. These calculations refer to the tax liabilities of a single man of working age with no special forms of tax relief.

Evans and Sezer (2005) use OECD data combining income tax data with NICs for 2001-2 and determine that the value of η for the UK is 1.5 when evaluated at the average production wage. Using the regression technique Evans et al. (2005) estimate η using tax schedules for the year 2002-3. A variety of estimates are reported based on different weighting schemes where the weights relate to the number of taxpayers and the amount of income. The resulting estimates are then compared to those arising from un-weighted calculations. Arguing that it serves as a specification test, Evans et al. analyse whether the intercept term in the regression equation is significantly different from zero. They also include estimates in which the dependent and the independent variables exchange places (thus providing an estimate of 1/ η). According to Evans et al the preferred results are those in which the order of the regression has been reversed and the regression is weighted by the average income of the nine different income categories included in their data. Although details are once again rather limited it appears that their paper uses Weighted Least Squares rather than (arguably more appropriate) 'analytical' weights. These concerns aside their preferred estimate of η is 1.63.

Evans (2008) reworks Stern's data after first deducting the personal tax allowance. The argument is that it is only reasonable to assume declining marginal utility of income after meeting basic living expenses. This causes the mean estimate of η across the different income categories to fall from 1.97 to 1.58. Evans notes that if the estimates were also weighted according to the number of tax payers in each category this estimate would fall even further (since most individuals are basic rate tax payers and the estimates using the average and marginal tax rates relevant to them would produce an estimate for η of close to unity). He also provides estimates for the tax year 2005-6 after the deduction of the single person's tax allowance. For 2005-6 the estimate of η measured at the average production wage is 1.06.

Table 3.1. Estimates of η from equal sacrifice studies

Study	η (including NICs)	η (excluding NICs)	Basic allowance deducted?	Weighting	Tax year
Stern (1977)	NA	1.97	No	No	1973-4
Cowell and Gardiner (2000)	1.29 1.28	1.43 1.41	Not clear	Not known	1998-9 1999-0
Evans and Sezer (2005)	1.5	NA	No	Point estimate at APW	2001-2
Evans et al (2005)	1.63	NA	No	Income weighting	
Evans (2008)	NA	1.06	Yes	Point estimate at APW	2002-3

Source: See text.

where the error term ε is assumed to be normally distributed with zero mean and constant variance, and *i* refers to a point on the distribution of earnings. Note that the regression equation has no constant term. Unlike the direct method the regression method provides a confidence interval for the estimate of η.

Equal Absolute Sacrifice Approach: Regression

We now provide updated estimates of η using the equal sacrifice approach. Data taken from Her Majesty's Revenue and Customs' (HMRC) website consists of 134 observations on all earnings liable to income taxation including: earnings arising from paid employment, self employment, pensions and miscellaneous benefits. These observations are drawn from the tax years 2000-1 through to the tax year 2009-10 excluding the tax year 2008-9 for which no data are available. Each tax year includes between 13 and 17 earnings categories. Together, these span almost the entire earnings distribution from earnings only slightly in excess of the tax allowance up to earnings of £1,940,000. Also included (and critical for our purposes) is the number of individuals in each earnings category.

For mean earnings in each earnings category we calculate the ATR and the MTR using the online tax calculator http://listentotaxman.com/.[19] The tax calculator separately identifies income tax and employee NICs. As with most previous papers the data generated assumes a single individual with no dependents or special circumstances (e.g. registered blind or student loan).[20]

Despite the in our view fundamental nature of the need to weight the observations according to the number of individuals in each earnings category this appears frequently to have been overlooked in the literature (although see Stern, 1977). The 'importance' weights we will employ refer to the number of individuals in each earnings category divided by the total number of individuals in the sample.[21] Hence, although our sample contains different numbers of observations in different tax years, each tax year receives equal weight.

In what follows we demonstrate that the direct method, the regression method employing un-weighted data and the regression method employing weighted data all yield different estimates of η. But only the regression method with weighted data yields an estimate of η which is representative of the population of income tax payers.

We also address the issue of whether or not to include NICs. Evans (2005) argues against their inclusion on grounds that "An income tax-only model seems more in keeping with the underlying theory concerning equal absolute sacrifice" (p.208). By contrast, Reed and Dixon (2005) find that there is no operational difference between them arguing that NICs are "increasingly cast as a surrogate income tax" (p. 110). These views are echoed by Adam and Loutzenhiser (2007) who survey the literature concerned with combining NICs and income tax. They assert that "NICs and national insurance expenditure proceed on essentially independent paths" (p. 21). Our view is that, whilst historically NICs embodied a contributory principle, this linkage has now all but disappeared, the key exception from this being the entitlement to a full state pension.[22] Nevertheless, in what follows we examine the sensitivity of estimates of η to omitting NICs from the calculations.

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[19] The majority of researchers have preferred to use the direct method presumably because this avoids calculating income tax and national insurance contributions at different points on the distribution of earnings. But using the online tax calculator referred to above this is easily accomplished.

[20] This is in contrast to Stern (1977) who considered a family with two children and a medium-sized mortgage. Since Stern's paper the tax system has been radically restructured such that there are no longer any tax allowances for married couples (except those over the age of 75) and Mortgage Interest Relief at Source (MIRAS) was abolished in April 2000.

[21] Stern (1977) also advised that the data should be weighted according to the number of income tax payers within each income category.

[22] Individuals are required to have 30 qualifying years, these are counted as years where you are paying National Insurance contributions, caring for someone for over 20 hours a week, getting child benefit, unemployed but actively seeking work, in full time training.

Finally, we note that several papers calculate average tax rates using 'supernumerary' income (estimated as gross income minus the tax free allowance) e.g. Evans (2005), Evans (2008), Evans and Sezer (2004) and Evans and Sezer (2005). These papers are moreover critical of others who do not follow the identical-same procedure (e.g. Stern, 1977). Such a step would in our view be appropriate only if (a) utility were a function of supernumerary income and (b) the tax free allowance were equal to subsistence income γ in which case utility would be given by:

(10) $$U = \frac{(Y-\gamma)^{1-\eta}-1}{1-\eta}$$

In this case it might seem that the parameter η can be obtained from the coefficient θ in the following regression where T(Y) represents total tax:

(11) $$\ln(1-MRT_i) = \theta \times \ln\left[1-\frac{T(Y_i)}{Y_i-\gamma}\right] + v_i$$

However, if utility is a function of supernumerary income the coefficient θ can no longer be interpreted as η. Instead, η is given by:

(12) $$\eta = \theta \times \left[\frac{Y_i}{Y_i-\gamma}\right]$$

which clearly varies with supernumerary income. Weaknesses of this approach include whether it is legitimate to interpret the tax allowance as an estimate of subsistence income. In any case, for most purposes economists rightly or wrongly desire an estimate of η which is constant so we do not pursue this matter any further.[23]

Table 3.2 contains regression estimates for η with the constant term suppressed. OLS Estimates from three models are reported. Model 1 is based on the un-weighted data and Model 2 is based on the weighted data. As discussed, the weights refer to the proportion of individuals contained in each particular income category. The un-weighted and weighted regression results are very different emphasising the importance of weighting the data to ensure that it is representative of the underlying population. But irrespective of whether the regression is weighted, if the constant elasticity and equal absolute sacrifice assumptions are correct, the hypothesis that η is equal to unity can be rejected at the one percent level of confidence. For Model 2 which is the preferred model, the estimate of η is 1.515 with a standard error of 0.047.

Model 3 interacts $\ln(1-ATR_i)$ with dummy variables identifying eight of the nine different years. These interacted variables are group insignificant even at the ten percent level of significance (F[8, 125] = 0.33). This shows that estimates of η are stable over the time period under scrutiny.

[23] In our view the main problem with assuming that utility is a function of supernumerary income however is that even if just one individual in the sample has earnings equal to subsistence income then the term in square brackets becomes infinity and the weighted average of elasticities across the sample also becomes infinity. In what follows therefore we concentrate on finding the best estimate of the elasticity of marginal utility using a functional form which presupposes that the elasticity of marginal utility is constant.

Table 3.3 presents two further OLS regression results based on the constant η model but excluding NICs. Once more there is a large difference between the results for the weighted (Model 4) and un-weighted (Model 5) regressions. But in either case, η is significantly greater than unity. In the case of Model 5, the 95 percent confidence interval overlaps with the 95% confidence interval corresponding to Model 2.

Equal Absolute Sacrifice: The 'Direct' Approach
Earlier, we drew attention to an alternative, less data-intensive, method of calculating the elasticity of marginal utility. The direct method calculates the MTR and ATR at a particular point on the distribution of incomes liable to income taxation, usually that point relating to a person of average income or average production wage (APW).

Table 3.4. presents estimates of η measured at the average production wage using this method. Two different sets of estimates first include and then exclude NICs. Although the estimates are stable over time and there is moreover, very little difference between them they nonetheless differ from the estimates contained in Tables 3.2 and 3.3 which are based on the regression method.

Table 3.2. Constant η OLS regression estimates

Dependent variable $\ln(1\text{-}MTR_i)$	Model 1	Model 2	Model 3
$\ln(1\text{-}ATR_i)$	1.274	1.515	1.530
	(45.03)	(32.22)	(14.64)
$\ln(1\text{-}ATR_i) \times$ DUM01			0.066
			(0.44)
$\ln(1\text{-}ATR_i) \times$ DUM02			0.081
			(0.54)
$\ln(1\text{-}ATR_i) \times$ DUM03			0.091
			(0.61)
$\ln(1\text{-}ATR_i) \times$ DUM04			-0.076
			(-0.41)
$\ln(1\text{-}ATR_i) \times$ DUM05			-0.083
			(-0.42)
$\ln(1\text{-}ATR_i) \times$ DUM06			-0.096
			(-0.48)
$\ln(1\text{-}ATR_i) \times$ DUM07			-0.102
			(-0.50)
$\ln(1\text{-}ATR_i) \times$ DUM08			0.003
			(0.02)
Weights	NO	YES	YES
No. Obs.	134	134	134
R-Squared	0.8621	0.8867	0.8888
F Statistic	F(1, 133) = 2027.59	F(1, 133) = 1038.05	F(9, 125) = 153.20

Note that the constant term has been suppressed. T-statistics in parentheses are robust.

