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Resolving intertemporal conflicts: Economics vs. Politics

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Abstract

Intertemporal conflicts occur when a group of agents with heterogeneous time preferences must make a collective decision about how to manage a common asset. How should this be done? We examine two methods – an 'Economics' approach that seeks to implement efficient allocations, and a 'Politics' approach in which agents vote over consumption plans. We compare these methods by varying two characteristics of the problem: are agents' preferences known or are they hidden information, and can they commit to intertemporal collective plans or not? We show that if commitment is possible the Economics approach always Pareto dominates the Politics approach, in both full and hidden information scenarios. By contrast, without commitment the group may be better off if the Politics approach is adopted. We investigate when Politics trumps Economics analytically, and then apply our model to a survey of economists' views on the appropriate pure rate of time preference for project appraisal. For a wide range of model parameters, and under both full and hidden information, the Politics approach is supported by a majority of agents, and leads to higher group welfare.

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1 Introduction

Many important decisions require groups of people with different time preferences to decide how to manage a common asset. Examples abound: family members must decide on savings and intra-household resource allocation, corporate partners must decide on investment and dividend policy, communities with property rights over a natural resource must decide how to manage it, and countries must decide how to manage sovereign wealth funds. In each of these examples an asset is held in common and requires a dynamic management plan, and the stake-holders in the decision very often have heterogeneous time preferences. How should such decisions be made, given people's different attitudes to time? This is the subject of this paper.

The heterogeneity in people's time preferences is now well documented. Frederick et al. (2002) summarize the empirical literature, which uses experimental and field studies to infer individuals' rates of time preference. Estimates vary from -6%/yr to infinity across the studies they cite, and within study variation in estimates is also large. These studies are positive in nature – they tell us how people behave, and not how they think they, or society, *should* behave for normative purposes. However, the time preferences economists prescribe for normative applications, e.g. in public project appraisal (Arrow et al., 2013), are also highly heterogeneous. This has been highlighted by the long-standing debate about the appropriate rate of time preference for the evaluation of climate change policy (Nordhaus, 2008; Stern, 2007; Weitzman, 2007). A recent survey of economists who have published on social discounting (Drupp et al., 2014) shows significant variation in prescriptions for the pure rate of time preference, as demonstrated in Figure 1.[1]

The pervasive heterogeneity in attitudes to time raises difficulties for collective decision-making. Strotz (1955) and Koopmans (1960) showed the necessity of a single constant discount rate[2] for time-consistent choice in the standard discounted utilitarian model. Jackson

[1]The debate over the appropriate pure rate of time preference for public choices stretches back at least as far as the seminal contribution of Ramsey (1928), who argued that 'discounting of future utilities is ethically indefensible and arises purely from a weakness of the imagination.' Nevertheless, Ramsey acknowledged that many might disagree with this ethical prescription (see also Arrow, 1999), and considered positive rates of time preference too. Koopmans (1969) famously observed that very low rates of time preference can lead to the 'paradox of the indefinitely postponed splurge', in which the current generation saves 100% of its income, bankrupting itself for the sake of its descendants. Subsequent commentators have argued the merits of a variety of discount rates without a clear 'best' value emerging, and different governments have adopted different values for public decision-making.

[2]Throughout the paper we use the term 'discount rate' to refer to the pure rate of time preference, i.e. the discount rate on utility. The discount rate on consumption will be referred to as the 'real discount rate'.

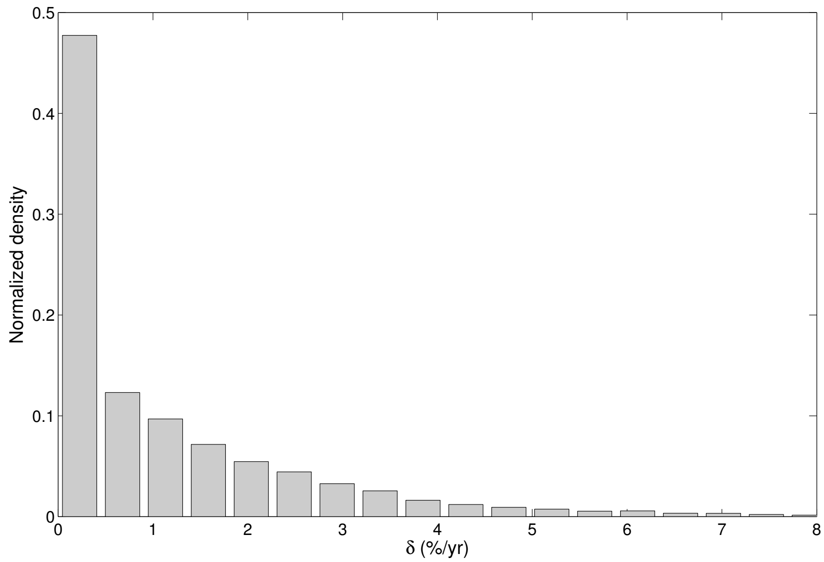

Figure 1: Distribution of the recommended pure rate of time preference δ for public project appraisal, from the Drupp et al. (2014) survey of economists. 180 responses were recorded in the original sample. A kernel density fit has been applied to smooth out the dataset.

& Yariv (2012, 2014) build on these insights to shows that methods of aggregating individual time preferences that are efficient cannot be time consistent.[3] Their work highlights the difficulties that arise in group decision making when people disagree about time; our paper examines possible solutions to these intertemporal conflicts.

We focus on a problem in which a group of agents who are identical except for their time preferences must decide how to manage a common asset. The group must choose a collective decision rule that specifies both how much is consumed in aggregate at each point in time, and how to divide consumption between individuals. We examine two natural methods for making such decisions: an 'Economics' method, in which the decision rule specifies all aspects of the allocation, and seeks to implement an efficient plan, and a 'Politics' method, in which the rule only specifies an income share for each agent, and agents then vote on the group's aggregate consumption plan.

If full information about agents' time preferences is available, and the group commits to always evaluate intertemporal plans from the perspective of social preferences at an initial time τ_0, the Economics approach implements first-best allocations, and thus trivially leads

[3] Our concept of time consistency is the same as that in e.g. Strotz (1955); Jackson & Yariv (2014). If $X^*(T,\tau)$ denotes an optimal action T units of time in the future as determined at calendar time τ, then time consistency requires that for all τ_1, τ_2 such that $\tau_2 > \tau_1$, and all $T \geq 0$, $X^*(T,\tau_2) = X^*(T + (\tau_2 - \tau_1), \tau_1)$.

	Full information	Hidden information
Commitment	Economics $\succ$ Politics	?
No commitment	?	?

Table 1: The four versions of our model of intertemporal conflict. Information about agents' preferences varies across the columns of the table, and ability to commit to intertemporal plans varies across the rows. Economics implements a first-best optimum under full information and if commitment is possible. However, in each of the three other versions of the model it is unclear *a priori* whether an Economics or Politics approach to resolving intertemporal conflicts will lead to higher group welfare.

to higher group welfare than the Politics approach. These efficient allocations, and the representative collective time preferences that emerge from them, have been studied by Gollier & Zeckhauser (2005) and Heal & Millner (2013). However, since this decision rule is efficient, it cannot be time consistent unless commitment is possible (Jackson & Yariv, 2012). In addition, implementing a given point on the Pareto frontier generically requires full information about agents' preferences.[4] It is thus natural to ask what happens if we relax the commitment and full information assumptions, one by one. This leads us to contrast Economics with Politics in the four versions of the model summarized in Table 1.

Retaining the commitment assumption, but assuming that agents' preferences are hidden information, allocations are tightly constrained by the requirements of incentive compatibility in both the Economics and Politics approaches. The effects of these constraints on individuals' preferences between the two approaches, and the group's welfare, could in principle go either way. Although the incentive compatible Economics approach is still Pareto efficient, we show that it corresponds to a particular choice of Pareto weights, and could thus lead to lower group welfare than Politics if a different set of weights is used to evaluate outcomes. Despite this possibility, we show that the Economics approach Pareto dominates the Politics approach in both full and hidden information versions of the model with commitment. Thus if commitment is possible, a 'technocratic' welfare economics based approach to resolving intertemporal conflicts is the clear winner, regardless of the weights assigned to individual agents' welfare.

Without the commitment assumption, the first-best allocations in the Economics approach are not implementable, and the group will rationally discard them. Rather, if they

[4] The efficient allocations in our model can replicate any market equilibrium in which agents trade claims on future consumption, assuming that complete futures markets exist (see further discussion below). The market mechanism does not however alleviate incentive problems, as agents' initial endowments determine the equilibrium. Any allocation of endowments will lead to an efficient equilibrium, but this may not be the collectively desired outcome if agents' preferences are hidden information.

seek to implement efficient allocations, they will choose the best current allocations they can, given the allocation strategy they will follow in the future. This induces a dynamic game between the current and future selves of the group (Phelps & Pollak, 1968). The Markov Perfect Equilibrium of this game is time-consistent and intratemporally efficient, but intertemporally inefficient. We show that the equilibrium of the Politics approach is intertemporally efficient, and thus also time consistent, but intratemporally inefficient. Thus without commitment the choice between Economics and Politics boils down to a horse-race between two different inefficiencies, one across time (Economics), the other between agents (Politics). The question is, under what conditions will one dominate the other? Asking this question under both full and hidden information scenarios leads to the two versions of the model in the second row of Table 1.

We derive conditions that determine when Politics trumps Economics under both informational scenarios, and investigate them analytically using a simple parametric assumption about agents' utility functions. Under full information we show that Politics is always preferred by a majority of agents, and leads to higher group welfare, for a large and empirically relevant class of distributions for agents' discount rates. Under hidden information we are able to show that for any distribution of discount rates, and any model parameters, Politics is always preferred by a majority of agents. However the welfare contest between the two approaches is tighter in this case. To investigate this further we apply our analysis to the empirical sample of discount rates in Figure 1 for a wider set of utility functions. We find that the Politics approach would win a majority rule 'meta-vote' between the two aggregation methods in both full and hidden information scenarios for all values of the model parameters. Moreover, for a choice of equitable welfare weights, which ensure that agents are treated equally on all constant consumption paths, we find that Politics always leads to higher group welfare under full information. This result also holds for a wide and empirically relevant range of model parameters under hidden information. Thus, the results we obtained under commitment are partially overturned – if commitment is not possible the group will often be better off if the collective decision rule does not seek to implement efficient plans, but instead submits aggregate consumption decisions to a vote.

Our work relates to three strands of literature. The most closely related work deals with the aggregation of time preferences, or real (i.e. consumption) discount rates more generally. The efficient approach to aggregating time preferences was originally explored by Gollier & Zeckhauser (2005). They consider a model in which an exogenous common stream of consumption is divided between heterogeneous agents efficiently, and derive the representative time preferences of the group. Generically the representative rate of time preference is non-constant, reflecting the implicit commitment assumption that is necessary for the efficient equilibrium to be implemented as a collective decision rule. Thus this work does not address the time inconsistency problem explicitly, and also assumes that agents' preferences are public information. Similarly, Li & Löfgren (2000) explore the consequences of efficient time preference aggregation in a public goods setting with endogenous resource management. They too assume full commitment, and information and incentive problems do not arise in their work, which is motivated via the purely normative intertemporal decision criterion of Chichilnisky (1996).[5] A parallel literature in finance investigates representative time preferences when agents make private decisions and interact through the market (e.g. Lengwiler, 2005; Cvitanić et al., 2012). Although representative time preferences are in general declining in this case, there is no time inconsistency problem, as each agents' individual decisions are time consistent. The tension between efficiency and time consistency arises only in collective decisions that attempt to reflect a variety of time preferences (Jackson & Yariv, 2012).

