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Dynamic Supply Adjustment and Banking under Uncertainty: the Market Stability Reserve*

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March 2, 2016

Abstract

The supply of allowances in the European Union Emissions Trading System is currently determined by a rigid allocation programme. The Market Stability Reserve (MSR) makes the allocation of allowances flexible and contingent on the aggregate bank, while preserving the overall emissions cap. We investigate under which conditions the MSR can alter the abatement and allowance price paths. We show that, for risk-neutral firms a cap-preserving MSR is largely irrelevant. The MSR will not influence the expected equilibrium allowance price or average price volatility. We then relax the risk neutrality assumption and investigate how the MSR can impact the risk of an unexpected breakdown of inter-temporal optimisation, ultimately affecting the equilibrium at different points in time. In this context, we show that changes to the timing of allowance allocation by the MSR can have a substantial impact on abatement and allowance price paths when firms are risk-averse.

Keywords: EU ETS Reform; Market Stability Reserve; Policy Design; Responsiveness; Resilience; Supply Management Mechanism; Risk-Aversion.

*Part of Kollenberg's research was supported by the University of Duisburg-Essen. Part of Taschini's research was supported by the Centre for Climate Change Economics and Policy, which is funded by the UK Economic and Social Research Council (ESRC). This project has received funding from the Stiftung Mercator and from the European Union's Seventh Programme for research, technological development and demonstration under grant agreement No 308481. Furthermore, the project was supported by the German Federal Ministry of Education and Research under promotional reference No 01LA1115A. The authors would like to thank Dallas Burtraw, Corina Comendant, Harrison Fell, Carolyn Fischer, Rüdiger Kiesel, Andrei Marcu, Steve Salant, Audun Sætherø, Peter Vis, Peter Zapfel, and other participants of the Workshop on EU ETS Market Stability Reserve policies held at DIW and CEPS. We thank Jennifer Mills for research assistance. The usual disclaimers apply.

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1 Introduction

In most existing cap-and-trade programmes the environmental reduction target (the cap) is fixed and the supply of allowances is inflexible and determined within a rigid allocation programme. In theory, as long as the regulator makes allowances available before they are needed, the programme will deliver a cost-effective solution ([Hasegawa and Salant, 2015]). However, observations from recent cap-and-trade schemes – in particular the European Union Emissions Trading System (EU ETS) – have raised concerns over excessive allowance price variability and price collapse. These maladies seem to stem from a problem of 'oversupply', wherein unexpectedly low levels of allowance demand have led to the accumulation of a significant *surplus* of allowances. An article in The Economist in 2013 lamented a surplus of allowances equivalent to a year's emissions. This surplus is often attributed to the financial crises of 2008 and to the promotion of energy from renewable sources that has reduced allowance demand, see [Grosjean et al., 2014] and [Ellerman et al., 2015]. The resultant drop in allowance prices has policy makers and other stakeholders concerned that the current imbalance in supply and demand, if left unchecked, could reduce incentives for low-carbon investment and ultimately impair the ability of the EU ETS to meet its targets.¹

There are already provisions within the cap-and-trade framework that, in theory, should compensate for unforeseen changes in allowance demand. For example, most ETSs have banking provisions that should provide firms with a tool to respond to demand shocks. Several studies have explored the effect of banking and borrowing provisions as cost 'smoothing' mechanisms which decrease allowance price variability; [Hasegawa and Salant, 2014] provide a comprehensive and critical review of the literature on bankable emissions allowances that has developed over the last two decades. Other studies demonstrate how hybrid systems, combinations of quantity- and price-based instruments, lower expected control costs ultimately mitigating allowance price variability. [Fell and Morgenstern, 2010], [Grüll and Taschini, 2011], [Fell et al., 2012b], [Fell et al., 2012a] describe the relative trade-off of these different policy designs.

However, these provisions alone may not be sufficient when the market is faced with severe demand shocks. This leads to the question of how to amend an existing ETS to deal with an unexpected under- or oversupply of allowances. Namely, how should the allowance allocation programme (supply, which can be controlled by regulators) be changed to better cope with unexpected changes in allowance demand. In the case of the EU ETS, the European Commission (EC) has proposed a structural reform of the ETS, including the implementation of a Market Stability Reserve (MSR) scheduled to be operative starting from 2019 ([EC, 2014a], [EC, 2014b] and [EP, 2015]). The MSR amends the allowance allocation programme. In particular, it adjusts the number of allowances auctioned based on the size of the aggregate bank. In a given year, if the total bank of allowances exceeds 833 million, 12% of the size of the aggregate bank

¹The European Parliament states that the surplus prevents "the EU ETS from delivering the necessary investment signal to reduce CO_2 emissions in a cost-efficient manner and from being a driver of low-carbon innovation..." [EC, 6th October 2015]

will be withheld from auctions and will be placed in a dedicated reserve. These allowances are returned to the market in batches of 100 million as soon as the aggregate bank drops below the threshold of 400 million. By design, the MSR changes the timing of the allowance allocation but leaves the total number of allocated allowances unchanged within the regulatory period. As such, the reserve is temporary in nature and the MSR preserves the original cap.