Table 3.3. Constant η OLS regression estimates excluding NICs

Dependent variable ln(1-MTR$_i$)	Model 4	Model 5
ln(1-ATR$_i$)	1.300 (38.87)	1.627 (34.75)
Weights	NO	YES
No. Obs.	134	134
R-Squared	0.9024	0.9053
F Statistic	F(1, 133) = 1510.81	F(1, 133) = 1207.43

Note that the constant term has been suppressed. T-statistics in parentheses are robust.

Table 3.4. Estimates of η at the Average Production Wage

Tax year	Average income liable to income tax (£)	η (including NICs)	η (excluding NICs)
2000-01	19,000	1.413	1.428
2001-02	20,100	1.428	1.433
2002-03	20,300	1.433	1.438
2003-04	20,500	1.430	1.434
2004-05	21,100	1.429	1.434
2005-06	22,400	1.413	1.417
2006-07	23,400	1.405	1.409
2007-08	24,400	1.401	1.406
2009-10	26,100	1.376	1.369

An obvious limitation of the foregoing analysis is that it utilises income tax schedules from the years 2000-01 to 2009-10. This period of time is quite short in relation to projects that might need to be analysed using CBA. It is also clear from Appendix 1 that incomes and income tax schedules have fluctuated considerably in the post-war period. Furthermore, it can be argued that a long-term view of inequality aversion is more relevant for intergenerational decision making. We now therefore provide a long-run analysis of inequality aversion in the post-war period.

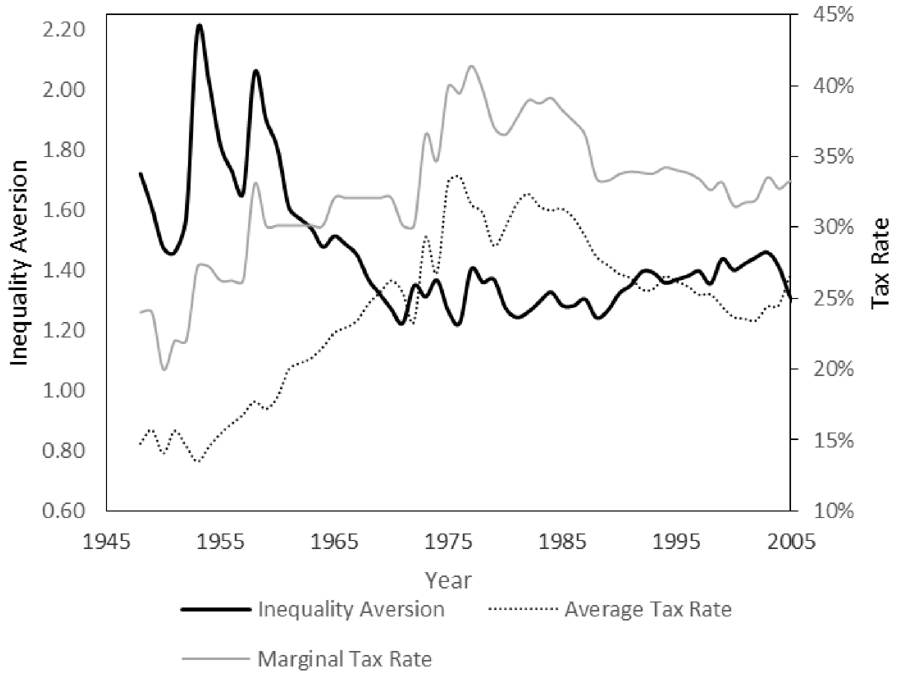
The analysis uses United Kingdom data drawn from the EuropTax database compiled by Lynch and Weingarten (2010). This historical database provides income tax schedules with options for evaluating the incidence of income tax on married, single person and other types of household. For the purposes of this analysis the results that follow consider a single person household. NICs are included in the analysis to increase comparability with the regression estimates.

Appendix 1 provides key information all United Kingdom income tax schedules since 1948. More specifically, it provides for every tax year the tax-free allowance, the starting rate of tax, the top rate of tax and the income threshold for the top rate of tax, together with the number of bands (as an indication of the overall complexity of each income tax schedule). In 1975 for example, there were no fewer than 11 different income tax bands starting with 25 percent rising to a top rate of 83 percent for those earning in excess of £24,000.

Using these data it is possible to use the direct method to calculate the implied value of η at different points on the income distribution. Figure 3.1 displays the MTRs and ATRs measured at the APW, along with the implied estimate of η using equation (9).

Figure 3.1 indicates that, over the period 1948-2007, these socially-revealed estimates of η, measured at the APW, declined significantly in the immediate aftermath of the Second World War. This was caused by a narrowing of the gap between the MTR and ATR measured at the APW, although this trend is also apparent at other income levels. The unweighted average value of η over the entire time period is 1.45, with a 95% confidence interval of 1.51 – 1.38 (standard error of 0.03). Appendix 2 shows that the direct estimates at +/- 20% the APW follow a similar trend over time. A simple t-test of these estimates under the assumption of independence, suggests that the average value at +/- 10% of the APW is not significantly different from those at the average at the 95% level. At +/- 20% of the APW the difference becomes statistically significant.[24] This casts doubt on one of the assumptions underpinning the equal absolute sacrifice approach, that estimates are independent of income levels.

Figure 3.1. MTRs and ATRs and the implied value of η in the United Kingdom 1948-2007



However, as noted by Cowell and Gardiner (1999), one of the positive aspects of this approach to estimating the elasticity of marginal utility is that it is rooted in the electoral process. The role of income taxes in elections has been the focus of analysis in public economics and political science for many years (e.g. Roberts, 1977; Johnson et al., 2005; Borck, 2007; Larcino, 2007). The theoretical models proposed in this literature suggest the presence of a degree of mean reversion in the time series of MRT and ATR, and hence η.[25] These theories motivate an analysis of the time series properties of η. Modelled as simple AR(1) process, the series reverts to a mean of 1.57 with a 95 percent confidence interval of 1.09-2.07.[26] With an autocorrelation coefficient of 0.98, the presence of persistence reduces the precision of the estimates. Nevertheless, the central estimate of 1.57 is close to the potentially more representative weighted regression estimates presented above.

[24] The t-test with unequal variance returns p-values of 0.005 and 0.0003 for +/- 20%, and 0.14 and 0.11 for +/- 10% of the APW.

[25] A simple median voter model with η as the dimension would introduce mean reversion for instance.

[26] An Augmented Dickey-Fuller test strongly rejects the presence of a unit root.

4. Life-cycle behavioural models: A macroeconometric approach

The preceding section sought to derive estimates of η by observing societal choices. Estimates of η however, can also be derived from individual households' observed saving decisions. More specifically, in the life-cycle model of household behaviour the household is viewed as allocating its consumption over different time periods in order to maximise a multi-period discounted utility function subject to an intertemporal wealth constraint. Consumption decisions are affected by the rate of interest and households' attempts to smooth consumption over time according to (a) the extent that deferred consumption is less costly than immediate consumption and (b) the curvature of the utility function. To some e.g. Pearce and Ulph (1995), this is the preferred method of estimating η since it avoids the untestable equal-sacrifice assumption and is thought to be conceptually closer to the dynamic context of social discounting than the equal sacrifice approach, which relates to intra-generational inequality aversion.

In the life-cycle model estimates of η are derived from the so-called Euler equation although in the macroeconomics literature this information is normally presented in terms of the elasticity of intertemporal substitution (EIS) which is equal to $1/\eta$. At the household level, when the EIS is high households readily reallocate consumption in response to changes in the interest rate and are less concerned about consumption smoothing. In the context of the social discounting and the Ramsey model, a high value of η will lead to a higher social discount rate and a flatter optimal growth path, other things equal. The long-run interpretation in this dynamic context is that η measures inter-generational inequality aversion. From here the positive argument is that households' observed inter-temporal decisions can inform the level of inter-temporal and inter-generational inequality aversion. The greater aversion individuals exhibit to inter-temporal consumption fluctuations, so the argument goes, the more averse society is to inter-generational inequalities and the less weight we place on future consumption – the higher the SRTP. Of course, not everyone agrees that individual behaviour reflects societal preferences and should inform government behaviour.

In order to derive the Euler equation let W represent an additively separable intertemporal welfare function, U represent utility, C represent consumption and β the utility discount factor:

(13) $$W = \sum_{t=0}^{t=\infty} \beta^t U(C_t)$$

This is maximised subject to the intertemporal wealth constraint where A is assets, r is the rate of interest and Y is labour income:

(14) $$A_{t+1} - A_t = r_t A_t + Y_t - C_t$$

Assuming a constant elasticity utility function where η is the elasticity of marginal utility, $U(C_t) = \left(C_t^{1-\eta} - 1\right)/(1-\eta)$, the first order conditions for optimality are:

(15) $$\beta^t \frac{\partial U(C_t)}{\partial C_t} = \lambda_t$$

(16) $$\lambda_t - \lambda_{t-1} = -\lambda_t r_t$$

where $\beta^t = (1+\delta)^{-t}$ and λ is the co-state variable. It can be shown that the following (Euler) equation holds:

(17) $$\ln(C_t) - \ln(C_{t-1}) = -\eta^{-1}\ln(1+\delta) + -\eta^{-1}\ln(1+r_t)$$

Using the Taylor series approximation, $r_t \approx \ln(1+r_t)$, this leads to the following empirical specification:

(18) $$\Delta\ln(C_t) = a + br_t + v_t$$

where the coefficient *b* is the EIS, *v* is an error term and the intercept yields information on the value of δ.[27]

The life-cycle model of household consumption behaviour rests on a large number of assumptions probably the most important of which is the presumed existence of perfect capital markets allowing households to borrow and lend in an unrestricted fashion. Estimates of the EIS may be impacted by periods of financial turbulence and prone to change following financial deregulation. And they may depend on the definition of consumption: e.g. whether consumption includes the purchase of durable goods. Where analysts use aggregate rather than microeconomic data this raises the question about how aggregate data accommodates changes in demographic composition and the changing 'needs' of households over the life-cycle. One might even question the convenient assumption that the intertemporal utility function is additively separable.