A related literature, stemming from the work of Weitzman (2001), focuses on aggregation of real discount rates, rather than pure rates of time preference. Weitzman takes a sample of opinions as to the appropriate (constant) real discount rate, treats these as uncertain estimates of the 'true' underlying rate, and takes expectations of the associated discount factors to derive a declining term structure for the 'certainty equivalent' real discount rate. As Freeman & Groom (2014) observe, opinions about real discount rates conflate ethical views about welfare parameters (e.g. the pure rate of time preference) with empirical estimates of consumption growth rates – they mix tastes and beliefs. This suggests that it is important to pursue approaches that treat preference aggregation as a distinct problem, before addressing the issue of empirical uncertainty. Our work contributes to this task.[6]

The second strand of literature studies time preference aggregation empirically. Several papers (e.g. Mazzocco, 2007; Adams et al., 2014) have developed revealed preference methodologies for identifying how households' collective decisions reflect their members' time preferences, and intra-household decision-making power. Microdata on household

[5] The importance of the commitment assumption in this context has recently been explored by Asheim & Ekeland (2014).

[6] Weitzman's insistence that his survey participants provide a constant value for the real discount rate has also been criticized (Dasgupta, 2001, pp. 187-190). We avoid these issues by focussing only on collective choices when people disagree about the pure rate of time preference. Individuals have good reasons for preferring constant values of this parameter in normative applications.

expenditures are used to estimate models of collective decision-making, and test which assumptions about individual time preferences and household decision-making rationalize the data. In a complimentary approach, Jackson & Yariv (2014) study a choice experiment in which representative 'planners' are asked to decide between different public consumption streams for a group of other participants. They find that most planners are time-inconsistent due to their desire to aggregate the group's time preferences. In contrast to these positive studies, our work takes a normative approach to the problem of collective dynamic choice, asking which methods groups might wish to use to resolve intertemporal conflicts.

Finally, our work relates to a diverse literature on disadvantageous power in second-best settings. In the version of our model without commitment the collective decision rule controls all aspects of decision making in the Economics approach, but control over aggregate consumption decisions is relinquished in the Politics approach. Nevertheless, the group may realize higher welfare under Politics than under Economics. Without commitment we are in a second-best world, and the constrained efficient equilibrium in the Economics approach may be improved upon by giving up some decision-making power to the agents themselves. This finding has analogues in the theory of market power (Salant et al., 1983; Maskin & Newbery, 1990), international cooperation (Rogoff, 1985), and government regulation (Krusell et al., 2002).

The paper is structured as follows. Section 2 sets out the basic structure of the model we employ. Section 3 develops the Economics approach to resolving intertemporal conflicts, first treating the case of commitment before characterizing the equilibrium of the dynamic game that arises in the version of the model without commitment. Both full and hidden information versions of the model are developed in each of these cases. Section 3 develops the Politics approach, and derives the equilibrium that emerges when agents vote over dynamic aggregate consumption plans. Again, the model is analyzed in both full and hidden information scenarios. Section 4 compares Economics to Politics in each of the four versions of the model in Table 1. Several general results are obtained, as well as some analytic results under specific parametric assumptions. Section 5 applies the results of the previous sections to the empirical sample of discount rates in Figure 1, and investigates when Politics trumps Economics over a wide range of model parameters. Section 6 concludes.

2 The setting

We assume a set of N agents indexed by i, each of whom has idiosyncratic discount rate $\delta_i \in [0, \infty)$.[7] To focus on the core issue of intertemporal conflict we assume away all other sources of heterogeneity between agents. In particular, they have a common utility function

$$U(c) = \begin{cases} \frac{c^{1-\eta}}{1-\eta} & \eta \neq 1 \\ \ln c & \eta = 1. \end{cases} \tag{1}$$

Thus if agent i is allocated a consumption path c_{it} at time t in the future, his realized welfare computed at time τ is

$$W_i = \int_{\tau}^{\infty} U(c_{it}) e^{-\delta_i (t-\tau)} dt. \tag{2}$$

The group of agents derives consumption from a common asset or resource S, which grows at constant rate r. The resource thus evolves according to

$$\dot{S}_t = r S_t - \sum_i c_{it} \tag{3}$$

and the initial value of the resource stock at time $\tau = 0$ is S_0.

A collective decision rule must specify how to distribute consumption between agents over time, taking into account the heterogeneity in their discount rates. We assume that the group has an opinion as to how much weight to place on agent i's welfare, denoted by $w_i > 0, (\sum_i w_i = 1)$. To ensure that the welfare integrals in (2) always converge we assume $\eta \geq 1$ in (1).[8]

In what follows it will sometimes be useful to use shorthand notation, which we collect

[7] The assumption of idiosyncratic discount rates greatly simplifies notation, but all our results can be easily extended to the case where agents share discount rates. A distribution of discount rates in which some agents share values of δ may also be arbitrarily closely approximated by a distribution in which discount rates are idiosyncratic by adding a small amount of of noise to each discount rate.

[8] All our results may be extended to $\eta < 1$ by placing appropriate restrictions on the model parameters, but no additional restrictions are needed for $\eta \geq 1$. Since $\eta \geq 1$ in empirical applications we focus on this case, but comment on the case $\eta < 1$ in the appendices.

here for convenience. We define

$$\alpha_i := \delta_i + (\eta - 1)r \tag{4}$$

$$\tilde{w}_i := \frac{w_i^{1/\eta}}{\sum_j w_j^{1/\eta}} \tag{5}$$

The weighted average of a quantity x_i ($i = 1..N$) taken with weights y_i ($y_i \geq 0, \sum_j y_j = 1$) will be written as

$$\langle x_i \rangle_{y_i} := \sum_i x_i y_i. \tag{6}$$

If the average operator doesn't have a subscript, this will mean that the weights y_i are equal:

$$\langle x_i \rangle := \frac{1}{N} \sum_i x_i. \tag{7}$$

3 Economics

In this section we describe an approach to resolving intertemporal conflicts based on traditional welfare economics. In this approach the group seeks to allocate consumption between agents and over time efficiently. The decision rule thus maximizes a weighted sum of agents' intertemporal welfare integrals. If commitment to intertemporal allocations is possible, and agents' discount rates are public information, any Pareto efficient allocation can be implemented. We initially describe this efficient equilibrium, and show that it is only implementable if commitment is possible. We then relax both the full information and commitment assumptions. If agents' true discount rates are private information, they will in general have incentives to lie about their discount rates. In the case of commitment we show that if a special rule for determining agents' consumption allocations is used they will be incentive compatible in dominant strategies. We then study the group's allocation problem without commitment. Following in the tradition established by Phelps & Pollak (1968), we treat the problem as a dynamic game – the group still seeks to maximize a weighted sum of agents' intertemporal welfare, but knows that its 'future selves' will make different decisions to the ones it would like. It thus chooses the best allocations it can, given the decisions these future selves will make. We describe the Markov Perfect Equilibrium of this game, and analyze the constraints incentive compatibility imposes on equilibrium allocations in this case. The section thus provides a complete description of the 'Economics' approach, both with and without commitment, and with full and hidden information.

3.1 Commitment

Assume that weight w_i is assigned to agent i's welfare. If the group can commit to evaluating collective welfare with respect to social preferences at time $\tau = 0$, agents' consumption c_{it} will be chosen to maximize

$$\sum_i w_i \int_0^\infty U(c_{it}) e^{-\delta_i t} dt \ \text{ s.t. } \dot{S} = rS - \sum_i c_{it}. \tag{8}$$

These allocations will be Pareto efficient. They are summarized by the following proposition:

Proposition 1. *1. The consumption path allocated to agent i is*

$$c_{it} = \left(\frac{w_i}{\lambda_0} e^{-(\delta_i - r)t} \right)^{1/\eta} \tag{9}$$

where λ_0, the initial shadow price of the resource, is given by

$$\lambda_0 = \left[\frac{\eta}{S_0} \sum_i \frac{w_i^{1/\eta}}{\delta_i + (\eta - 1)r} \right]^\eta . \tag{10}$$

2. Agent i's realized welfare is:

$$W_i^E = \begin{cases} \frac{\eta}{1-\eta} \left[\frac{S_0}{\eta} \tilde{w}_i \langle \alpha_i^{-1} \rangle_{\tilde{w}_i}^{-1} \right]^{1-\eta} \frac{1}{\alpha_i} & \eta > 1 \\ \frac{1}{\delta_i} \ln(S_0 w_i \langle \delta_i^{-1} \rangle_{w_i}^{-1}) + \frac{1}{\delta_i^2}(r - \delta_i) & \eta = 1 \end{cases} \tag{11}$$

3. The aggregate consumption plan $C_t = \sum_i c_{it}$ is equivalent to the plan that would be chosen by a representative agent with utility function $U(C_t)$, and pure rate of time preference

$$\hat{\delta}(t) = \frac{\sum_i \delta_i (w_i e^{-\delta_i t})^{1/\eta}}{\sum_i (w_i e^{-\delta_i t})^{1/\eta}}. \tag{12}$$

Proof. See Appendix A.1. □

The superscript E in W_i^E reminds us that this is the welfare agent i receives when consumption allocations are *efficient*.

This proposition relies on the assumption that the group commits to implementing an efficient plan at time $\tau = 0$.[9] To see this, let agent L have the lowest discount rate, δ_L. From (9) we see that the ratio of agent $i \neq L$'s consumption to agent L's consumption at time t is proportional to $e^{-(\delta_i - \delta_L)t/\eta}$. This ratio is decreasing in time for all $i \neq L$. Thus, regardless of the choice of welfare weights, an increasing share of aggregate consumption is allocated to the agent with the lowest discount rate as time passes.[10] While such an allocation is efficient at time $\tau = 0$, it is not time-consistent without the commitment assumption, as the group will want to allocate consumption to impatient agents if it is allowed to revise its plan at a later time. The necessity of commitment is also reflected in the representative discount rate (12), which decreases monotonically to the lowest discount rate (Heal & Millner, 2013). Only constant discount rates lead to time consistent optimal plans in the absence of commitment (Strotz, 1955).

Sticking with the commitment assumption for the moment, we now ask how hidden information about agents' time preferences constrains which efficient allocations are implementable.[11] If *full information* about agents' discount rates is available the choice of consumption allocations is unconstrained. By contrast, under *hidden information*, agents have incentives to lie about the value of their discount rate. As an example suppose that $w_i = 1/N$ for all i. In this case (9) shows that agents' consumption paths are Pareto ordered, with the agent with the lowest discount rate receiving a path that Pareto dominates the allocation of the agent with the next lowest discount rate, and so on. Thus, all agents have an incentive to announce the lowest possible discount rate, regardless of their true discount rate. If the group wishes to avoid such incentive problems, while still implementing Pareto efficient allocations, its choice of allocations will be tightly constrained. It needs to ensure that all agents cannot improve their welfare (computed with respect to their true discount rate δ_i) by announcing a different discount rate δ_i'. This can be achieved if a spe-

[9] The efficient allocations in this proposition can be decentralized if complete futures markets exist. The commitment assumption is not necessary in a market context, as decisions are made by individual time consistent agents, and not by a decision rule that aggregates agents' preferences to make a social decision. The welfare weights $\{w_i\}$ can be mapped to agents' initial endowments, and trading at $t = 0$ leads to an efficient and time consistent market equilibrium. While this equilibrium will clearly be Pareto efficient, it will not be welfare optimal from the perspective of a future social planner unless we commit to evaluating allocations using $\tau = 0$ social preferences. If a future planner at $\tau > 0$ continued to evaluate agents' intertemporal welfare according to (2), she would want to reallocate their current endowments and reopen the market. If she rationally anticipates all such future reallocations the resulting set of market equilibria will correspond to the equilibrium of the dynamic game we study in the following sub-section.

[10] Heal & Millner (2013) prove this in much more general models than the one we're considering.