In effect, a cap-preserving MSR means that the entire reserve can be expected to be depleted before the end of the banking period.² As it will be clearer later, this allows us to abstract from the operational details of the EC MSR and study the impact of a generalised, cap-preserving supply management system on emissions abatement and allowance price paths.³ As such, we draw from and contribute to the literature on inter-temporal permit trading under uncertainty and to the emerging literature on the assessment of the MSR and similar allowance supply adjustment mechanisms.

The results of previous theoretical and empirical analyses of intertemporal trading of emission allowances reveal that, under the usual assumption that marginal abatement costs are increasing in emissions reduction, firms start accumulating allowances and then draw them down, see [Rubin, 1996], [Schennach, 2000], [Ellerman and Montero, 2007], and [Ellerman et al., 2015]. Banking of allowances is thus a manifestation of the inter-temporal trading problem. The rationale for banking is quite intuitive: if tomorrow's discounted expected cost is higher than today's cost, it is worth banking allowances, whether obtained by abating more emissions today or by purchase, and either using them to cover some of tomorrow's emissions or selling them later on. The expected duration of the banking period, i.e. the period of time during which firms hold allowances, depends on the amount of abatement implied by the cap and, anticipating our discussion, is independent of the allowance allocation programme when the total amount of allowances issued over the banking period does not change (and firms are risk-neutral).

A banking model with no uncertainty and perfect competition would predict that during the banking period $[0, \tau)$ the price P_t of allowances will rise at the risk-free rate r, $dP_t/P_t = rdt$, where τ identifies the first instance when the aggregate bank is completely depleted and t represents time.⁴ In practice, however, firms cannot perfectly predict the number of allowances they will require in the future and, consequently, the market equilibrium price of allowances becomes subject to uncertainty. Investment in abatement is no longer risk-free and the evolution of allowance prices during the banking period is now governed by the no-arbitrage condition $\mathbb{E}[dP_t]/P_t = \mu_t dt$, where μ_t includes the possibly time-dependent risk premium

 $^{^{2}}$ In practice, there is a time lag between the reference year when the withdrawal or the re-injection of allowances is determined and the adjustment of yearly allocation plans. This, combined with the limited amount of allowances re-injected, implies the possibility of a reserve that is not fully depleted when the aggregate bank is zero. However, this does not qualitatively affect our results. Also, this event can be included in a larger set of events we discuss in Section 2.3.

 $^{^{3}}$ The Deutsches Institut für Wirtschaftsforschung (DIW Berlin) coordinated an international model comparison exercise where different sets of parameters have been tested using economic models and laboratory experiments. We refer interested readers to the final report, [Neuhoff et al., 2015].

 $^{^{4}}$ Hereafter, we refer to quantities concerning the entirety of regulated firms as 'aggregate' whereas the respective quantities for each firm are referred to as 'individual'.

([Ellerman and Montero, 2007]). In effect, allowance prices will respond to changes in firms' expectation about future allowance demand and supply during the banking period, the length of which depends in turn on these expectations.

In the analysis that follows, we explore the conditions under which the MSR to be implemented in the EU ETS will have an effect on allowance prices and abatement strategies, using a model of the inter-temporal pollution control and allowance trading. Regulated firms face an inter-temporal optimisation problem where, at each point in time, they have to decide by how much they want to offset their emissions, considering current and future costs of reducing emissions, as well as their existing bank of allowances and future allowance demand and allocations. The chief decision parameter is the firm's expected required abatement, the difference between counterfactual emissions (cumulative emissions in the absence of emissions restrictions) and the number of allowances allocated. Firms adjust their abatement and trading strategies at each time t based upon this parameter, taking into account their current bank of allowances and any change in the required abatement. Under uncertainty, changes in firms' expectation about the required abatement affect how much abatement and banking will occur in the future – and for how long. We thus frame our analysis of the impact of the MSR in terms of two main parameters: the firm's expectations about the required abatement and the length of the banking period.

We first consider the inter-temporal optimisation problem for risk-neutral firms and then expand our analysis to risk-averse firms. Previous studies have demonstrated that firms' strategy adjustments (and the overall efficacy of the MSR) are highly dependent on the constraints on temporal provisions (i.e. limitations on borrowing) and on the design of the mechanism implemented to adjust the allowance allocation, see [Salant, 2015], [Perino and Willner, 2015], [Fell, 2015]. In the absence of borrowing constraints, abatement decisions are independent of the temporal distribution of allowances. If firms can always borrow from future allocations, any change to the allocation programme that maintains the overall emissions cap is irrelevant. Firms will simply borrow the required allowances needed to remain on their original costminimised emissions path and the MSR will have no influence. Only under borrowing constraints can the MSR change abatement and allowance price trajectories.

Moreover, we demonstrate that even under limited borrowing, the MSR can only change abatement and allowance price paths if and only if the onset of the MSR changes the expected required abatement. Our model indicates that when neither the expectation about the length of the banking period (τ) nor the total number of allowances allocated until τ change, the expected required abatement is the same. This result reproduces Proposition 4 in [Perino and Willner, 2015].