Besley and Meghir (1998) review the international literature whilst for the United Kingdom there appear to be 10 papers which provide estimates of the EIS (and one further empirical paper from which estimates of the EIS could not be recovered).[28] Each of these papers typically provides more than one estimate usually arising out of attempts to assess the sensitivity of results to changes in estimation technique, minor changes in the specification and different periods of time. For every paper we have tried to identify a single 'preferred' model although this is inevitably a subjective process.

In the earliest of these papers Kugler (1988) notes that the majority of (US) studies estimating the EIS utilise the Euler equation but that any attempt to analyse the (presumably stationary) variables is rendered difficult by the presence of serial correlation, measurement and aggregation errors. In Kugler's paper the representative consumer maximises an intertemporal additive utility function containing both consumption and leisure as its arguments. Making rather strong assumptions concerning the structure of preferences Kugler

[27] Hansen and Singleton have a more advanced treatment which allows the state equation (16) to contain an error term. This has the effect of adding an additional constant term which in turn has implications for the measurement of delta. We omit this treatment for the sake of brevity.

[28] These studies were identified using ECONLIT and searching for the terms "intertemporal elasticity of substitution" or "elasticity of intertemporal substitution" or "intertemporal substitution elasticity" combined with "United Kingdom" or "UK" anywhere in the text.

derives a relationship between two nonstationary variables permitting him to estimate the EIS by means of the cointegrating regression. The estimate for the United Kingdom is obtained using quarterly data from 1966 to 1985. Although Kugler finds only weak evidence for cointegration the estimates for EIS are 0.64 or 0.85 (depending on which way round the cointegrating regression is run).

Attanasio and Weber (1989) present a model which somewhat presciently allows for different estimates of the coefficient of relative risk aversion and the EIS. This model is estimated using average consumption data from a large cohort of households whose (married) heads were between 25 and 40 years of age. The data is from the Family Expenditure Survey from 1970-1984. Attanasio and Weber remark that an Euler equation estimated on aggregate data will necessarily reflect changes in the demographic composition of the population. The "most convincing" estimates point to a value of 0.514 for the EIS with a standard error of 0.183 (these results are taken from Table 2 in Attanasio and Weber's paper). But whatever specification is selected it is sobering to see that the coefficient of relative risk aversion and η are always very different.

Campbell and Manikw (1991) estimate the Euler equation using data on non-durable consumption for the United Kingdom from 1955Q1 to 1988Q2. Their model allows for excess sensitivity to both current and lagged income growth. The EIS from their preferred model (using seasonally adjusted data) has the 'wrong' sign with a value of -0.159. This estimate is taken from those contained in Table 3 in Campbell and Mankiw.

Using aggregate data on nondurables and services Patterson and Pesaran (1992) analyse the conventional Euler equation. Their model however, is extended to allow for the possibility that a fraction of households consume out of their current rather than permanent income. Taking the results of their Model 8 in Table 2 suggests that the EIS is 0.390. This is estimated using a variety of instrumental variables lagged (t-1) and (t-2). They find no evidence of a structural break in the proportion of the individuals who do not consume out of permanent income.

Using seasonally adjusted nondurable consumption data Robertson and Scott (1993) construct annual consumption growth rates from quarterly consumption data. Detecting an MA(5) process they use as instrumental variables the (t-6) and (t-7) lags of real interest rates. They present estimates of the EIS derived using six different estimation techniques but it is the IV estimates which have the lowest standard error.

Attanasio and Weber (1993) compare estimates of the EIS derived using aggregate data, average FES with controls for demographic variables and cohort data. Using data from 1970Q1 to 1986Q4 Attanasio and Weber analyse the growth in non-durable goods and services (excluding housing services). Unusually the rate of interest used is the return on Building Society deposit accounts. Using aggregate data with a term intended to capture excess sensitivity to current income growth the EIS is 0.415. Using FES data with demographic controls and a term to capture excess sensitivity this increases to 0.483. But when cohort data is used (specifically those households whose heads were born between 1930 and 1940 and who are accordingly between the ages of 40 and 56) the term for excess sensitivity becomes insignificant and the EIS increases markedly to 0.775.

In an influential paper Blundell et al (1994) argue that analyses assuming only one good 'consumption' and which ignore demographic variables are potentially misleading. Blundell et al present estimates for the EIS using microeconomic data taken from the FES 1970 to 1986. The estimate of the EIS taken from their paper is 1.11 with a standard error of 0.17. This estimate is taken from Model 6 in their Table IV. Note however, that Blundell et al find evidence that the EIS is *acutely* sensitive to the inclusion of a dummy variable identifying the period 1970-1980. Without this dummy variable their estimate of the EIS falls to 0.64 with a standard error of 0.11. Blundell et al also provide evidence that the EIS is not constant but instead varies with income.

Van Dalen (1995) estimates the EIS using remarkably, data from 1830 to 1990. Amongst other things this span of data permits the author to test whether the EIS is stable over time. Van Dalen specifies a utility function containing as its arguments both private and Government consumption. The Euler equation is estimated using instrumental variables. Given concerns about parameter stability we take the parameter estimates obtained from analysing the latest period (1948 to 1990) to obtain a value of 0.439 with a T-statistic of 4.323 (displayed in row 13 of Table 2 in Van Dalen's paper).

Attanasio and Browning (1995) further assess the empirical validity of the life-cycle hypothesis using data taken from the FES 1970 to 1986. The analysis includes only households containing married couples where the male was born between 1920 and 1949. Attention is focussed on nondurable consumption excluding housing and vehicles. Attanasio and Browning contend that the excess sensitivity to income growth noted by earlier researchers disappears when suitable controls for age and demographic factors are included. Moreover the EIS is a function of both age and demographic factors as well as the level of consumption. Unfortunately the paper does not provide information on the value of the EIS although the authors remark that this parameter is not estimated very precisely.

Like Blundell et al (op cit) Berloffa (1997) includes household characteristics as possible determinants of intertemporal allocation decisions. More specifically, she attempts to identify the manner in which household characteristics enter the Euler equation: purely additively (in which case changes in household characteristics have only a temporary effect on the consumption growth rate), or by allowing the EIS itself to be a function of household characteristics (implying that changes in household composition have a permanent impact on the consumption growth rate). Using FES data from 1970 to 1986 it appears that neither method is unambiguously superior to the other. The results from the 'constant' EIS model including only statistically significant household characteristics point to a value of 0.363 and a standard error of 0.099. These estimates are drawn from Model 3 in Table 2 of Berloffa's paper.

Yogo (2004) addresses the consequences of using weak instruments in the context of the Euler equation. He employs a number of estimation techniques which are, in terms of bias, potentially less affected by the use of weak instruments than TSLS. This also necessitates consideration of what normalisation to invoke (i.e. whether the growth rate of consumption or the interest rate should be the dependent variable). Using data from 1970Q3 to 1999Q1 Yogo finds that normalising on the growth of consumption is best and that the size of the T-test provided by TSLS is distorted. Despite this all estimators yield very similar values for the EIS namely 0.16 with a standard error of 0.13 (this estimate is taken from Table 2 using quarterly data and normalising on the consumption growth rate).

Table 4.1. Estimates of the EIS using United Kingdom data

Authors	Time period	Data	EIS	Std. Err.	Remarks
Attanasio and Weber (1989)	1970-1984	Family Expenditure Survey	0.514	0.183	Data include only heads of households who are married and between 25-40 years of age.
Campbell and Mankiw (1991)	1955Q1 to 1988Q2	Aggregate non-durable consumption	- 0.159	NA	Model allows for sensitivity to both current and lagged income growth, and uses seasonally adjusted data
Berloffa (1997)	1970 to 1986	FES	0.363	0.099	Model assumes that household attributes do not permanently impact the growth rate
Blundell et al (1994)	1970 to 1986	FES	1.11	0.17	Results are sensitive to the inclusion of income growth and a dummy variable identifying the period prior to 1981
Patterson and Pesaran (1992)	1955Q1 to 1989Q4	Aggregate non-durable consumption	0.390	NA	Includes the change in the log of household incomes and is estimated using first lags of instruments
Yogo (2004)	1970Q3 to 1999Q1		0.16	0.13	Estimates appear insensitive to estimation technique
Van Dalen (1995)	1830 to 1990	Private consumption	0.439	0.102	Estimates obtained from analysing the period 1948 to 1990
Kugler (1985)	1966Q1 to 1985Q4	Aggregate consumption	0.75	NA	Depending on the normalisation adopted estimates are 0.64 and 0.85
Robertson and Scott (1993)	1968Q1 to 1989Q1	Seasonally adjusted nondurable consumption	0.290 3	0.0734	Six alternative estimates provided but IV estimates provide the lowest standard error
Attanasio and Weber (1993)	1970Q1 to 1986Q4	FES Non-durables and services (excl. housing services)	0.483	NA	Estimate refers to average FES data including demographic variables and a term for excess sensitivity
Attanasio and Browning (1985)	1970Q1 to 1986Q4	FES Non-durables (excluding housing and vehicles)	NA	NA	The EIS varies with age, demographic variables and the level of consumption but is overall not precisely determined

Source: see text.