[11] We focus on incentive compatibility in dominant strategies. Aside from being the simplest, strongest, and most practically useful form of incentive compatibility, this ensures comparability between the Economics and Politics approaches, as we show below.

cific rule for mapping agents' announced discount rates into their consumption allocations is used:

Proposition 2. *The efficient allocations in (9) will be incentive compatible in dominant strategies if and only if an agent who announces discount rate δ_i is assigned Pareto weight*

$$w_i = \frac{(\delta_i + (\eta - 1)r)^\eta}{\sum_j (\delta_j + (\eta - 1)r)^\eta}. \tag{13}$$

Proof. See Appendix B. □

The intuition for the qualitative features of this result can be understood as follows. In order to avoid the Pareto ordering of consumption streams that obtained in our example of efficient allocation when the welfare weights are equal, it is necessary for the weights to be adjusted in favor of agents with high discount rates. In particular, it should always be the case that an agent with a high discount rate δ_h receives a larger initial share of consumption than an agent with a lower discount rate $\delta_l < \delta_h$, and agent h's consumption should fall faster than l's. Inspection of the formula (9) shows that this constrains the welfare weights to be increasing in δ_i, as in (13). But this is not sufficient for incentive compatibility in dominant strategies. We also require that agents' realized welfare should not depend on the discount rate announcements of other agents. Appendix B demonstrates that the weights in (13) are the unique Pareto weights that achieve this.

3.2 No Commitment

We now consider the case where intertemporal commitments are not possible. In this case a planner at time τ_1 who wants to implement an efficient policy knows that at time $\tau_2 > \tau_1$ her 'future self' will implement what she views as the best consumption allocation. However, the allocation that the time τ_2 planner makes will not coincide with what the τ_1 planner would have liked. For example, the planner at τ_1 would like to assign weight $w_i e^{-\delta_i(\tau_2 - \tau_1)}$ to agent i's welfare at time τ_2 when deciding on consumption allocations, but from the perspective of the planner at τ_2 the appropriate weight on agent i's welfare at time τ_2 is just w_i. Hence the time inconsistency problem.

The traditional economics approach to solving models of intertemporal choice with time inconsistent preferences is to treat them as a dynamic game between current and future selves (Phelps & Pollak, 1968). This method has become the norm in a host of applications including consumer behavior (Laibson, 1997), growth theory (Barro, 1999), political economics (Persson & Tabellini, 2000), and environmental economics (Karp, 2005). In this approach the planner at τ_1 rationally anticipates the consumption allocations of her future selves at all $\tau_2 > \tau_1$. She makes the best decision she can, subject to what these future selves will do. This induces a dynamic game between current and future selves, and we look for Markov Perfect Equilibria (MPEs) of this game. Since equilibrium strategies are conditioned only on payoff relevant state variables, and not on calendar time, they will be time consistent.

A Markov allocation rule in our context is a profile of state dependent functions $P = \{\sigma_i(S)\}$ such that $c_{it} = \sigma_i(S_t)$ for all $t \geq \tau$. A profile is an MPE if, in the limit as $\epsilon \to 0$, when planners at times $t \in [\tau + \epsilon, \infty)$ in the future use profile P, the best response of the current planner in $t \in [\tau, \tau + \epsilon)$ is also to use P. The fact that we are after a profile of N allocation rules raises considerable computational problems if we take a direct approach to deriving the equilibrium conditions.[12] However, it is possible to reduce the problem to one that only requires us to solve for a single unknown function. The key observation is that the current planner only influences what happens in the future through her choice of *aggregate* consumption – how she distributes a given level C_τ of current aggregate consumption across agents has no effect on the actions of future planners, and her stream of future welfare. This is so since future allocations depend only on the state S_t, which is affected only by aggregate consumption decisions. Since the actions of future selves are independent of the distribution of current consumption, the current planner will rationally allocate whatever consumption C_τ she chooses between the agents so as to maximize their current collective welfare. Specifically, given any (as yet unknown) choice of aggregate consumption C_τ, the current planner chooses $c_{i\tau}$ to maximize

$$\sum_i w_i U(c_{i\tau}) \text{ s.t. } \sum_i c_{i\tau} = C_\tau.$$

A simple calculation then shows that

$$c_{i\tau} = \frac{w_i^{1/\eta}}{\sum_i w_i^{1/\eta}} C_\tau = \tilde{w}_i C_\tau. \tag{14}$$

Thus, in equilibrium, the planner will always allocate a share $\tilde{w}_i$ of aggregate consumption to agent i at each point in time. This implies that the equilibrium allocation to agent

[12]For example, if we look for a linear equilibrium in which the profile of allocation rules takes the form $\sigma_i(S) = a_i S$, the equilibrium conditions for the set of N coefficients $\{a_i\}$ are a coupled nonlinear system of N algebraic equations.

i must satisfy $\sigma_i(S) = \tilde{w}_i \sigma(S)$ for some function $\sigma(S_t)$, where aggregate consumption is given by $C_t = \sigma(S_t)$. Clearly, for each τ, the planner views her current allocations at time τ as intratemporally efficient.

Now we need to solve for the equilibrium aggregate consumption rule $\sigma(S)$. To do this, note that the value the planner at $t = \tau$ gets from aggregate consumption C_t consumed by the group at time $t \geq \tau$ is:

$$\sum_i w_i e^{-\delta_i(t-\tau)} U(\tilde{w}_i C_t) = U(C_t) \left(\sum w_i^{1/\eta} \right)^\eta \left(\sum_i \tilde{w}_i e^{-\delta_i(t-\tau)} \right) \tag{15}$$

Defining

$$\tilde{U}(C) = \frac{C^{1-\eta}}{1-\eta} (\sum_i w_i^{1/\eta})^\eta \tag{16}$$

$$\beta(t) = \sum_i \tilde{w}_i e^{-\delta_i(t-\tau)} \tag{17}$$

the problem is to find the MPE of a modified single agent problem with utility function $\tilde{U}(C)$ and discount factor $\beta(t)$, and where the resource stock evolves according to $\dot{S}_t = rS_t - C_t$.

An immediate question that arises is whether there is a unique MPE in this dynamic game. In general models with non-constant discount rates admit a continuum of MPEs (Krusell & Smith, 2003; Karp, 2007; Ekeland & Lazrak, 2010). If however we view our model as the infinite time horizon limit of a set of finite horizon games, the equilibrium is unique, and coincides with the linear MPE (see e.g. Krusell et al., 2002).[13] We thus focus on the linear equilibrium, which is characterized in the following proposition:

Proposition 3. *1. The linear MPE of the modified planner's problem without commitment is given by an aggregate consumption rule $C_t = \sigma(S_t) = AS_t$, where A satisfies*

$$\left\langle \frac{A}{\delta_i + (\eta - 1)(r - A)} \right\rangle_{\tilde{w}_i} = 1. \tag{18}$$

2. The aggregate consumption path in the linear MPE is equivalent to the optimal path of a time-consistent planner with discount rate

$$\delta_{NC} := r + \eta(A - r). \tag{19}$$

[13]A proof of this claim for our specific model is available on request.

3. Agent i's welfare in equilibrium is given by

$$W_i^{NC} = \begin{cases} \frac{\eta}{1-\eta}\left[\frac{S_0}{\eta}\tilde{w}_i\alpha_{NC}\right]^{1-\eta}\frac{1}{\alpha_{NC}+\eta(\alpha_i-\alpha_{NC})} & \eta > 1 \\ \frac{1}{\delta_i}\ln\left(S_0 w_i \langle\delta_i^{-1}\rangle_{w_i}^{-1}\right) + \frac{1}{\delta_i^2}\left(r - \langle\delta_i^{-1}\rangle_{w_i}^{-1}\right) & \eta = 1. \end{cases} \tag{20}$$

where $\alpha_{NC} := \delta_{NC} + (\eta - 1)r$.

Proof. See Appendix C. □

The superscript "NC" on W_i^{NC} reminds us that this is the welfare agent i realizes in the 'No Commitment' version of the model. Note that although the equilibrium is observationally equivalent to the time-consistent equilibrium associated with discount rate δ_{NC}, it is not intertemporally efficient, as originally observed by Phelps & Pollak (1968). δ_{NC} is an artificial construct, and the equilibrium does not correspond to the optimal consumption plan of any of the individuals the planner is attempting to represent.

Understanding the constraints that incentive compatibility imposes on consumption allocations is simpler in this case than in the version of the model with commitment. Under *full information* the choice of welfare weights is again unconstrained. To analyze incentives under *hidden information*, notice that each agent gets a constant share $\tilde{w}_i$ of aggregate consumption at each point in time in equilibrium. Thus the consumption streams allocated to agents will be Pareto ranked if the $\tilde{w}_i$ are not equal, leading to incentive problems. If we set $\tilde{w}_i = 1/N$, although all agents now receive the same allocation, they may still have incentives to lie about their discount rates so as to manipulate the aggregate consumption path to their advantage. We will assume that N is large, so that each agent's announced discount rate has a negligible effect on the value of A, and thus the aggregate consumption path.[14] In this case, provided that $\tilde{w}_i = 1/N$, all agents' realized welfare is exogenous to their announced discount rate, and they gain nothing by lying.

4 Politics

In this section we present an alternative method for resolving intertemporal conflicts. In contrast to the Economics approach described above, this method requires the collective decision rule to cede some control over consumption allocations to agents themselves. In-

[14]Technically, we also require an (arbitrarily large, but finite) upper bound on permissible discount rate announcements.

stead of prescribing the group's aggregate consumption, a political institution is established for deciding on *intertemporal* consumption plans in this approach.

The political approach requires the group to choose a set of constant income shares s_i, so that agent i receives a share s_i of aggregate consumption at all times. The s_i are endogenously determined, either from welfare considerations or from incentive constraints, as we explain below. Agents are then allowed to nominate an aggregate consumption plan for the group to follow. All agents vote over each pair of nominated plans, and the plan that gets a majority of votes wins each pairwise contest. A Condorcet winner (if it exists) is a plan that wins every pairwise contest. If there is a Condorcet winner, it is implemented.

In general, voting over arbitrary consumption plans can lead to intransitive collective choices (Jackson & Yariv, 2012). If however we restrict attention to votes over aggregate consumption plans that are *optimal* for a given agent, the problem has a lot more structure, and a unique Condorcet winner exists under mild conditions, as we now show.

Let the optimal aggregate consumption plan of an agent with discount rate δ_i be $C(\delta_i) = (C_t^{\delta_i})_{t\geq 0}$. Agent i's consumption at time t under such a plan is $s_i C_t^{\delta_i}$. Thus $C(\delta_i)$ is the solution of

$$\max_{C_t} \int_0^\infty U(s_i C_t) e^{-\delta_i t} dt \text{ s.t. } \dot{S}_t = rS_t - C_t. \tag{21}$$

We are interested in agents' preferences over the set of optimal plans $\{C(\delta_i)\}$. We begin with a lemma:

Lemma 1. *Suppose that:*

1. *Initial optimal aggregate consumption C_0^δ is an increasing function of δ.*

2. *Each pair of aggregate consumption paths $\{C(\delta), C(\delta')\}$ has exactly one intersection point, i.e. for any $\delta' > \delta$, there exists a time T such that*

$$\forall t > 0, (T - t)(C_t^{\delta'} - C_t^{\delta}) > 0.$$

Then all agents have single-peaked preferences over optimal aggregate consumption paths.

Proof. See Appendix D. □

Using this lemma, we have the following result:

Proposition 4. *The unique Condorcet winner of a vote over optimal aggregate consumption plans is the optimal plan of the median agent, with discount rate δ_m.*

Proof. Since the utility function is iso-elastic, agents' preferences over optimal aggregate consumption plans are independent of their income shares s_i. So we can set $s_i = 1$ for all i, and focus on individually optimal consumption plans. Theorem 2 in Becker (1983) shows that optimal initial aggregate consumption is an increasing function of δ for any concave production function. As $t \to \infty$, the path $C(\delta)$ tends to zero if $\delta > r$, and $+\infty$ if $\delta < r$. Thus the limiting value of C_t^δ is non-increasing in δ. In addition, all optimal consumption paths are monotonic functions of time (see e.g. Kamien & Schwartz, 1991). All pairs of consumption paths must therefore cross exactly once. The two conditions of Lemma 1 are thus satisfied, and agents have single-peaked preferences over optimal plans. Application of the classic results of Black (1948) then yields the result.[15] □

Our analysis of the voting equilibrium has thus far assumed that there is a once-off vote on the aggregate consumption plan at time $\tau = 0$. One could also allow agents to vote over aggregate consumption in each period. In this case the problem of proving that agents' preferences over aggregate consumption are single peaked is simpler, and it can be shown that the same equilibrium would obtain in this case.[16] The political mechanism for resolving intertemporal conflicts is time consistent, as aggregate consumption decisions are determined by a single discount rate δ_m. Moreover, since the aggregate consumption path associated with δ_m is also an optimal path for the median agent, the equilibrium is intertemporally efficient. In contrast to allocations determined by the Economics mechanism however, allocations determined by the Politics mechanism are not intratemporally efficient – the group's instantaneous welfare at any time t could be increased by redistributing aggregate consumption between the agents. Doing so however would require agents' consumption shares to vary with time, and thus lead us back to a time inconsistency problem.

Given that aggregate consumption is determined by the median agent's preferred policy

[15]The proof is no more difficult for an arbitrary concave production function $F(S)$ that admits an interior steady state. In this case the steady state value of consumption on a path $C(\delta)$ is given by $F((F')^{-1}(\delta))$, which by the concavity of F, is again a non-increasing function of δ. The rest of the proof goes through unchanged.

[16]Consider a discrete time, finite horizon version of our model. Then by using backwards induction the problem reduces to proving single peakedness for a sequence of static consumption choices, rather than for a full dynamic plan (Boylan & McKelvey, 1995). Taking an infinite horizon and infinitesimal time step limit of this result extends it to our continuous time model.

we can calculate agent i's welfare under Politics (see Appendix A.2):

$$W_i^P(s_i) = \begin{cases} \frac{\eta}{1-\eta}\left[\frac{S_0}{\eta}\alpha_m s_i\right]^{1-\eta}\frac{1}{\alpha_m+\eta(\alpha_i-\alpha_m)} & \eta > 1 \\ \frac{1}{\delta_i}\ln\left(S_0\delta_m s_i\right) + \frac{1}{\delta_i^2}(r-\delta_m) & \eta = 1 \end{cases} \tag{22}$$

where $\alpha_m := \delta_m + (\eta - 1)r$.

Incentive compatibility imposes constraints on the choice of income shares $\{s_i\}$ in the political mechanism. As in the Economics mechanism, any income shares are feasible when agents' discount rates are public information. If the group is rational, it will choose these shares to maximize collective welfare computed with the welfare weights w_i, given that aggregate consumption will be determined by the median voter's preferred plan:

$$\max_{s_i}\sum_i w_i W_i^P(s_i) \text{ s.t. } \sum_i s_i = 1.$$

These optimal income shares can be computed explicitly from (22):

$$s_i^* = \frac{\left(w_i[\alpha_m + \eta(\alpha_i - \alpha_m)]^{-1}\right)^{1/\eta}}{\sum_i \left(w_i[\alpha_m + \eta(\alpha_i - \alpha_m)]^{-1}\right)^{1/\eta}} \tag{23}$$

These income shares are As Efficient As Possible (AEAP), given that control of aggregate consumption decisions has been given up.[17] The set of AEAP income shares is uniquely specified, given any set of welfare weights $\{w_i\}$. Crucially, the AEAP income shares are independent of the current level of the resource stock S_t. This means that the time consistency of the voting equilibrium is preserved with this choice of income shares. If we allowed revision of the choice of income shares at a later period they would not change.

Under *hidden information*, the choice of income shares is tightly constrained if incentive problems are to be avoided. Since the consumption allocated to each agent is Pareto ordered (higher income shares Pareto dominate lower income shares), the only way to ensure incentive compatibility is to choose $s_i = 1/N$ for all i. In this case, the median voter equilibrium is well known to be incentive compatible in dominant strategies (e.g. Moulin, 1980).[18]

[17] s_i^* is always real and positive if $r \geq \delta_m$ when $\eta \geq 1$.

[18] Note that even though agents' preferences over *aggregate* consumption are single peaked for any s_i, individual *allocations* are only incentive compatible when income shares are equal. This is different to the standard result on the strategy-proofness of the median voter equilibrium with single-peaked preferences. In the setup of the standard result all agents always receive the same outcome from a policy selection, whereas in our case agents' outcomes are differentiated for non-equal income shares.

5 Economics vs. Politics

In this section we compare the Economics and Politics mechanisms in each of the four versions of our model in Table 1. We will show that if commitment is possible Economics dominates Politics, under both full and hidden information. By contrast, Politics may well be preferred by the majority of agents, and yield higher group welfare than Economics, if commitment is not possible. In the case of no commitment, our task will be to characterize the conditions under which one mechanism trumps the other.

We will compare Economics with Politics using two metrics: which mechanism does the majority of agents prefer, and which mechanism leads to higher group welfare? The first criterion is an ordinal measure that reflects the group's political preferences – which method would win a majoritarian 'meta-vote' between the two alternatives? The second criterion is a cardinal measure that tells us which mechanism leaves the group better off, given a choice of welfare weights $\{w_i\}$.

5.1 Commitment

In the case of *full information*, it is trivially true that regardless of the choice of welfare weights $\{w_i\}$, the group will be better off under Economics than under Politics. In the Economics mechanism an unconstrained optimum can be implemented, which dominates all other allocations by definition. While we immediately know that Economics guarantees higher group welfare, the following result tells us something much stronger:

Proposition 5. *Commitment, full information: All agents strictly prefer the Economics mechanism to the Politics mechanism with AEAP income shares (23).*

Proof. See Appendix E. □

This result holds for *any* welfare weights $\{w_i\}$. Thus, not only does the Economics mechanism lead to higher group welfare, it also has the unanimous support of all agents.

Under *hidden information*, both the Economics mechanism and the Politics mechanism are tightly constrained. There is a unique set of welfare weights in the Economics mechanism, and a unique set of income shares in the Politics mechanism, that induce dominant strategy incentive compatible consumption allocations. Given these constraints on feasible allocations, it is *a priori* possible that if the group's welfare is evaluated using a set of weights w_i', it could be better off under Politics than Economics, despite the former's inefficiency. However, the following proposition rules out this possibility:

Proposition 6. *Commitment, hidden information: All agents strictly prefer the incentive compatible Economics mechanism to the incentive compatible Politics mechanism, except the median agent, who is indifferent.*

Proof. See Appendix F. $\square$

Once again, Economics receives unanimous support, regardless of the choice of welfare weights. Thus, the superiority of Economics over Politics holds under both full and hidden information versions of the model when intertemporal commitments are possible.

5.2 No commitment

The previous sub-section showed that Economics is a clear winner under the assumption that commitment is possible. We now consider a setting in which the group cannot commit to evaluating collective welfare with respect to social preferences at time $\tau = 0$. We again compare the two mechanisms in both full and hidden information scenarios, and show that the results we obtained under commitment are often reversed.

5.2.1 Full Information

In order to analyze the tradeoff between Economics and Politics in detail in this case it is necessary to specify the welfare weights w_i. We use a specific choice, which we will refer to as 'equitable' weights:

$$w_i = \frac{\delta_i}{\sum_j \delta_j}. \tag{24}$$

This choice ensures that if every agent is allocated the same constant consumption path $c_{it} = c$, each agent makes an equal contribution to group welfare. This follows since agent i's welfare on constant consumption paths is

$$W_i = \int_\tau^\infty U(c) e^{-\delta_i (t-\tau)} dt = U(c)/\delta_i. \tag{25}$$

Thus $w_i W_i$ is equal for all i on constant paths when the equitable welfare weights (24) are chosen.

To make initial analytical headway it is also helpful to specialize to the case $\eta = 1$, as the equilibrium condition (18) is analytically solvable in this case. Substituting $\eta = 1$ and (24) into the expression (23) for the AEAP income shares in the Politics model, we see that

$$s_i^* = \frac{1}{N}. \tag{26}$$

Thus, for this choice of welfare weights, the welfare maximizing income shares in the Politics mechanism distribute income equally amongst all agents.

Using this finding, the formula for the equitable welfare weights (24), and the expressions (20) and (22) for agent's welfare under Economics and Politics, we can calculate agents' individual welfares, and the group's collective welfare, under the two approaches. The results of these calculations are summarized in the following proposition:

Proposition 7. *No commitment, full information: Assume that $\eta = 1$, and equitable welfare weights (24). Define*

$$z_i := \delta_i/\delta_m. \tag{27}$$

1. *Politics is preferred to Economics by a majority of agents if and only if $\langle z_i \rangle > 1$ (i.e. the δ_i are positively skewed). If $\langle z_i \rangle = 1$ (i.e. the δ_i are symmetrically distributed), the population is evenly split between the two mechanisms.*

2. *Group welfare is higher under Politics than under Economics if and only if*

$$\langle \ln z_i \rangle < (\langle z_i \rangle - 1)\langle z_i^{-1} \rangle. \tag{28}$$

3. *A mean-preserving spread in z_i **decreases** the difference between group welfare under Economics and group welfare under Politics.*

Proof. See Appendix G. □

The first point in the proposition show that the group's ordinal preferences between the two mechanisms are entirely captured by the skewness of the distribution of δ_i – at least a majority prefers Politics (Economics) if δ_i is positively (negatively) skewed. Welfare preferences between the two mechanisms are characterized by points 2 and 3 of the proposition. As condition (28) demonstrates, welfare comparisons depend on the distribution of δ_i in a complex manner in general. The following proposition shows however that for a large class of distributions sharper results are possible:

Proposition 8. *No commitment, full information: Assume that $\eta = 1$, equitable welfare weights, and that δ_i is log-symmetrically distributed. Then*

1. *The majority of agents prefers Politics to Economics.*

2. Group welfare is higher under Politics than Economics.

Proof. Since $\ln \delta_i$ is symmetrically distributed by assumption, $\ln z_i = \ln(\delta_i/\delta_m)$ is also symmetrically distributed. In addition, we must have $\langle \ln z_i \rangle = 0$. This follows since, by definition, the median of z_i is 1. Since $\ln z_i$ is symmetric, the median of z_i is also equal to $e^{\langle \ln z_i \rangle}$, and hence $\langle \ln z_i \rangle = 0$. Thus since $0 = \langle \ln z_i \rangle < \ln \langle z_i \rangle$ by the concavity of the log, we must have $\langle z_i \rangle > 1$. By Proposition 7.2, we conclude that the majority always prefers Politics. The condition (28), which determines when Politics yields higher group welfare, becomes

$$0 < (\langle z_i \rangle - 1)\langle z_i^{-1} \rangle.$$

But since $z_i > 0$ for all i, and we have shown $\langle z_i \rangle > 1$, this inequality is always satisfied. □

Thus for log-symmetric distributions of δ_i – a classic example being the log-normal distribution – the group would favor a Political approach to resolving intertemporal conflicts, both in welfare terms, and in a majoritarian meta-vote between the two mechanisms.

The feature of log-symmetric distributions that delivers this result is that they are always strongly positively skewed, with a long upper tail of values above the median. Figure 1 shows that this is an empirically plausible property. Large outliers are a significant presence for these distributions. To gain intuition for why Politics will tend to be favored for distributions with large outliers, it is helpful to study the group's representative time preferences under the two approaches. Substituting $\eta = 1$ into (18) and (19), we see that the group's aggregate consumption path in the Economics approach is equivalent to the optimal path of a time consistent agent with discount rate

$$\delta_{NC}|_{\eta=1} = \langle \delta_i^{-1} \rangle_{w_i}^{-1}. \tag{29}$$

With equitable welfare weights, this becomes

$$\delta_{NC}|_{\eta=1} = \langle \delta_i \rangle. \tag{30}$$

Thus aggregate behavior is controlled by the *mean* discount rate in the Economics approach, and the *median* discount rate in the Politics approach. It is well known that outliers have a disproportionately large effect on the mean of a distribution, but will have no effect on its median. Thus, if the distribution of discount rates is highly positively skewed, the representative discount rate in the Economics approach will be biased upwards relative to the median. By contrast, the Politics approach is comparatively unaffected by outliers –

the median is robust to their presence. Now consider how the two mechanisms determine intratemporal allocations: income shares are equal under Politics, but proportional to δ_i under Economics. Thus the Economics mechanism is doubly slanted towards high values of δ_i when the distribution is strongly positively skewed – both intra- and intertemporally. Since each agent's realized welfare contributes to group welfare roughly equally for equitable welfare weights, this bias means that Economics is too sensitive to the needs of agents with high δ_i.