A simple mean over the 10 different estimates yields a value of 0.434 for the EIS with a standard deviation of 0.337 pointing to a 95 percent confidence interval of 0.193 to 0.674. In terms of η these estimates correspond to a central estimate of 2.304 with a 95 percent confidence interval of 1.483 to 5.181.

These studies differ considerably in terms of their sophistication. And although there is some indication that studies employing microeconomic data return higher estimates of the EIS (0.618 versus 0.311), the difference is not statistically significant. Employing a T-test for a difference in means whilst allowing for potentially unequal variances the null hypothesis that the means are equal cannot be rejected against the alternative hypothesis that the means are different (prob = 0.196).

Perhaps the most important feature of the literature is the finding of statistically significant parameter instability uncovered by Blundell et al, Campbell and Mankiw and Van Dalen. Only Patterson and Pesaran find no evidence of structural instability of the Euler equation.[29] Such instability is unsurprising: the period 1970-1986 witnessed oil price shocks, record levels of inflation and an experiment with monetary policy. Both the Building Societies Act and the Financial Services Act were passed by Parliament in 1986. It is hard to argue that these events will not have had some impact on intertemporal consumption allocation decisions.

Evidence of instability and the fact that the latest available study was published in 2004 casts some doubt as to whether the currently available evidence on the EIS is entirely satisfactory. We now therefore update the empirical evidence on the EIS for the United Kingdom. Data for the Euler equation approach is taken from the ONS and the Bank of England (BOE) websites. Quarterly data is available from 1975Q1 through to 2011Q1. We employ data which has not been seasonally adjusted. Domestic spending in both current prices and 2006 prices is available for durable goods, semi-durable goods, non-durable goods and services. Following convention, we omit durable goods and form a price index P_t for all other goods and services using the share-weighted geometric mean of the price series for semi-durable goods, non-durable goods and services. Henceforth we refer to this as 'consumption'. Consumption is measured in constant 2006 prices. For the real interest rate i_t we take the official Bank of England base rate minus the previously created price index. A quarterly series for population is created from mid-year population estimates using linear interpolation.[30]

Table 4.2. OLS estimates of the Euler equation

	Model 1	Model 2
Dependent Variable $\Delta Ln(C_t)$	Coefficient (T-statistic)	Coefficient (T-statistic)
CONS	0.043	0.043
	(19.77)	(18.18)
r_t	0.631	0.632
	(9.35)	(9.06)
DUM_Q1	-0.134	-0.134
	(-35.22)	(-34.54)
DUM_Q2	-0.009	-0.009
	(-3.08)	(-3.03)
DUM_Q3	-0.025	-0.025
	(-7.62)	(-7.45)
DUM_1993Q2		-0.000
		(-0.23)
DUM_1993Q2 x i_t		-0.026
		(-0.21)
R-squared	0.9484	0.9485
F-statistic	F(4, 140) = 643.70***	F(6, 138) = 423.44***
Breusch-Pagan	chi2(1) = 1.06	chi2(1) = 0.91
Durbin-Watson	2.00	2.01
RESET	F(3, 137) = 2.52*	F(3, 135) = 2.88**

*Note that ***, ** and * imply significance at the one, five and ten percent level respectively.*

[29] All studies but one use data prior to 1991 which was a period of marked financial deregulation in the United Kingdom. We have taken those estimates referring to the latest time period.

[30] More specifically we use the data series EBAQ, UTIQ, UTIS, UTIK, UTIO, UTIA, UTIQ, UTII, UTIM from the ONS and IUQABEDR from the BOE.

The Table displays an OLS regression of the per capita consumption growth rate ($\Delta Ln(C_t)$) against a constant term, r_t and three dummy variables (DUM_Q1, DUM_Q2 and DUM_Q3) to account for seasonal effects. The regression displays no evidence of heteroscedasticity or autocorrelation and the test for functional form is significant only at the 10 percent level of significance. In Model 2 we interact the real rate of interest with a dummy variable DUM which takes the value unity for observations from 1993Q2 onwards. An F-test cannot reject the hypothesis of structural stability i.e. that the dummy variable and the coefficient on the interacted term are both simultaneously zero [$F(2, 138) = 0.07$] with a p-value = 0.935. This is surprising in light of the problems encountered by earlier analyses.

Table 4.3 re-estimates the equation using instrumental variables. The instrumental variables chosen are the lagged consumption growth rate, $\Delta Ln(C_{t-1})$, the lagged real rate of interest, i_{t-1}, and the lagged inflation rate, $\Delta Ln(P_{t-1})$. The coefficient on the real rate of interest is very similar to that obtained by the OLS regression and remains significant at the one percent level of confidence. The hypothesis of under-identification is easily rejected at the one percent level of significance whereas the Sargan test of over-identification is not statistically significant even at the 10 percent level of significance. The Durbin-Wu-Hausman (DWH) test for exogeneity is likewise statistically insignificant at the 10 percent level of significance. Tests of heteroscedasticity and autocorrelated errors are statistically insignificant at the five percent level of significance whilst a test for functional form is statistically insignificant at the 10 percent level. All these tests are appropriate for IV estimation. In light of the tests for the Sargan statistic and the DWH test statistic we take the estimates from the OLS regression.

To obtain an estimate of η (the inverse of the coefficient on the real rate of interest, r_t) we use bootstrap techniques with 1,000 replications. This procedure results in a point estimate of 1.584 for the elasticity of marginal utility with a standard error of 0.205 and associated 95 percent confidence interval of 1.181-1.987.

Table 4.3. Instrumental variable estimates of the Euler equation

Dependent Variable $\Delta Ln(C_t)$	**Model 3**
CONS	0.043 (15.77)
r_t	0.593 (5.01)
DUM_Q1	-0.133 (-23.83)
DUM_Q2	-0.008 (-2.43)
DUM_Q3	-0.024 (-5.58)
R-squared (uncentred)	0.9475
F-statistic	F(4, 139) = 606.06***
Under-identification	Chi-sq(3) = 47.471***
Sargan	Chi-sq(2) = 3.691
DWH	Chi-sq(1) = 0.22561
Pagan-Hall	Chi-sq(6) = 11.833*
Cumby-Huizinga	Chi-sq(1) = 3.4726693*
Pesaran-Taylor	Chi-sq(1) = 2.11

*Note that ***, ** and * imply significance at the one, five and ten percent level respectively.*

5. Additive preferences and the Frisch formula

A further technique for estimating η relies on the presumed existence of additive preferences (elsewhere this property is referred to as 'strong separability' or 'wants independence').[31] Additivity implies that the marginal utility obtained from consuming infra-marginal units of the additively separable commodity is independent of the quantity consumed of any other commodity. Given additivity all the information necessary for estimating η can be obtained by analysing the demand for the additively separable commodity.

For goods that enter the utility function in an additive fashion it can be shown that the following relationship holds (Frisch, 1959):

$$\eta = \frac{\kappa_i(1 - w_i\kappa_i)}{\varepsilon_{ii}} \tag{19}$$

Where η is as before, w is the budget share, κ is the income elasticity of demand and ε is the own compensated elasticity of demand for good *i*. A derivation of this formula (along with an alternative formula) is provided in the Appendix.

Evans (2008) provides a review of estimates of η obtained using this technique.[32] In fact there are two types of study: primary studies analyse food and non-food commodity expenditures using a system of demand equations, from which it is possible to retrieve the income and compensated price elasticities, with the specific intention of estimating η.[33] Obviously in order to invoke the Frisch formula it is has to be assumed that the food and non-food commodities are additively separable. Secondary studies simply obtain estimates of the relevant elasticities from existing studies of consumer demand undertaken for purposes other than estimating η (e.g. Blundell et al, 1993 and Banks et al, 1997). And although such studies invariably identify several commodities, food is once more presumed to be additively separable (even if the validity of this assumption is often not explicitly tested).[34]

So far as the United Kingdom is concerned there seem to be three studies of the demand for food undertaken primarily for the purposes of obtaining estimates to use in the Frisch formula.

Evans and Sezer (2002) estimate the demand for food using a constant elasticity model (CEM).[35] Using annual aggregate data from 1967 to 1997 Evans and Sezer estimate an error correction model using parameters from the long run cointegrating regression. Their estimates of the income and compensated own price elasticities of demand for food imply a value of 1.64 for η.

Using annual aggregate data from 1965 to 2001 Evans (2004) estimates demand equations for food using the CEM, AIDS and QUAIDS models. Although the restrictions associated with homogeneity are not accepted in any of these models the differences in the income and

[31] In fact the preceding section also relies on the assumption of an additively separable utility function.
[32] In the survey presented by Evans the only available evidence is in the form of point estimates.
[33] In fact many studies find that taking a more ad hoc approach i.e. estimating a demand equation for food which cannot be derived from any system of preferences provides superior results.
[34] The Blundell et al (op cit) and Banks et al (op cit) studies employ microeconomic data whereas all those studies undertaken specifically with a view to estimating the elasticity of marginal utility utilise aggregate data.
[35] Although convenient the CEM cannot be derived from an underlying system of preferences.

compensated price elasticities resulting from imposing this constraint are negligible. But further tests reveal that only for the CEM is the hypothesis of a cointegrating relationship acceptable leading Evans to prefer the estimate of 1.60 provided by that model. For the QUAIDS model the implied estimates of η are deemed to be implausible.

Evans et al (2005) present estimates of η based on the demand for food once more using the CEM, AIDS and QUAIDS models. Compared to Evans (2004) the data period is now extended from 1963 to 2002. Once again only the CEM yields a cointegrating relationship but now the homogeneity restriction is acceptable. The parameter estimates from the CEM once more point to an estimate of 1.60 for η using the Frisch formula.

Table 5.1 contains details of these three studies along with references to studies undertaken by other authors whose findings with regards to the demand for food have been used to construct secondary estimates of the elasticity of marginal utility.