This finding is further confirmed by point 3 of Proposition 7, which holds regardless of the distribution of δ_i. To understand this result note that implementing a mean-preserving spread in z_i requires us to hold $\langle \delta_i \rangle / \delta_m$ fixed. We are thus holding a measure of the skewness of the distribution of δ_i fixed, while increasing the spread in the distribution of δ_i/δ_m. A simple example of a transformation of the distribution of δ_i that implements this is as follows: Take an empirical sample of δ_i, divide each value by δ_m, apply a mean preserving spread to the result, and finally multiple the resulting values by δ_m. It is clear that this procedure pushes out the upper tails of the distribution of δ_i. Roughly speaking, as the proportion of 'outliers' in the sample increases (holding the ratio of the mean to the median fixed), Politics looks increasingly attractive relative to Economics.

Propositions 7.3 and 8 thus suggest that the more positively skewed the distribution of δ_i, the more the Politics approach provides a better means of achieving consensus on intertemporal plans, as it is less subject to the whims of large outliers. This will be born out in our empirical application in Section 6, which also considers the case $\eta > 1$.

5.2.2 Hidden information

If agents' discount rates are private information, allocations must be determined by choosing $w_i = 1/N$ in the Economics mechanism, and $s_i = 1/N$ in the Politics mechanism, if incentive problems are to be avoided. Thus, since agents receive equal constant income shares under both mechanisms, their preferences between the approaches depend only on the aggregate consumption plan that is implemented. Using this fact, we immediately obtain the following result, which holds for all values of the model parameters:

Proposition 9. *No commitment, hidden information: The majority of agents* ***always*** *prefers Politics to Economics.*

Proof. By Proposition 4, agents have single-peaked preferences over time consistent optimal aggregate consumption plans. Under Politics, agents' allocations are determined by an optimal plan corresponding to the median discount rate δ_m, and under Economics allocations are determined by an optimal plan corresponding to discount rate δ_{NC}. But the median plan is a Condorcet winner, so at least a majority of agents prefer it to any other time-consistent plan, including the plan corresponding to discount rate δ_{NC}. $\square$

Even though the majority of agents always prefers Politics, this doesn't mean that the group always realizes higher welfare under this mechanism, as the next proposition shows.

Proposition 10. *No commitment, hidden information: Set $\eta = 1$, and suppose that incentive compatible allocations are evaluated using welfare weights q_i. Then, Politics yields higher group welfare than Economics if and only if*

$$\left[\frac{\langle z_i^{-1}\rangle_{q_i}}{\langle z_i^{-2}\rangle_{q_i}}\right] \ln\langle z_i^{-1}\rangle > 1 - \langle z_i^{-1}\rangle^{-1} \tag{31}$$

In the case of equitable welfare weights ($q_i \propto \delta_i$), Economics always yields higher group welfare than Politics.

Proof. See Appendix H. $\square$

The proposition shows that the choice of $\eta = 1$ and equitable welfare weights for evaluating incentive compatible allocations is very special – Economics always yields higher welfare than Politics in this case. To understand this result note that with equitable weights, equation (29) shows that the group's representative discount rate is $\delta_{NC}|_{\eta=1} = \langle \delta_i^{-1}\rangle^{-1}$ under Economics, and the representative discount rate is again δ_m under Politics. Since $\langle \delta_i^{-1}\rangle^{-1}$ is a generalized mean, Economics is again more sensitive to outliers than Politics. However in the hidden information case aggregate consumption is necessarily split equally between agents in both mechanisms – there is no 'double bias' in the Economics approach. This can mean that sensitivity to outliers is a good thing – it can shift the aggregate consumption path in a welfare enhancing direction.

There is however a complex interaction between the fact that more weight is placed on the welfare of agents with high δ_i, that agents with high δ_i have intrinsically lower welfare values when consumption shares are equal, and the different representative discount rates that the two mechanisms prescribe. Only for equitable welfare weights do things work out so that, regardless of the distribution of δ_i, Economics is always preferred. To illustrate the non-robustness of this result, consider an example in which δ_i is log-normally distributed, and the weights q_i are chosen proportional to δ_i^2. In this case one can show that Politics will always yield higher welfare than Economics.[19] The dominance of Economics is also highly non-robust to changes in the value of η. When $\eta > 1$, the group's welfare is often higher under Politics even for equitable welfare weights. We demonstrate this empirically in the next section. Nevertheless, these results do suggest that in the hidden information case the contest between Economics and Politics will be tighter than in the full information case, even for strongly positively skewed distributions. This will be born out in our empirical analysis.

6 Empirical Application

In order to move beyond the case $\eta = 1$, which was required for some of the results in the previous section, it is necessary to use numerical methods to solve the equilibrium condition (18) in the version of the Economics approach without commitment, given an empirical distribution of discount rates. The distribution of discount rates we will use is taken from the Drupp et al. (2014) survey. Economists who published on social discounting since the year 2000 were asked for their opinions as to the appropriate values of the parameters of the social discount rate, including the pure rate of time preference. 180 responses were collected, and the distribution of opinions is shown in Figure 1.

To begin our analysis of the data, we plot the group's representative discount rate in each of the four versions of our model, assuming that the δ_i are distributed according to the distribution in Figure 1, and for $\eta = 2, r = 2\%/\text{yr}$, in Figure 2. The figure shows the declining term structure of the representative discount rate under commitment, and the constant representative rates δ_{NC} without commitment, assuming equitable welfare weights. The group is more patient under hidden information than under full information in both commitment scenarios. It is straightforward to understand this finding in the case of no commitment. In this case, the welfare weights are proportional to δ_i under full information, and uniform under hidden information. Thus more weight is placed on high discount rates, and less on low discount rates, under full information.[20] This causes the group to be more impatient under full information. The result is more complex in the case of commitment, as there is no simple stochastic dominance relationship between the equitable welfare weights (24) and the incentive compatible weights (13). Empirically,

[19] Log-normal distributions are log-symmetric, so by Proposition 8 we know that $\langle z_i \rangle > 1$. For $q_i \propto z_i^2$, $\frac{\langle z_i^{-1}\rangle_{q_i}}{\langle z_i^{-2}\rangle_{q_i}} = \langle z_i \rangle$. For log-normal distributions we also know that $\langle z_i^{-1} \rangle = \langle z_i \rangle > 1$, so that $\ln\langle z_i^{-1}\rangle > 0$. Substituting these relationships into (31), and using the fact that $\ln x > 1 - x^{-1}$ for all x, yields this result.

[20] The two sets of weights are ranked by the monotone likelihood ratio order.

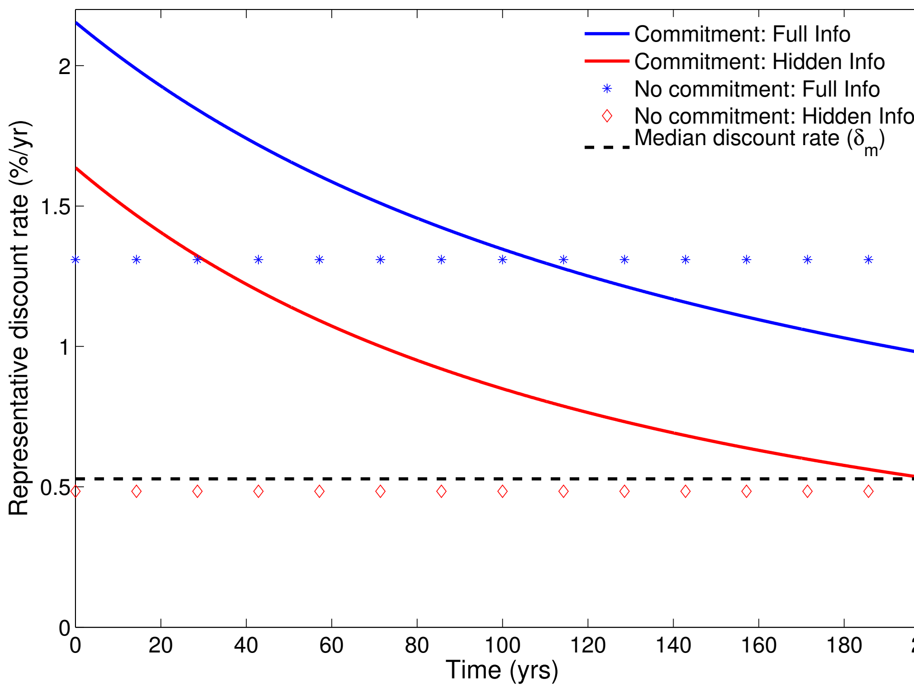

Figure 2: Representative discount rates for aggregate consumption decisions, both with and without commitment, for $\eta = 2, r = 2\%/\text{yr}$. Equitable welfare weights (24) were used under full information, incentive compatible welfare weights (13) under hidden information with commitment, and equal welfare weights under hidden information without commitment. Economics yields higher group welfare under both informational scenarios with commitment, and Politics yields higher group welfare under both informational scenarios without commitment.

we find that the incentive compatible weights are greater than the equitable weights for $\delta_i \in [0, 0.65\%] \cup [6.17\%, \infty)$, while the converse is true for $\delta_i \in [0.065\%, 6.17\%]$, when $\eta = 2, r = 2\%/\text{yr}$. Thus the hidden information welfare weights emphasize low and very high discount rates, and down-weight intermediate values, relative to the full information case. Since there are many more low discount rates than very high discount rates in the sample (the median is $0.53\%/\text{yr}$), this suggests that the hidden information case will correspond to less collective impatience than the full information case, as is born out in Figure 2.

From Propositions 5 and 6, we know that all agents prefer Economics to Politics under commitment. Without commitment, Proposition 7 showed that when $\eta = 1$, and with equitable welfare weights (24), the majority of agents will prefer Politics if the distribution

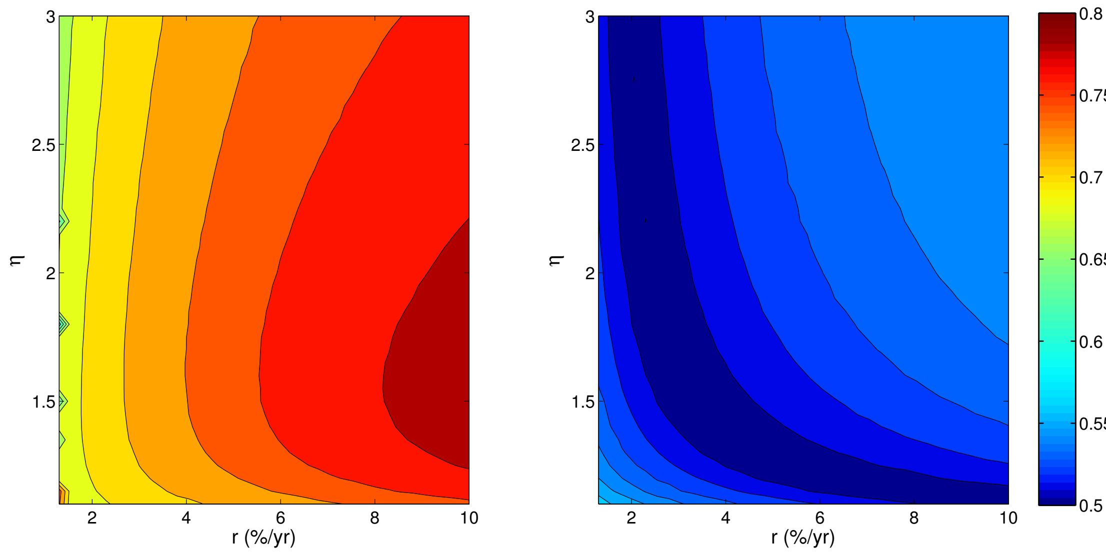

Figure 3: No commitment: Proportion of agents who prefer Politics to Economics.

of δ_i is positively skewed. The empirical distribution of δ_i is strongly positively skewed ($\delta_m = 0.53\%/\text{yr} < \langle \delta_i \rangle = 1.15\%/\text{yr}$), so we might expect this result to continue to hold. Proposition 9 showed that the majority of agents always prefers Politics when information is hidden. Figure 3 plots the percentage of agents who prefer Politics in both full and hidden information versions of the model, for a range of values of r and η. A significant supermajority prefers Politics in the full information version of the model for all parameter values. Politics is also always preferred by the majority in the hidden information model, as we expect, but the contest between the mechanisms is tighter in this case. The ordinal preferences of the group are nevertheless clear: Politics would always win a 'meta-vote' between the two approaches if commitment is not possible.

We also find empirically that Politics leads to higher group welfare for all values of η and r when commitment is not possible but full information is available about agents' preferences. Thus the analytical result we obtained in Proposition 8 for log-symmetric distributions and $\eta = 1$ continues to hold for our empirical sample, and when $\eta > 1$. Figure 4 demonstrates this by plotting the percentage welfare loss, relative to the efficient allocation under commitment, of the allocations under Economics without commitment, and Politics. Politics always achieves a lower welfare loss. Both mechanisms perform better relative to the efficient allocation for larger values of r.

Finally, Figure 5 plots the regions of the (η, r) parameter space where Politics yields higher welfare under hidden information and no commitment, assuming that allocations are evaluated using equitable welfare weights. The tradeoff between the two mechanisms is tighter in this case, with Economics yielding higher welfare for some parameter values, and Politics yielding higher welfare for others. Nevertheless, in contrast to the findings of Proposition 10 with $\eta = 1$, Politics does better for many empirically plausible values of η and r (e.g. $\eta = 1.5, r \approx 1 - 4\%/\text{yr}$, see Drupp et al. (2014)).

7 Conclusions

Collective decisions that involve groups of people with heterogeneous time preferences must face up to an inherent tension between efficiency and time consistency if it is not possible to make unbreakable commitments (Jackson & Yariv, 2012). Yet such decisions are encountered in a host of real world contexts, from the smallest scales (families) to the largest (countries). The democratic requirement that collective decisions should not be executed by a *de facto* dictator leads us to seek methods of reaching agreement that reflect the distribution of individuals' views, despite the acknowledged difficulties of doing so. Our paper has proposed two such methods, each rooted in long traditions in Economics and Politics respectively. We have analyzed which of these approaches would be preferred both when agents' preferences are known and when they are not, and commitment to intertemporal plans is, and is not, possible. The findings are stark: Economics trumps Politics if commitment is possible. Without commitment, under many empirically plausible model specifications, Politics trumps Economics.

These results have applications in many group decision problems. Take as an example the problem of choosing a social (i.e. consumption) discount rate for public project appraisal. This parameter is probably the single most important economic input to many public investment decisions. Many countries (including e.g. the USA, EU member states, China, and India), have project evaluation guidelines that stipulate its value. These rules affect billions of dollars of investment annually, and play an important role in determining the set of public assets our generation will leave for the next. Owing to the restrictive perfect market assumptions that are necessary to identify market interest rates with social shadow prices, many government bodies (e.g. the US Congressional Budget Office, the Environmental Protection Agency, and the UK Treasury) use explicit welfare computations to set the social discount rate via the famous Ramsey formula (see e.g. Gollier, 2012). The pure rate of time preference is a crucial input to this formula, and can have a significant im-

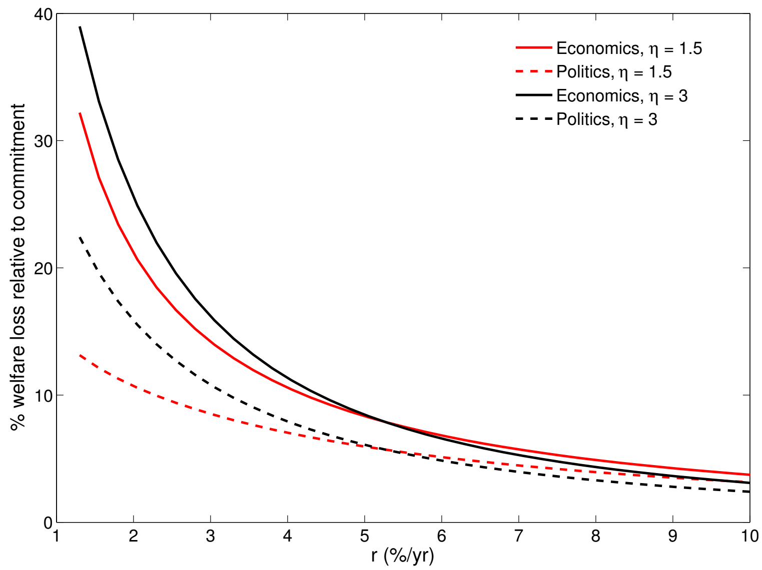

Figure 4: No commitment, Full information: Group welfare under Economics and Politics. Politics yields higher welfare for all parameter values, but here we show results for two sample values of η.

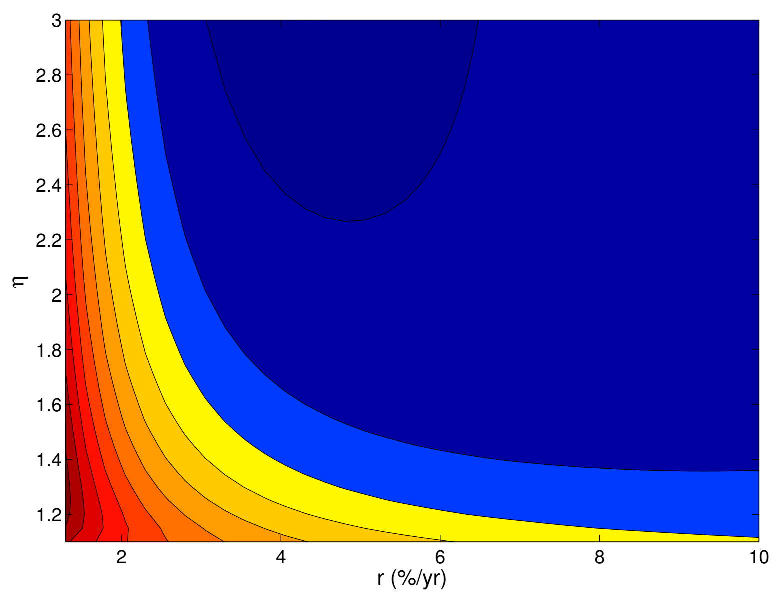

Figure 5: No commitment, Hidden Information: Group welfare under Economics and Politics. Warm colors are where Politics yields higher welfare, cool colors where Economics yields higher welfare. Incentive compatible allocations are evaluated using the equitable welfare weights (24).

pact on a country's choice of social discount rate. Yet until now, prescriptions for its value have been based on dictatorial value judgements, with one form of ethical reasoning given precedence over all others. Given the importance of this choice for intertemporal resource allocation, it would seem natural for governments to attempt to represent a variety of legitimate viewpoints, rather than putting all their ethical eggs in a single basket (Sen, 2010). Doing so requires an engagement with the difficulties of temporal preference aggregation, and a method for cutting through them. Our paper offers precisely such methods, and allows us to select the 'best' approach, given the nature of the decision problem at hand. In common resource settings, and when the government cannot commit (which is often the relevant case), our analysis suggests that it may be appropriate to adopt a representative pure rate of time preference of roughly 0.5%/yr. This is the median discount rate from the Drupp et al. (2014) study, which corresponds to the equilibrium of the Politics approach. We have shown that this leads to higher group welfare than the Economics approach for all model parameters under full information, and also for empirically plausible parameter values under hidden information. Similar exercises could inform the management of sovereign wealth funds, and natural resources.

A Agents' welfare under the Economics and Politics mechanisms

A.1 Economics: Commitment

1)The Hamiltonian of the planner's problem with commitment is:

$$H = \sum_i w_i U(c_{it}) e^{-\delta_i t} + \lambda_t (r S_t - \sum_i c_{it}) \tag{32}$$

where λ_t is the shadow price of the resource, and S_t evolves according to

$$\dot{S}_t = r S_t - \sum_i c_{it} \tag{33}$$

A standard application of the Maximum principle yields

$$w_i U'(c_{it}) e^{-\delta_i t} = \lambda_t \tag{34}$$

$$\dot{\lambda}_t = -r \lambda_t. \tag{35}$$

Solving the equation for λ_t, and using the functional form (1) for the utility function, we have

$$c_{it} = \left(\frac{w_i}{\lambda_0} e^{-(\delta_i - r)t} \right)^{1/\eta} \tag{36}$$

where λ_0 is the initial shadow price, which we need to solve for. With this solution we can write the evolution equation for the stock in equilibrium as:

$$\dot{S}_t - r S_t = - \sum_i \left(\frac{w_i}{\lambda_0} e^{-(\delta_i - r)t} \right)^{1/\eta} \tag{37}$$

Multiplying through by an integration factor e^{-rt} and integrating from 0 to t:

$$S_t e^{-rt} - S_0 = - \int_0^t e^{-rt} \sum_i \left(\frac{w_i}{\lambda_0} e^{-(\delta_i - r)t} \right)^{1/\eta} dt \tag{38}$$

where S_0 is the initial resource stock. The transversality conditions on these solutions require:

$$\lim_{t\to\infty} S_t \lambda_t = \lim_{t\to\infty} S_t \lambda_0 e^{-rt} = 0 \tag{39}$$

Hence, the initial value of the shadow price λ_0 must satisfy:

$$S_0 = \int_0^\infty e^{-rt} \sum_i \left(\frac{w_i}{\lambda_0} e^{-(\delta_i - r)t} \right)^{1/\eta} dt \tag{40}$$

from which we find

$$\lambda_0 = \left(\frac{\eta}{S_0} \sum_i \frac{w_i^{1/\eta}}{\delta_i + (\eta - 1)r} \right)^{\eta} \tag{41}$$

2)Computing agent i's equilibrium welfare for $\eta \neq 1$, we find

$$\begin{aligned} W_i &= \int U(c_{it}) e^{-\delta_i t} dt \\ &= \frac{\eta}{1-\eta} \left(\frac{w_i}{\lambda_0} \right)^{\frac{1-\eta}{\eta}} \frac{1}{\delta_i + (\eta - 1)r} \\ &= \frac{\eta}{1-\eta} \left[\frac{S_0}{\eta} \tilde{w}_i \langle \alpha_i^{-1} \rangle_{\tilde{w}_i}^{-1} \right]^{1-\eta} \frac{1}{\alpha_i}. \end{aligned}$$

where in the last line we have substituted in for λ_0 from (41), and used the definitions of α_i and $\tilde{w}_i$. A similar calculation yields the result for $\eta = 1$.
3) For a proof of this result see Heal & Millner (2013).

A.2 Politics

To solve for the agents' equilibrium welfare under Politics, notice that agents' income shares s_i don't affect their optimal aggregate consumption plans. To find the optimal plan of the median agent, we may thus simply set $w_m = 1, w_i = 0, i \neq m$ in (9) to find that aggregate consumption is given by

$$C_t = \frac{S_0 \alpha_m}{\eta} \left(e^{-(\delta_m - r)t} \right)^{1/\eta}. \tag{42}$$

The welfare agent i realizes in the political equilibrium is

$$W_i^P = \int_0^\infty U(s_i C_t) e^{-\delta_i t} dt. \tag{43}$$

Computing the integral explicitly using the utility function (1) yields (22).