Table 5.1. Estimates of η based on additive preferences

Study	Model	Period	η
Blundell (1988)	AIDS	1970-1984	1.97
Blundell et al (1993) Aggregate model (GMM)	QUAIDS	1970-1984	1.06
Micro model (OLS)	QUAIDS	1970-1984	1.06
Micro model (GMM)	QUAIDS	1970-1984	1.37
Banks et al.(1997)	QUAIDS	1970-1986	1.07
Evans and Sezer (2002)	CEM	1970-1997	1.64
Evans (2004)	CEM	1965-2001	1.6
Evans et al (2005)	CEM	1963-2002	1.6

Source: Adapted from Evans (2008).

The obvious limitation of all these estimates is the almost complete absence of any evidence suggesting that food is additively separable from all other commodities.[36] But if food is not additively separable from other commodities then the Frisch formula does not apply. Furthermore, it is important to avoid attaching too much significance at this stage to the fact that the estimates appear to take 'plausible' values: as Deaton and Muellbauer (1980) note, even with the failure of additivity, it is not even very surprising that so-called 'estimates' of η should fall into the range 1 to 3. This is because the Frisch parameter will always be estimated as approximately equal to the average ratio of uncompensated own price elasticities and income elasticities. And with the typical level of aggregation adopted in consumer demand studies, an estimate of 1 to 3 is entirely plausible. Furthermore there is no indication as to the precision of these elasticity estimates.

[36] The only evidence cited in favour of the hypothesis of additivity is Selvanathan (1988).

The approach taken here is to analyse the demand for food using first the Rotterdam demand system and then the CEM whilst explicitly testing the validity of the restrictions imposed by the assumption additivity.

The Rotterdam demand system has long served as a vehicle for testing theoretical postulates of consumer demand theory. Furthermore the Rotterdam system has been found comparable to the more modern AIDS system in terms of its ability to estimate the value of key parameters. And of particular interest to us is the fact that the Rotterdam system may be used to test and impose the restrictions associated with additivity in a relatively straightforward manner.[37] And once they have been imposed an estimate of η is directly available along with its associated standard error.

The Rotterdam system is defined by the equation:

(20) $$w_i d\ln(Q_i) = a_i + b_i d\ln(R) + \Sigma_j c_{ij} d\ln(P_{ij})$$

where:

(21) $$d\ln(R) = d\ln(Y) - \Sigma_i w_i d\ln(P_i)$$

And w is the commodity share, Q is quantity and P is price. The variable R can be interpreted as real income. Note the existence of an intercept allowing for autonomous changes in the demand for food. This system is then implemented using time series data and the following approximations:

(22) $$w_i = 0.5(w_{it} + w_{it-1})$$

(23) $$d\ln(Q_i) = \ln(Q_{it}) - \ln(Q_{it-1})$$

(24) $$d\ln(P_i) = \ln(P_{it}) - \ln(P_{it-1})$$

(25) $$d\ln(Y) = \ln(Y_t) - \ln(Y_{t-1})$$

The system of demand equations can be used to test homogeneity, symmetry, convexity and additivity, the last of which involves imposing the following constraints on the substitution matrix:

(26) $$c_{ii} = \phi b_i(1 - b_i)$$

and:

(27) $$c_{ij} = -\phi b_i b_j$$

Where ϕ is the so-called Frisch parameter (equal to $1/\eta$).

[37] The Rotterdam model has in the past frequently been used to test for the assumption of additivity e.g. Barten (1969) and Deaton (1974). And although the evidence obviously depends on the commodity classification and the time period under the assumption of additivity is not generally found to be acceptable.

Consistent with previous research we analyse United Kingdom food and non-food commodity expenditures. Annual data are available from the ONS from 1964 to 2010. All variables are taken in per capita terms and prices are indexed such that the year 2006 = 100. We calculate the price of the non-food commodity by assuming that the logarithm of the implied price index for all household expenditure P is equal to the share weighted sum of the logarithm of P_F and the logarithm of P_N where these represent the price of the food and the non-food commodities respectively.[38]

Table 5.2. Non-Linear Least Squares Estimates of the Rotterdam System

Dependent Variable wdlnQ$_F$	**Model 1**	**Model 2**
Variable	Coefficient (T-statistic)	Coefficient (T-statistic)
Constant	-0.000346	-0.0000379
	(-0.77)	(-1.30)
dlnR$_t$	0.0578	0.0581
	(5.53)	(5.93)
dlnP$_{Ft}$	-0.0156	
	(-1.64)	
dlnP$_{Nt}$	0.0151	
	(1.59)	
η		3.57
		(1.63)
R-squared	0.473	0.473
No. Obs.	46	46
Log Likelihood	239.32	239.31

The results from the econometric analysis are displayed in Table ~~4~~5.2. The results of the unrestricted model (Model 1) do not impose additivity (or homogeneity). The estimate of the income elasticity of demand for food is 0.393 with an average budget share for food of 0.147. The uncompensated own price elasticity is -0.164 whilst the compensated own price elasticity of demand is -0.106. Turning to Model 2, the restrictions associated with additivity are imposed and, surprisingly perhaps, these are accepted even at the 10 percent level of significance as shown by Likelihood Ratio test ($\chi^2[1] = 0.0105$). The estimate for η of 3.57 is however, imprecise having a standard error of 2.188. Note that if the estimate of η were based on the Frisch formula using estimates of the income and own compensated elasticities for food taken from Model 1 the results would have been only a little different (3.493).

Given that the recent researchers have chosen not to utilise the Rotterdam system and have instead chosen to base their estimates on models which do not impose additivity it is, for the purposes of comparison, desirable to provide an alternative set of estimates (even if it is hard to impose the restrictions associated with additivity) and we now analyse the demand for food using the CEM.[39]

[38] More specifically the analysis uses the data series ADIP, ABZV, ABQJ, ABQI and EBAQ.
[39] We note in passing that although they may appear different the Rotterdam system and the CEM are actually both based on the log linear model of Leser (1954). But whereas the CEM model is estimated in levels (and allows for dynamic adjustment) the Rotterdam system is estimated using growth rates (and is parameterised differently, specifically by multiplying through by budget shares). It would not therefore be surprising that these two approaches emerge with different estimates of key parameters (which is indeed what occurs).

The CEM demand equation is given by:

$$(29) \qquad Log(Q_{Ft}) = a_F + b_F YEAR_t + \kappa_F Log(Y_t) + e_{FF} Log(P_{Ft}) + e_{FN} Log(P_{Nt}) + v_{Ft}$$

Where Q is the quantity of food, Y is nominal income and P_F and P_N are price indices for the food and the non-food commodity respectively, κ_F is the income elasticity of demand for food and e_{FF} and e_{FN} are respectively the uncompensated own and cross price elasticities of demand.

Using the Slutsky decomposition this equation may be rewritten as:

$$(30) \qquad Log(Q_{Ft}) = a_F + b_F YEAR_t + \kappa_F Log(R_t) + \varepsilon_{FF} Log(P_{Ft}) + \varepsilon_{FN} Log(P_{Nt}) + v_{Ft}$$

In which ε_{FF} and ε_{FN} are now the compensated elasticities of demand and R is once more real income. The equation includes a linear time trend to capture any autonomous developments in the demand for food.

All variables are tested for unit roots using the Dickey-Fuller GLS test assuming a linear trend and choosing the lag length according to the Schwarz Criterion. The results shown in Table 4.3 indicate that the Logarithms of R, P_F and P_N are I(2). The Logarithm of Q_F by contrast is I(1). The demand equation is then tested for cointegration using the Johansen test procedure once more allowing for a linear time trend. The results suggest that the hypothesis of no cointegrating vector can be rejected at the 5 percent level of confidence. The null hypothesis of at most one cointegrating vector cannot however be rejected against the alternative.[40]

Table 5.3. DFGLS tests for stationarity

	Level Value is I(1)	First Diff. is I(1)	Second Diff. is I(1)
Log(Q_F)	-1.272	-4.542***	-7.482***
Log(R)	-2.897	-3.323*	-5.517***
Log(P_F)	-1.425	-2.391	-5.244***
Log(P_N)	-1.669	-2.789	-6.876***

Table 5.4. The Johansen test of cointegration for the food equation

Maximum Rank	Trace Statistic	5 Percent Critical Value
0	59.531	47.21
1	28.111	29.68
2	11.441	15.41

Finally the residuals u_t from the long run cointegrating regression were used in the Error Correction Model (ECM):

$$(31) \qquad \Delta Log(Q_{Ft}) = b_F + \kappa_F \Delta Log(R_t) + \varepsilon_{FF} \Delta Log(P_{Ft}) + \varepsilon_{FN} \Delta Log(P_{Nt}) + c v_{Ft-1} + \psi_t$$

Where ψ is the error term of the ECM.

The resulting elasticities are similar to those already encountered in the literature (see Table 45.6) and, along with the sample average budget share for food of 0.147, these can be used to

[40] According to the Johansen test procedure there is also a long run cointegrating relationship between ΔLog(R), ΔLog(P_F) and ΔLog(P_N).

calculate η using the Frisch formula.[41] Before doing so however, we must first address the question of whether the assumption of additivity is valid in the case of the CEM. The implied parameter restrictions are (see the Appendix):

$$\frac{\varepsilon_{FF}}{\kappa_F - \kappa_F^2 w_F} = \frac{\varepsilon_{FN}}{-\kappa_F \kappa_N w_N} \tag{32}$$

Note that the estimate of the income elasticity for the non-food commodity is obtained by the adding up restriction. The resulting F-test once more suggests that the assumption of additivity is not invalid (F[1, 41] = 2.55). Because these restrictions are awkward to impose we simply apply the Frisch formula to the estimates of the income and compensated price elasticities displayed in Table 45.6. Given that we have not imposed additivity we obtain two obviously not dissimilar estimates of η: 1.110 (with a standard error of 0.461) and 1.553 (with a standard error of 0.706).