A.3 Economics: No commitment

In the version of the Economics mechanism without commitment, we know that in equilibrium agent i receives a share $\tilde{w}_i$ of aggregate consumption, and that the aggregate consumption path corresponds to the optimal path of a time-consistent agent with discount rate δ_{NC}. Thus the welfare agent i achieves in this mechanism is identical to her welfare under Politics, except that we need to send $s_i \to \tilde{w}_i$, and $\delta_m \to \delta_{NC}$ in (22). Thus agent i's welfare in the no commitment model for $\eta > 1$ is simply

$$W_i^{NC} = \frac{\eta}{1-\eta} \left[\frac{S_0}{\eta} \tilde{w}_i \alpha_{NC} \right]^{1-\eta} \frac{1}{\alpha_{NC} + \eta(\alpha_i - \alpha_{NC})}. \tag{44}$$

When $\eta = 1$ the equilibrium condition (18) may be solved explicitly, to find

$$A = \langle \delta_i^{-1} \rangle_{w_i}^{-1}. \tag{45}$$

Substituting $\eta = 1$ into (19), we see that $\delta_{NC} = \langle \delta_i^{-1} \rangle_{w_i}^{-1}$. Using this expression in (22), and sending $s_i \to w_i$, $\delta_m \to \delta_{NC}$ yields the result for $\eta = 1$.

B Proof of Proposition 2

We know from (9) that an agent who announces discount rate δ_i will receive allocation

$$c_{it} = \left(\frac{w_i}{\lambda_0} e^{-(\delta_i - r)t} \right)^{1/\eta} \tag{46}$$

where λ_0 is the initial shadow price. The welfare an agent with discount rate δ' receives from this allocation is:

$$W(\delta_i, \delta') := \int U(c_{ti}) e^{-\delta' t} dt = \frac{1}{1-\eta} \left(\frac{w_i}{\lambda_0} \right)^{\frac{1-\eta}{\eta}} \frac{\eta}{(r - \delta_i)(\eta - 1) + \eta \delta'}. \tag{47}$$

Consider the factor w_i/λ_0 in (47):

$$\frac{w_i}{\lambda_0} = \frac{w_i}{\left[\frac{\eta}{S_0}\sum_i \frac{w_i^{1/\eta}}{\delta_i+(\eta-1)r}\right]^\eta} \tag{48}$$

Notice that the welfare weights only enter (47) through this factor, which is homogeneous of degree zero in the w_i, and thus independent of the normalization of the welfare weights. For the choice of w_i in (13), we have

$$\frac{w_i}{\lambda_0} \propto (\delta_i + (\eta - 1)r)^\eta. \tag{49}$$

Plug (49) into (47), to find that

$$W(\delta_i, \delta') \propto \frac{1}{1-\eta}\frac{(\delta_i + (\eta-1)r)^{1-\eta}}{(r-\delta_i)(\eta-1)+\eta\delta'}, \tag{50}$$

where the proportionality constant is positive. Differentiating explicitly with respect to δ_i, one finds that

$$\text{sgn}\frac{\partial W}{\partial \delta_i} = \text{sgn}(\delta' - \delta_i) \tag{51}$$

This implies a unique maximum at $\delta_i = \delta'$, and hence we have incentive compatibility in dominant strategies.

To see that this solution is the unique set of weights that induces incentive compatibility in dominant strategies, notice from (48) that the choice of welfare weights (13) is the only one that will make each individual's realized welfare independent of everyone else's announced discount rate. For any other choice agent i's realized welfare depends not only on his announced discount rate δ_i and his true discount rate δ', but also on the discount rate announcements of other agents. This means that dominant strategy implementation of an efficient allocation cannot be possible.

C Proof of Proposition 3

1) Let the aggregate consumption policy function be $C_t = \sigma(S_t) = AS_t$. Suppose that planners from $t \in [\tau + \epsilon, \infty)$ follow strategy $\sigma(S)$. The planner at τ's welfare from this is just

$$V(C_\tau, \epsilon, A) = \int_{\tau+\epsilon}^{\infty} \tilde{U}(AS_t)\beta(t)dt \tag{52}$$

where S_t is the solution of the differential equation

$$\dot{S} = rS - AS;\ S(\tau+\epsilon) = S_\epsilon \tag{53}$$

$$\Rightarrow S_t = S_\epsilon e^{(r-A)t}, \tag{54}$$

and S_ϵ is the stock the current planner bequeaths to his future self at $t = \tau + \epsilon$. Using the state equation, and assuming that ϵ is small, we find

$$S_\epsilon \approx S_\tau(1 + \epsilon(r - C_\tau/S_\tau)). \tag{55}$$

A straightforward calculation then shows that

$$V(C_\tau, \epsilon, A) = \frac{(\sum w_i^{1/\eta})^{\eta-1}}{1-\eta}(AS_\epsilon)^{1-\eta}\sum_i w_i^{1/\eta}\frac{e^{-[\delta_i+(\eta-1)(r-A)]\epsilon}}{\delta_i + (\eta-1)(r-A)} \tag{56}$$

The planner's total welfare is

$$\int_\tau^\infty \tilde{U}(C_t)\beta(t)dt \tag{57}$$

$$= \int_\tau^{\tau+\epsilon} \tilde{U}(C_t)\beta(t)dt + \int_{\tau+\epsilon}^\infty \tilde{U}(C_t)\beta(t)dt \tag{58}$$

$$\approx \epsilon\tilde{U}(C_\tau) + V(C_\tau, \epsilon, A) \tag{59}$$

where the approximation becomes exact as $\epsilon \to 0$. We wish to solve for the optimal C_τ in the limit as $\epsilon \to 0$. We can expand $V(C_\tau, \epsilon, A)$ in powers of ϵ as follows:

$$V(C_\tau, \epsilon, A) = V_0 + \epsilon\left.\frac{\partial V}{\partial \epsilon}\right|_{\epsilon=0} + \mathcal{O}(\epsilon^2)$$

Since the contribution of C_τ to welfare in the current period is first order in ϵ, we care only about the part of $V(C_\tau, \epsilon, A)$ which is also first order in ϵ, and which depends on C_τ. Computing the derivative, evaluating at $\epsilon = 0$, and keeping only the terms that depend on C_τ, we find that

$$\left.\frac{\partial V}{\partial \epsilon}\right|_{\epsilon=0} \sim -C_\tau A(AS_\tau)^{-\eta}(\sum_i w_i^{1/\eta})^{\eta-1}\sum_i \frac{w_i^{1/\eta}}{\delta_i + (\eta-1)(r-A)}.$$

Thus in the limit as $\epsilon \to 0$, C_τ must be chosen such that

$$C_\tau = \text{argmax}\left[(\sum_i w_i^{1/\eta})^\eta \frac{(C_\tau)^{1-\eta}}{1-\eta} - C_\tau A(AS_\tau)^{-\eta}(\sum_i w_i^{1/\eta})^{\eta-1}\sum_i \frac{w_i^{1/\eta}}{\delta_i + (\eta-1)(r-A)}\right]$$
$$\Rightarrow C_\tau = AS_\tau[A\langle(\delta_i + (\eta-1)(r-A))^{-1}\rangle_{\tilde{w}_i}]^{-1/\eta}$$

In equilibrium, $C_\tau = AS_\tau$, which implies that the equilibrium condition for A is:

$$\left\langle \frac{A}{\delta_i + (\eta-1)(r-A)} \right\rangle_{\tilde{w}_i} = 1. \tag{60}$$

2) Straightforward calculations show that the equilibrium aggregate consumption path will be

$$C_t^{NC} = S_0 A \exp\left[-(A-r)t\right]. \tag{61}$$

Similar calculations show that the optimal consumption path of a time consistent planner with discount rate δ will be

$$C_t^\delta = S_0\left[\frac{\delta + (\eta-1)r}{\eta}\right]\exp\left(-\left(\left[\frac{\delta+(\eta-1)r}{\eta}\right] - r\right)t\right). \tag{62}$$

Solving for δ such that $C_t^\delta = C_t^{NC}$ for all t yields the result 3).

3) See Appendix A.3.

D Proof of Lemma 1

Let the optimal aggregate consumption path for an individual with discount rate δ be $C(\delta) = (c_t^\delta)_{t\geq 0}$, and let her (fixed) income share be s. Denote the preferences over aggregate consumption paths of an agent with discount rate δ by $\prec_\delta$.

We first prove that under the conditions of the lemma, given any pair of discount rates $\delta' < \delta''$, for any $\delta < \delta'$ we must have $C(\delta'') \prec_\delta C(\delta')$, and for any $\delta > \delta''$, we must have $C(\delta') \prec_\delta C(\delta'')$. Consider the case $\delta > \delta''$, and let $\delta = \delta'' + \epsilon$, where $\epsilon > 0$. We will evaluate the difference in welfare of agent δ under the two consumption paths $C(\delta')$ and $C(\delta'')$. Let

T be the intersection point of the two consumption streams. We have:

$$
\begin{aligned}
&\int_0^\infty U(sc_t^{\delta''})e^{-\delta t}dt - \int_0^\infty U(sc_t^{\delta'})e^{-\delta t}dt \\
&= \int_0^T [U(sc_t^{\delta''}) - U(sc_t^{\delta'})]e^{-\delta t}dt - \int_T^\infty [U(sc_t^{\delta'}) - U(sc_t^{\delta''})]e^{-\delta t}dt \\
&= \int_0^T [U(sc_t^{\delta''}) - U(sc_t^{\delta'})]e^{-\delta'' t}e^{-\epsilon t}dt - \int_T^\infty [U(sc_t^{\delta'}) - U(sc_t^{\delta''})]e^{-\delta'' t}e^{-\epsilon t}dt \\
&\geq e^{-\epsilon T}\int_0^T [U(sc_t^{\delta''}) - U(sc_t^{\delta'})]e^{-\delta'' t}dt - e^{-\epsilon T}\int_T^\infty [U(sc_t^{\delta'}) - U(sc_t^{\delta''})]e^{-\delta'' t}dt \\
&= e^{-\epsilon T}\int_0^\infty [U(sc_t^{\delta''}) - U(sc_t^{\delta'})]e^{-\delta'' t}dt \\
&\geq 0.
\end{aligned}
$$

The last inequality follows from the optimality of $C(\delta'')$. Thus we have shown that for any $\delta > \delta''$, $C(\delta') \prec_\delta C(\delta'')$. A similar argument shows that for any $\delta < \delta'$, we must have $C(\delta'') \prec_\delta C(\delta')$.

It is straightforward to see that with these properties in hand, agents' preferences over consumption paths must be single peaked. Consider two paths $C(\delta')$ and $C(\delta'')$, and an agent with discount rate δ, where $\delta < \delta' < \delta''$. From the above properties, we must have $C(\delta'') \prec_\delta C(\delta')$. Similarly, for any paths $C(\delta')$ and $C(\delta'')$ with $\delta'' < \delta' < \delta$, we must have $C(\delta'') \prec_\delta C(\delta')$. Thus all agents' preferences are single peaked.

E Proof of Proposition 5

Using (11) and (22), the ratio of agent i's welfare under Economics to his welfare under Politics is

$$R_i = \frac{W_i^E}{W_i^P} = \left(\alpha_m \langle \alpha_j^{-1} \rangle_{\tilde{w}_j}\right)^{\eta-1} \left(\frac{s_i^*}{\tilde{w}_i}\right)^{\eta-1} \frac{\alpha_m + \eta(\alpha_i - \alpha_m)}{\alpha_i} \tag{63}$$

Using the expression (23) for s_i^*, we have

$$\frac{s_i^*}{\tilde{w}_i} = \left[\frac{(w_i[\alpha_m + \eta(\alpha_i - \alpha_m)]^{-1})^{1/\eta}}{\sum_j (w_j[\alpha_m + \eta(\alpha_j - \alpha_m)]^{-1})^{1/\eta}}\right] \frac{\sum_i w_j^{1/\eta}}{w_i^{1/\eta}} \tag{64}$$

$$= [\alpha_m + \eta(\alpha_i - \alpha_m)]^{-1/\eta} \langle [\alpha_m + \eta(\alpha_j - \alpha_m)]^{-1/\eta} \rangle_{\tilde{w}_j}^{-1} \tag{65}$$

Substituting this expression back into (63) and simplifying, we find

$$R_i = \frac{[\alpha_m + \eta(\alpha_i - \alpha_m)]^{1/\eta}}{\alpha_i} \left(\alpha_m \langle \alpha_j^{-1} \rangle_{\tilde{w}_j} \langle [\alpha_m + \eta(\alpha_j - \alpha_m)]^{-1/\eta} \rangle_{\tilde{w}_j}^{-1}\right)^{\eta-1} \tag{66}$$

Consider the first factor in this expression, which depends on α_i. Differentiating it with respect to α_i, one can show that

$$\operatorname{sgn} \frac{d}{d\alpha_i} \frac{[\alpha_m + \eta(\alpha_i - \alpha_m)]^{1/\eta}}{\alpha_i} = \operatorname{sgn}(\alpha_m - \alpha_i) \tag{67}$$

Thus this factor attains its maximum value at $\alpha_i = \alpha_m$. The second factor in (66) is common for all i, so if we can show that $R_i < 1$ when $i = m$, we will be done. Note that, since $\eta > 1$, all welfare integrals are negative, which means that $R_i < 1$ implies agent i prefers Economics to Politics.

Substituting $\alpha_i = \alpha_m$ into (66), we find

$$R_m = \left(\alpha_m^{1-1/\eta} \langle \alpha_j^{-1} \rangle_{\tilde{w}_j} \langle [\alpha_m + \eta(\alpha_j - \alpha_m)]^{-1/\eta} \rangle_{\tilde{w}_j}^{-1}\right)^{\eta-1} \tag{68}$$

Thus $R_m < 1$ if

$$\left\langle \frac{\alpha_m}{\alpha_j} \right\rangle_{\tilde{w}_j} < \left\langle \left(\frac{\alpha_m}{\alpha_m + \eta(\alpha_j - \alpha_m)} \right)^{1/\eta} \right\rangle_{\tilde{w}_j} \tag{69}$$

For this inequality to be satisfied it is clearly sufficient for

$$\frac{\alpha_m}{\alpha_j} \leq \left(\frac{\alpha_m}{\alpha_m + \eta(\alpha_j - \alpha_m)} \right)^{1/\eta} \tag{70}$$

$$\iff \alpha_j^\eta - \alpha_m^{\eta-1}(\alpha_m + \eta(\alpha_j - \alpha_m)) \geq 0 \tag{71}$$

for all j, with the inequality being strict for at least one j. When $\alpha_j = \alpha_m$, the left hand side of (71) is zero, so the inequality is satisfied. Now differentiate the left hand side with respect to α_j, to find

$$\frac{d}{d\alpha_j}[\alpha_j^\eta - \alpha_m^{\eta-1}(\alpha_m + \eta(\alpha_j - \alpha_m))] = \eta(\alpha_j^{\eta-1} - \alpha_m^{\eta-1}). \tag{72}$$

The sign of the derivative is positive for $\alpha_j > \alpha_m$, and negative for $\alpha_j < \alpha_m$. Thus the left hand side of (71) achieves its global minimum at $\alpha_j = \alpha_m$, at which its value is zero. (71) is thus satisfied for all j, and strictly satisfied for any $j \neq m$. This establishes the result.

The case $\eta < 1$ follows analogously, however in this case we need to worry about the convergence of welfare integrals. We require $\delta_i + (\eta - 1)r > 0$ for all i in order to be able to compare all agents' welfare under the two mechanisms. If this condition is satisfied, the proof goes through with only minor modifications, noting that now the Economics mechanism is preferred if $R_i > 1$. If there are some agents with $\delta_i + (\eta - 1)r < 0$ their welfare is undefined under both mechanisms, so we cannot determine their preferences. To avoid these complications we focus on $\eta \geq 1$.

F Proof of Proposition 6

With the choice of welfare weights in (13), we have $\tilde{w}_i \langle \alpha_i^{-1} \rangle_{\tilde{w}_i} = \alpha_i / N$. Thus from (11), agent i's welfare under the Economics mechanism is

$$W_i^E = \frac{\eta}{1-\eta} \left(\frac{S_0 \alpha_i}{N\eta} \right)^{1-\eta} \frac{1}{\alpha_i} \tag{73}$$

Substituting $s_i = 1/N$ into (22), we find that under Politics agent i's welfare is

$$W_i^P = \frac{\eta}{1-\eta} \left(\frac{S_0 \alpha_m}{N\eta} \right)^{1-\eta} \frac{1}{\alpha_m + \eta(\alpha_i - \alpha_m)}. \tag{74}$$

Let R_i be the ratio of agent i's welfare under Economics to her welfare under Politics in the incentive compatible case:

$$R_i = \frac{W_i^E}{W_i^P} = \left(\frac{\alpha_i}{\alpha_m} \right)^{1-\eta} \frac{\alpha_m + \eta(\alpha_i - \alpha_m)}{\alpha_i} \tag{75}$$

It is clear by inspection that $R_i = 1$ if $\alpha_i = \alpha_m$. Now

$$\frac{\partial R_i}{\partial \alpha_i} = -\frac{\eta(\eta-1)(\alpha_i - \alpha_m)(\alpha_m/\alpha_i)^\eta}{\alpha_i \alpha_m}. \tag{76}$$

Now since $\alpha_i, \alpha_m > 0$ when $\eta > 1$, this means that

$$\mathrm{sgn} \frac{\partial R_i}{\partial \alpha_i} = \mathrm{sgn}\left(\alpha_m - \alpha_i\right). \tag{77}$$

Thus, when $\eta > 1$, R_i has a global maximum at $\alpha_i = \alpha_m$ (i.e. $\delta_i = \delta_m$), at which it takes the value 1. This implies $R_i \leq 1$ for all i. Since welfare values are *negative* for $\eta > 1$, if $R_i \leq 1$ for all i, the Economics mechanism is preferred by everyone expect the median agent, who is indifferent. The case $\eta < 1$ follows analogously, with the same caveats as required in Appendix E.

G Proof of Proposition 7

1) Substituting the equitable welfare weights (24) and $\eta = 1$ into (20) we have:

$$W_i^{NC} = \frac{1}{\delta_i} \ln \left(\frac{S_0 \delta_i}{N} \right) + \frac{1}{\delta_i^2} (r - \langle \delta_i \rangle). \tag{78}$$

Similarly, substituting $s_i = 1/N$ and $\eta = 1$ into (22) yields:

$$W_i^P = \frac{1}{\delta_i} \ln \left(\frac{S_0 \delta_m}{N} \right) + \frac{1}{\delta_i^2} (r - \delta_m). \tag{79}$$

Using (78) and (79), we can determine when agent i prefers his allocation under Politics to his allocation under Economics:

$$W_i^P \geq W_i^{NC} \iff \delta_i \ln \left(\frac{\delta_i}{\delta_m} \right) \leq \langle \delta_i \rangle - \delta_m. \tag{80}$$

To analyze this condition it is useful to work with the dimensionless variable $z_i = \delta_i / \delta_m$. The condition (80) then reduces to

$$\ln z_i \leq \frac{\langle z_i \rangle - 1}{z_i} \tag{81}$$

In what follows we make use of the identities:

$$\ln z \geq 1 - z^{-1} \tag{82}$$

$$\ln z \leq z - 1. \tag{83}$$

Using the identity (82) in (81), in order for agent i to prefer Economics to Politics, it is sufficient for

$$1 - z_i^{-1} \geq \frac{\langle z_i \rangle}{z_i} - z_i^{-1} \tag{84}$$

$$\iff z_i \geq \langle z_i \rangle. \tag{85}$$

If $\langle z_i \rangle < 1$, this means that at least a majority of agents will prefer Economics to Politics (since $z_i = 1$ is the median agent).

Next, using identity (83) in (81) implies that in order for agent i to prefer Politics to Economics, it is sufficient for

$$z_i - 1 \leq \frac{\langle z_i \rangle - 1}{z_i} \tag{86}$$

$$\iff z_i^2 - z + (1 - \langle z_i \rangle) \leq 0. \tag{87}$$

Assume that $\langle z_i \rangle > 1$. Since $z_i \geq 0$, this inequality is solved for any $z_i \in [0, z_+]$, where

$$z_+ = \frac{1 + \sqrt{1 + 4(\langle z_i \rangle - 1)}}{2} > 1 \tag{88}$$

Thus at least a majority will prefer Politics to Economics when $\langle z_i \rangle > 1$.

Finally, substituting $\langle z_i \rangle = 1$ into (81) shows that the population is evenly split between the two mechanisms in this case.

2) When does Politics lead to higher group welfare than Economics? Calculating the group's welfare under the two mechanisms we find:

$$V^{NC} = \sum_i w_i W_i^{NC} = \frac{1}{\sum_i \delta_i} \left(\sum_i \ln \left(\frac{S_0 \delta_i}{N} \right) + (r - \langle \delta_i \rangle) \sum_i \delta_i^{-1} \right) \tag{89}$$

$$V^{P} = \sum_i w_i W_i^{P} = \frac{1}{\sum_i \delta_i} \left(N \ln \left(\frac{S_0 \delta_m}{N} \right) + (r - \delta_m) \sum_i \delta_i^{-1} \right) \tag{90}$$

Hence we have

$$V^{NC} - V^{P} = \frac{1}{\langle \delta_i \rangle} \left(\langle \ln \delta_i \rangle - \ln \delta_m - \langle \delta_i^{-1} \rangle (\langle \delta_i \rangle - \delta_m) \right) \tag{91}$$

Politics thus leads to higher group welfare if and only if:

$$\langle \ln z_i \rangle < (\langle z_i \rangle - 1) \langle z_i^{-1} \rangle. \tag{92}$$

3) Consider the effect of a mean-preserving spread in z_i on the terms in (28). Since $\ln z$ is concave in z, the term on the left hand side decreases under a mean-preserving spread. The first factor on the right hand side of (28) is constant by assumption, and the term $\langle z_i^{-1} \rangle$ increases, since z^{-1} is convex in z. Thus we find that a mean-preserving spread in z_i

must necessarily *decrease* the welfare difference $V^{NC} - V^P$. Increasing the spread in the distribution of z_i thus increases the appeal of the Politics approach.

H Proof of Proposition 10

Setting $w_i = 1/N$ and $\eta = 1$ in (20), we find that agent i's welfare in the incentive compatible Economics mechanism is

$$W_i^{NC} = \frac{1}{\delta_i} \ln\left(\frac{S_0}{N}\langle\delta_i^{-1}\rangle^{-1}\right) + \frac{1}{\delta_i^2}(r - \langle\delta_i^{-1}\rangle^{-1}). \tag{93}$$

Equation (79) gives agent i's welfare under the incentive compatible Politics mechanism. Subtracting group welfare under Economics from group welfare under Politics, using welfare weights q_i, we find

$$\begin{aligned} V_P - V_{NC} &= \sum_i q_i(W_i^P - W_i^{NC}) \\ &= \langle\delta_i^{-2}\rangle_{q_i}(\langle\delta_i^{-1}\rangle^{-1} - \delta_m) - \langle\delta_i^{-1}\rangle_{q_i}\ln(\langle\delta_i^{-1}\rangle^{-1}\delta_m^{-1}) \end{aligned}$$

Using the definition $z_i = \delta_i/\delta_m$, some simple manipulations show that $V_P - V_{NC} > 0$ if and only if (31) is satisfied.

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