Table 5.5. The cointegrating OLS regression for the food equation

Dep. Var. Log(Q_F)	Coefficient
CONSTANT	-7.837
YEAR$_t$	0.006
ln(R_t)	0.229
ln(P_{Ft})	-0.174
ln(P_{Nt})	0.091

Table 5.6. The OLS ECM for the food equation

Dep. Var. Δln(Q_{Ft})	Coefficient (T-statistic)
CONSTANT	0.003
	(1.18)
Δln(R_t)	0.274
	(3.74)
Δln(P_{Ft})	-0.237
	(-3.70)
Δln(P_{Nt})	0.169
	(2.75)
v_{Ft-1}	-0.391
	(-3.04)
R-squared	0.594
F-statistic	F(4, 41) = 15.05
Durbin-Watson	d-statistic (5, 46) = 1.88
Breusch-Pagan	Chi-sq(1) = 0.39
RESET	F(3, 38) = 0.62

[41] Note that lagged values of the dependent or independent variables were statistically insignificant.

6. Subjective well-being

Layard et al (2008) present a method of estimating η based on surveys of subjective wellbeing. In such surveys individual respondents are invited to respond to questions such as:

All things considered how satisfied (or happy) are you on a 1-10 scale where 10 represents the maximum possible of satisfaction and 1 the lowest level of satisfaction?

Life satisfaction is taken as being synonymous with utility and it is assumed that survey respondents are able accurately to map their utility onto an integer scale:

(33) $$S_i = g_i(U_i)$$

Where S_i is the reported satisfaction of individual *i* and g_i describes a monotonic function used by individual *i* to convert utility U_i to reported S. It is further necessary to assume all survey respondents use a common function *g* to convert utility to reported S: $g_i = g \forall i$
The functional relationship *g* between S and *U* determines the appropriate estimation technique. The least restrictive approach is to assume only an ordinal association between reported life satisfaction and utility. So if an individual reports an 8 one ought merely to assume that they are more satisfied than if they had reported a 7. This entails use of the ordered logit model. Analysing such data using ordinary least squares by contrast assumes a linear association between the utility of each respondent and their reported life satisfaction.

Layard et al (2008) analyse six separate surveys variously containing questions on happiness and life satisfaction. The estimates for η remain surprisingly consistent across datasets and are robust to different estimation techniques. Finally, Layard et al (2008) test the stability of η by splitting observations into various population subgroups according to age, gender, educational attainment and marital status and estimates of η are found to remain constant across these subgroups. Layard et al (2008) acknowledge however that income reported in household surveys may contain measurement error (especially when respondents are required only to identify a range within which their income falls rather than the exact value). The estimate of η for the British Household Panel survey is 1.32 with a confidence interval of 0.99-1.65 implying a standard error of 0.168 (these estimates are taken from the slightly less restrictive ordered logit model).

7. Meta-Analysis of η

The preceding sections investigated four alternative methodologies for estimating η. We reviewed the evidence and in the case of two methodologies (the Euler equation approach and the Equal Sacrifice approach) generated more up to date estimates along with their associated standard errors. Our analysis of the demand for food tested the assumption of additivity and resulted in several estimates of η. The final methodology utilised data on Subjective Wellbeing. Here we merely identified a single estimate for Britain provided by Layard et al. These four estimates and their associated standard errors are summarised in Table 7.1.

Table 7.1. Meta-analysis of estimates of η

Methodology	η	Standard error
Equal sacrifice (Regression)	1.515	0.047
Equal sacrifice (Direct)	1.573	0.481
Euler equation	1.584	0.205
Additive preferences (Estimate #1)	1.110	0.461
Additive preferences (Estimate #2)	1.553	0.706
Subjective wellbeing	1.320	0.168
Pooled estimate	1.501	
Parameter homogeneity	Chi-sq(4) = 2.14 (p=0.710)	

The pooled random effects estimator for η is 1.501 with a 95 percent confidence interval of 1.415 to 1.587. This clearly excludes the current Green Book estimate of unity. Furthermore the hypothesis of parameter homogeneity cannot be rejected. This latter finding is of interest because at least two of our estimates arise out of very different situations where others e.g. Atkinson et al (op cit) have suggested that the value of η might differ: the intertemporal allocation of consumption and societal inequality aversion. One important caveat is required though. The precision of the estimate from the meta-analysis embodies only sampling error and ignores modelling error.

8. Implications for the social discount rate

The UK Social Rate of Time Preference

In this section we demonstrate the implications of our new estimate of η for the SRTP for the United Kingdom using the simple and the extended Ramsey rules. The Ramsey rule requires inputting an assumption about expected growth rates and the extended Ramsey rules require evidence on the historical variance and autocorrelation of growth rates. The evidence for these things comes from historical observations on per capita consumption in 2005 prices stretching from 1830-2007.[42] More specifically, we estimate the following equation:

(34) $$\ln(C_{t+1}) - \ln(C_t) = \mu + y_t$$

Where C is consumption and

(35) $$y_t = \phi y_{t-1} + \varepsilon_{yt}$$

and ε_{yt} is assumed to be distributed N(0, σ_ε^2).[43]

[42] The series is constructed from historical data on per capita consumption and inflation available from the Bank of England: www.bankofengland.co.uk/publications/.../threecenturiesofdata.xls. Last accessed 16/08/13.

[43] Note that this description of the growth of consumption is somewhat simpler than the one given in Gollier (2012) in which:

(34b) $$\ln(C_{t+1}) - \ln(C_t) = \mu + \phi Y_{t-1} + \varepsilon_{xt} + \varepsilon_{y_t}$$

Several decisions have to be made when estimating this system. Perhaps the most important is the choice of time series. In the context of long-term social discounting, Newell and Pizer (2003) analyse a 200 year period of interest rates in the US (1798-1999). They smooth the data using a 3 year moving average arguing that their interest is in the long-term phenomena and smoothing removes irrelevant short-term fluctuations. Using the same data, Freeman et al. (2013) show that this procedure can affect the empirical term structure by introducing greater persistence. They prefer to model interest rate uncertainty using the unsmoothed time series.

This raises the question of which procedure is appropriate for the analysis of growth. On the one hand one should be careful not to introduce arbitrary data transformations to the analysis, yet on the other, the term structure of discount rates is arguably a long-term issue which should not be obscured by short-term fluctuations. Rather than resolve these issues here, we opt to present several results to illustrate the sensitivity to these decisions.

We estimate the model above using a state space approach on three transformations of consumption growth data. Model 1 uses the difference in the logarithm of real consumption growth. Models 2 and 3 use data which has been purged of short-term fluctuations. Model 2 uses a simple 5-year moving average of growth (MA5). Since the choice of window for the moving average is somewhat arbitrary, in Model 3 we use the Hodrick-Prescott filter (HP) to separate the trend from the cyclical component of growth, and use this trend as the basis for the analysis of the growth model. This has the benefit of using a well-known structure, hence providing a less arbitrary means of smoothing the data.[44] Figure A4 in Appendix 4 depicts the resulting time series of growth in each case.

Concern about long-run issues also raises questions about the appropriate period of growth data that should be used. Once again we provide sensitivity analysis by distinguishing between the Green Book assumptions for growth, and a longer run approach. The Green Book assumes that average growth is 2% based on analysis of the period 1950-1998, yet consumption data are available for the 180 year period 1830-2009. It is a moot point as to which period is more appropriate in the context of social discounting and long-run analysis. Space does not allow a deep discussion of these issues, but Tables 8.1 and 8.2 show the implications of these decisions for the estimated parameters of the AR(1) process, while Table 8.3 shows the implications for the social discount rate.

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and where ε_{xt} is assumed to be distributed N(0, $\sigma^2_{x\varepsilon}$). Data limitations prevent separate estimates of $\sigma^2_{x\varepsilon}$ and $\sigma^2_{y\varepsilon}$.

[44] For the HP filter we use a smoothing parameter of 6.25, which is typical for annual data. We use the "one-sided" HP filter to avoid issues associated with estimation using a Kalman Filter.

Table 8.1. AR(1) model of per capita consumption growth 1950-1998 (Green Book)

	Model 1 (Unsmoothed)	**Model 2 (MA5)**	**Model 3 (HP-Filter)**
Dependent Variable $\ln(C_{t+1})$-$\ln(C_t)$	Coefficient (T-statistic)	Coefficient (T-statistic)	Coefficient (T-statistic)
μ	0.0234 (4.24)	0.0233 (2.55)	0.0222 (2.45)
ϕ	0.431 (3.26)	0.912 (14.19)	0.938 (18.73)
σ_ε^2	0.000488 (4.90)	0.0000452 (4.94)	0.000025 (4.88)
Log L	114.78	174.66	185.25
Wald Test	10.65	201.48	350.93
p-value	0.001	0.00	0.00
No. Obs.	49	49	48

Table 8.2. AR(1) model of per capita consumption growth1830-2009 (Long-run)

	Model 1 (Unsmoothed)	**Model 2 (MA5)**	**Model 3 (HP-Filter)**
Dependent Variable $\ln(C_{t+1})$-$\ln(C_t)$	Coefficient (T-statistic)	Coefficient (T-statistic)	Coefficient (T-statistic)
μ	0.0117 (3.26)	0.0111 (2.25)	0.0113 (1.89)
ϕ	0.237 (3.13)	0.849 (21.09)	0.921 (33.07)
σ_ε^2	0.00135 (9.46)	0.000103 (9.49)	0.0000451 (9.46)
Log L	337.18	569.53	640.72
Wald Test	9.78	445.00	1093.48
p-value	0.002	0.00	0.00
No. Obs.	179	179	178

In Table 8.3, the panel entitled "Green Book", provides estimates of the social discount rate under the parameter assumptions used by the Treasury, except using the elasticity of marginal utility is 1.5. The parameters for prudence and the autocorrelation process are estimated over the period 1948-1998, the period used by the Green Book to estimate average growth. The panel entitled "Long-Run" uses estimates for growth and the autocorrelation using data from the period 1830-2009. In each case estimates of $\sigma_{y\varepsilon}^2$ and ϕ are used from each model to obtain the SDR under certainty, with prudence, and in the long-run with persistence according to equations (5) and (6).

The Green Book panel of Table 8.3 shows that, if one uses the value of η derived from the meta-analysis and combines it with the current assumption concerning the pure rate of time preference and empirically based estimates of the growth of per capita consumption, one obtains a central value for the SDR more than 1 percent higher than that recommended in the Treasury Green Book. As elsewhere (e.g. Gollier, 2012), the prudence effect is very small due to the relatively low variance of per capita growth, and this decreases the smoother the time series deployed. The main surprise however, is that the long run estimate of the SDR differs by at most 0.75 percentage points and with the unsmoothed data, for which there is no persistence to speak of, 0.17 percentage points. This is much smaller than the decline of 2.5 percentage points in the forward rate that is currently recommended over the long-term in the Treasury Green Book.

Table 8.3. The Social Discount Rate for the United Kingdom (Forward Rates)

Data	SDR (certainty)	SDR (with prudence)	SDR (long run)
Growth = 2% measured between 1950-1998 (Green Book)[a]			
Unsmoothed	**4.5**	4.45	4.33
Smoothed (MA5)	**4.5**	4.49	3.84
Smoothed (Hodrick-Prescott)	**4.5**	4.5	3.77
Growth = 1.1% measured between 1830-2009 (Long-run)[b]			
Unsmoothed	**3.26**	3.10	2.99
Smoothed (MA5)	**3.17**	3.15	2.66
Smoothed (Hodrick-Prescott)	**3.20**	3.19	2.38

Notes: [a] The 'Green Book' panel assumes that g = 2%, ρ = 1.5%, η = 1.5, and estimate the AR(1) model using data from 1950-1998; [b] The 'Long-run' panel assumes, ρ = 1.5%, η = 1.5 and uses the estimate of g associated with the estimates shown in Table 8.2, which were approximately 1.1%. In each case estimates of $\sigma^2_{y\varepsilon}$ and ϕ are taken from the estimates in Tables 8.1 and 8.2.

When long-run growth is analysed, average growth over the period 1830-2009 is about 1.1%. This reduces the SRTP commensurately. The persistence of growth in the long-run is also more pronounced. Where the HP-filter data is used this leads to a term structure of the SRTP which begins at 3.20% and declines to 2.38% at the limit.

The results should be treated as an example of the implications for discounting policy of some of the simpler theories of the term structure of social discount rates. There are clearly further empirical issues that could be dealt with in more depth at some future stage, such as model selection, or the appropriate transformation of the data. These issues have been dealt with in relation to the interest rate (Freeman et al., 2013; Groom et al., 2007). Perhaps a more pertinent question is which estimate of growth is relevant? The long run of 1.1% or the medium term growth rate of around 2% currently used by the Treasury? Indeed, with the estimated level of persistence, it is the level effects of growth and η that are the main factors in determining the term structure of the SRTP.[45]

[45] With the estimated level of persistence the impact on the decline of the term structure arising from a movement from $\eta = 1$ to $\eta = 1.5$ is minimal. Larger values of η, e.g. $\eta = 4$, raise the short-term discount rate, but increase the decline of the term structure and the lower the limiting value. See Appendix 5 for an illustration.

9. Conclusions

In terms of the nomenclature of social discounting since Arrow et al. (1996), this paper has taken a *positive* approach to the estimation of the elasticity of marginal utility, rather than a *normative* approach. Four separate theoretical and empirical methods have been deployed, each of which uses revealed preference data.

The title of our paper is a deliberately provocative reference to Atkinson et al. (2009), whose stated preference estimation of risk aversion, inter-temporal substitution and inequality aversion earned the description of “siblings” rather than “triplets”, meaning related via the Ramsey Rule, but not numerically identical. Our paper, on the other hand, suggests that with regard to revealed preference data, the conceptual and empirical approach taken in the UK is not materially important. Despite being conceptually different, estimates of the four concepts of inter-temporal substitution, inequality aversion, intra-temporal substitution and elasticity of marginal subjective well-being, are not statistically different from one another in the UK. Statistically speaking, the description “non-identical quadruplets” seems more appropriate.

The meta-analysis suggests that a value of 1.5% is defensible for the UK, with confidence intervals which exclude 1%, the value currently favoured in the UK Treasury Green Book and preferred by the Stern Review (HMT 2003; Stern 2007). With no other changes to the parameters of the SRTP this would recommend a value for the UK of 4.5%, a 1% increase from the current value of 3.5%. Given this, analysis of long-run UK consumption growth further shows that simple persistence in growth justifies a term structure which declines to no less than 3.75% in the long-run, compared to the currently recommended 1%. Estimates of long-run growth (1830-2009) lead to much lower annual growth rates of around 1.1%. The associated term structure starts at 3.2% and falls to no less than 2.4%. This raises the question of which growth horizon is appropriate to which policy analysis.

More generally, questions remain about the validity of the theoretical approaches underpinning these empirical estimates, and the suitability of the theory underpinning them to the policy question at hand. For instance, for the purpose of long-run discounting, objections have been raised about the use of market based estimates, such as the estimates of the elasticity inter-temporal substitution presented here. Dietz et al. (2008) argue that such revealed preferences are unduly influenced by the rich due to their plutocratic “one dollar one vote” characteristics. In the context of the Ramsey Rule, such criticisms may lead one to favour the equal absolute sacrifice estimates of inequality aversion. These are more democratic in the sense that tax schedules arise from a democratic process. But even here one must hope that inequality aversion was in mind when each vote was cast for this method to be superior in this regard. Even if we accept this premise, whether plutocratic or democratic, future generations’ preferences are ignored by revealed preference approaches, and may well receive little attention from decision makers in the current market.

Ultimately, no estimate of the elasticity of marginal utility is without criticism, whether normative or positive, but the time horizon being considered may favour some concepts and empirical measures over others even in the world of revealed preference. The practical message of this paper is, however, that in the UK case this would make very little difference to the final policy recommendation of raising the elasticity of marginal utility from 1 to 1.5.

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Appendices

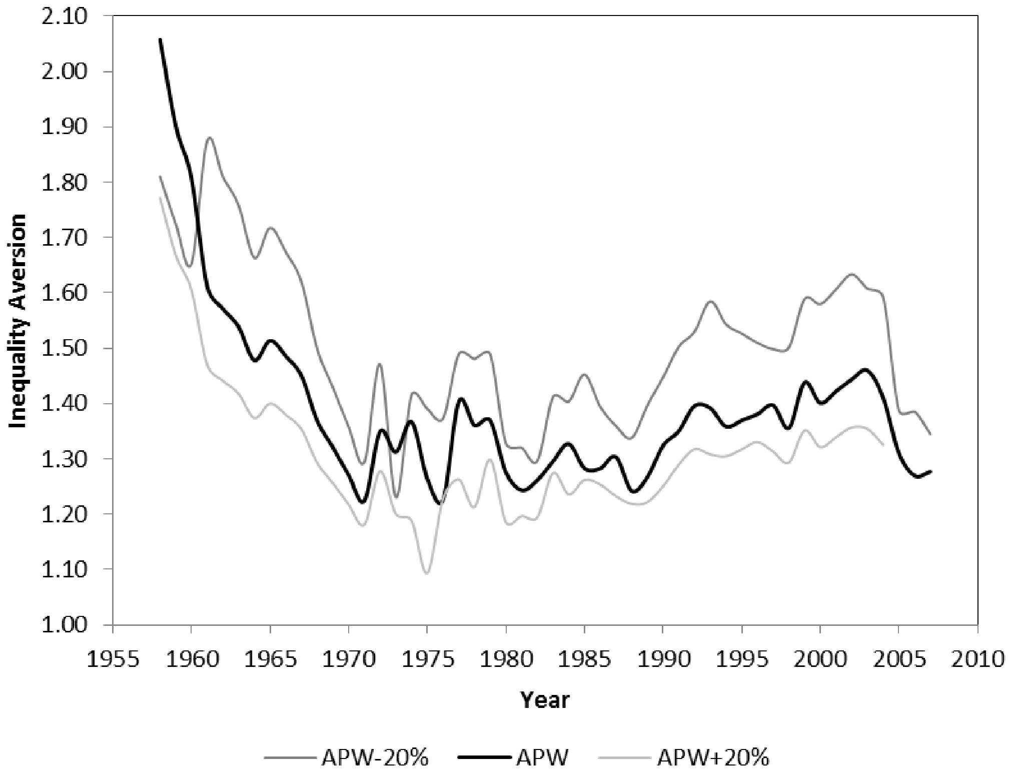
Appendix 1. Income taxes in the United Kingdom 1948-2007

Year	Tax-free allowance (£)	Lowest rate (%)	Highest rate income threshold (£)	Top rate (%)	Number of bands	MTR (%)	ATR (%)
1948	50	15.00	250	45.00	3	24.00	14.73
1949	50	15.00	250	45.00	3	24.00	14.73
1950	50	12.50	250	45.00	3	20.00	14.04
1951	50	15.00	250	47.50	3	22.00	15.65
1952	100	15.00	400	47.50	4	22.00	14.58
1953	100	12.50	400	47.50	4	27.22	13.46
1954	100	12.50	400	47.50	4	27.22	13.46
1955	60	11.25	360	42.50	4	26.25	15.43
1956	60	11.25	360	42.50	4	26.25	15.43
1957	60	11.25	360	42.50	4	26.25	15.43
1958	60	11.25	360	42.50	4	26.25	15.43
1959	60	8.75	360	38.75	4	30.14	17.18
1960	100	20.00	300	38.75	3	30.14	18.01
1961	100	20.00	300	38.75	3	30.14	18.01
1962	100	20.00	300	41.25	3	30.14	20.41
1963	100	20.00	300	41.25	3	30.14	20.41
1964	100	20.00	300	41.25	3	30.14	20.41
1965	100	20.00	300	41.25	3	30.14	20.41
1966	260	30.00	260	41.25	2	32.08	22.93
1967	0	41.25	-	-	1	32.08	23.42
1968	0	38.75	-	-	1	32.08	24.62
1969	0	38.75	-	-	1	32.08	24.62
1970	5000	30.00	20000	75.00	9	32.08	26.25
1971	4500	33.00	20000	83.00	10	30.14	25.39
1972	4500	35.00	20000	83.00	10	30.14	23.34
1973	5000	35.00	20000	83.00	10	36.54	29.28
1974	6000	34.00	21000	83.00	10	34.71	26.79
1975	750	25.00	24000	83.00	11	39.94	33.18
1976	750	25.00	25000	60.00	7	39.46	33.59
1977	12800	30.00	>31500	60.00	6	41.39	31.66
1978	11250	30.00	>27750	60.00	6	39.71	31.05
1979	12800	30.00	>31500	60.00	6	37.07	28.70
1980	14600	30.00	>36000	60.00	6	36.54	29.99
1981	15400	30.00	>38100	60.00	6	37.69	31.64
1982	16200	30.00	>40000	60.00	6	38.95	32.37
1983	17200	29.00	>41200	60.00	6	38.75	31.49
1984	17900	27.00	>41600	60.00	6	39.13	31.20
1985	20700	25.00	>20700	40.00	2	38.26	31.33
1986	20700	25.00	>20700	40.00	2	38.26	31.33
1987	20700	25.00	>20700	40.00	2	38.26	31.33
1988	23700	25.00	>23700	40.00	2	33.44	27.94

1989	2000	20.00	>25700	40.00	3	33.30	27.35
1990	2500	20.00	>26200	40.00	3	33.76	26.73
1991	3000	20.00	>26700	40.00	3	33.93	26.42
1992	3900	20.00	>29400	40.00	3	33.85	25.62
1993	4100	20.00	>30200	40.00	3	33.80	25.65
1994	4300	20.00	>31400	40.00	3	34.23	26.54
1995	4300	20.00	>31400	40.00	3	34.23	26.54
1996	1500	10.00	>28000	40.00	3	33.83	25.85
1997	1520	10.00	>28400	40.00	3	33.41	25.25
1998	1520	10.00	>28400	40.00	3	32.65	25.29
1999	1920	10.00	>29900	40.00	3	33.18	24.45
2000	1960	10.00	>30500	40.00	3	31.54	23.69
2001	2020	10.00	>31400	40.00	3	31.75	23.56
2002	2090	10.00	>32400	40.00	3	31.97	23.42
2003	2150	10.00	>33330	40.00	3	33.54	24.41
2004	2230	10.00	>34600	40.00	3	32.73	24.50
2005	2090	10.00	>32400	40.00	3	33.25	26.54
2006	2150	10.00	>33330	40.00	3	32.75	26.83
2007	2230	10.00	>34600	40.00	3	33.17	27.06

Source: Adapted from Lynch and Weingarten (2010).

Appendix 2: Estimates of Inequality Aversion at +/- 20% of the Average Production Wage



Appendix 3. How additive preferences facilitate the measurement of η

Assume the existence of an explicitly additive direct utility function:

(A1) $$U = U_1(Q_1) + U_2(Q_2) + ... + U_n(Q_n)$$

Maximising this expression subject to a budget constraint yields:

(A2) $$\frac{\partial U(Q_i)}{\partial Q_i} = \lambda P_i$$

This is then differentiated first with respect to income and then with respect to P_j and P_i yielding respectively:

(A3) $$\frac{\partial^2 U(Q_i)}{\partial Q_i^2}\frac{\partial Q_i}{\partial Y} = \frac{\partial \lambda}{\partial Y}P_i$$

(A4) $$\frac{\partial^2 U(Q_i)}{\partial Q_i^2}\frac{\partial Q_i}{\partial P_j} = \frac{\partial \lambda}{\partial P_j}P_i$$

(A5) $$\frac{\partial^2 U(Q_i)}{\partial Q_i^2}\frac{\partial Q_i}{\partial P_i} = \lambda + \frac{\partial \lambda}{\partial P_i}P_i$$

Next, define the following elasticities:

(A6) $$\eta = \frac{\partial \lambda}{\partial Y}\frac{Y}{\lambda}$$

(A7) $$\theta_i = \frac{\partial \lambda}{\partial P_i}\frac{P_i}{\lambda}$$

Rewriting the above equations using these elasticities yields:

(A8) $$\frac{\partial^2 U(Q_i)}{\partial Q_i^2}\frac{\partial Q_i}{\partial Y}\frac{Y}{\lambda} = \eta P_i$$

(A9) $$\frac{\partial^2 U(Q_i)}{\partial Q_i^2}\frac{\partial Q_i}{\partial Y}\frac{P_j}{\lambda} = \theta_j P_i$$

(A10) $$\frac{\partial^2 U(Q_i)}{\partial Q_i^2}\frac{\partial Q_i}{\partial P_i}\frac{P_i}{\lambda} = P_i + \theta_i P_i$$

This demonstrates the following holds:

(A11) $$\eta e_{ij} = \kappa_i \theta_j + \mu_{ij}\kappa_i$$

Where μ_{ij} is the Kronecker delta. Multiplying both sides by w_i yields:

A(12) $$\eta w_i e_{ij} = w_i \kappa_i \theta_j + \mu_{ij} w_i \kappa_i$$

Summing over i and applying the Cournot aggregation:

A(13) $$\sum_i w_i e_{ij} + w_j = 0$$

And adding up:

(A14) $$\sum_i w_i \kappa_i = 1$$

Yields the following solution for θ_i:

A(15) $$\theta_i = -w_i(\eta + \kappa_i)$$

Inserting this expression yields

A(16) $$\eta e_{ij} = -\kappa_i w_j(\eta + \kappa_j) + \mu_{ij}\kappa_i$$

Rearranging this expression yields:

A(17) $$e_{ii} = \phi\kappa_i - \kappa_i w_i(1 + \phi\kappa_i)$$

A(18) $$e_{ij} = -\kappa_i w_i(1 + \phi\kappa_i)$$

Finally, using the Slutsky equation, noting that $\eta_i = 1/\phi_i$:

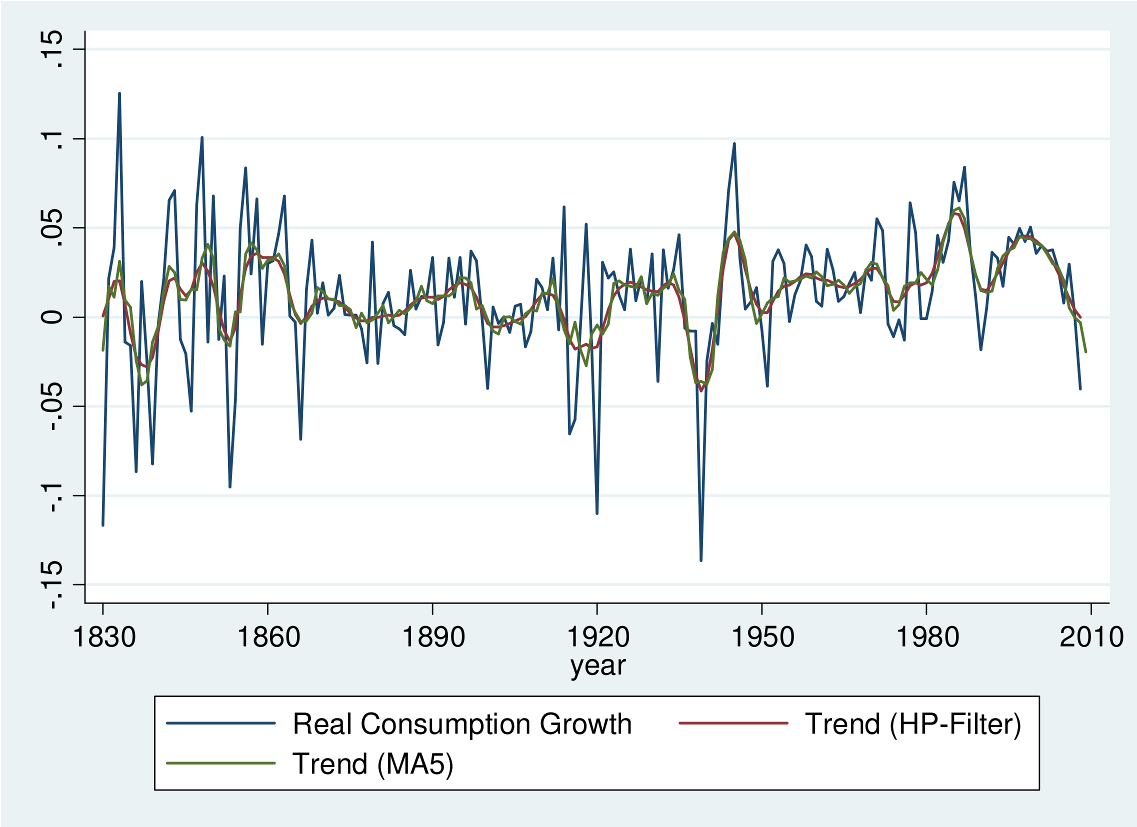
A(19) $$e_{ii} = \varepsilon_{ii} - \kappa_i w_i$$

The first of these equations is most often shown as:

A(20) $$\eta = \frac{\kappa_i(1 - w_i\kappa_i)}{\varepsilon_{ii}}$$

Appendix 4

Figure A4.1. UK per capita consumption Growth (1830-2009)



Appendix 5. The impact of eta on the term structure of Social Discount Rates (Forward Rates from the HP model for period 1950-1998)