Building upon this result, it is crucial to note that although the changes in the allowance allocation programme determined by the cap-preserving MSR leave the expected length of the banking period and the expected required abatement unchanged, the distribution of this time τ differs.⁵ When considering the impact of previously unexpected changes (e.g. demand shocks), we note that changes to the timing of allowance allocation and the distribution of τ affects the likelihood of the event of an *instantaneous depletion of the bank*. We term this instantaneous breakdown. That is to say, changes to the distribution of τ by the MSR can raise the probability that firms are not able to compensate for a demand shock with their current bank of allowances. This is related to the discussion of price variability in Perino and Willner's analysis. They posit that the short-term scarcity produced by the (binding) MSR can drive prices higher in the short term, lead to lower prices in the long term and induce overall higher price variability. Our findings support the conclusion that the MSR increases price volatility when unexpected changes occurs in the period when the reserve is building up (short term). But we find that price volatility decreases in the longer term, leaving the average price volatility unaltered. Thus on average, the MSR generates the same abatement and allowance price paths.⁶

When firms are indifferent to the change in the probability of the event of an instantaneous depletion of the bank, i.e. firms are risk neutral, changes in price volatility due to the MSR leave emission abatements unaffected. However, for risk-averse entities, short-term differences in price volatility matter and are reflected in the risk premium demanded by firms. Thus, changes in the distribution of τ brought on by the MSR that lead to higher short-term price variability lead to higher risk premiums. The higher the risk premium associated with holding allowances, the more quickly firms will deplete their bank, which leads to lower levels of abatement. This has significant implications for the overall impact of an MSR when the regulated firms are not perfectly risk-neutral. While one of the goals some stakeholders attribute to the MSR is to increase prices during periods of over-supply, for risk-averse firms, the building up of the allowance reserve by the MSR will have the opposite effect. The rise in price volatility will lead to higher risk premiums, accelerated depletion of the bank and lower abatement, and consequently lower allowance prices. Thus the influence of an MSR can actually be counter-productive when the behaviour of risk-averse firms is considered.

The remainder of the paper is organised as follows. In Sections 2.0 and 2.1 we introduce our main assumptions and define the key decision making variables. In Section 2.2 we present the market equilibrium in terms of aggregate quantities and discuss the effect of the MSR on abatement and allowance price paths. Both static and dynamic views of our results are discussed. In Section 2.3 we relax the assumption of risk neutrality and explore the effect of the MSR on a time-dependent risk premium. Section 3 concludes.

 $^{^{5}}$ We use the term distribution in the sense of a distribution of variables that are subject to uncertainty.

 $^{^{6}}$ Our analytical findings support the numerical outcome in Fell (2015) – 'MSR designs that withdraw about as many permits in high-bank states as they inject in low-bank states do little to increase permit prices or decrease permit price variation relative to the EU ETS system as is.' [Fell, 2015]

2 The model

Regulated firms are assumed to be atomistic in a perfectly competitive market for emission allowances. Firms face an inter-temporal optimisation problem where, at each point in time, they have to decide how much they want to offset their emissions (either by abating or by trading allowances), considering the current and future costs of reducing emissions. Firms account for their current bank of allowances and the number of allowances they expect to be allotted in the future. In this context, the *required abatement*, the difference between the cumulated amount of emissions without abatement requirements (counterfactual emissions) and their future allocation, is the key quantity each firm has to assess at each point in time. Under uncertainty, changes in firms' expectation about the required abatement affect how much abatement only during the banking period. Once the bank is depleted, the inter-temporal problem breaks down: firms use every allowance available to cover contemporaneous emissions and instantaneously abate their residual emissions. Thus, we focus our analysis on the banking period $(0, \tau)$ and investigate under which conditions the MSR can alter the length of the banking period τ .⁷

The firm's dynamic cost minimisation problem is⁸

 \mathbf{S}

$$\min_{\alpha^{i},\beta^{i}} \mathbb{E}\left[\int_{0}^{\tau} e^{-rt} v^{i}(\alpha^{i}_{t},\beta^{i}_{t})dt\right],$$
(1)
i.t.
$$B^{i}_{t} = B^{i}_{0} + A(0,t) - E(0,t) + \int_{0}^{t} \alpha^{i}_{s} ds - \int_{0}^{t} \beta^{i}_{s} ds,$$

$$B^{i}_{t} > 0, \quad \text{and} \quad B^{i}_{\tau} = 0.$$

where r is the risk-free rate; v^i denotes the cost function; B_0^i represents the firm's initial bank of allowances; A(0,t) represents the sum of allowances allocated from time 0 to t; and E(0,t) represents pre-abatement cumulated emissions during the same period. With an MSR the allowance allocation programme changes, thus both allocation and emissions may be subject to uncertainty. Finally, let α_t^i denote instantaneous abatement and β_t^i be the number of allowances sold ($\beta_t^i > 0$) or bought ($\beta_t^i < 0$). Later we will assume a specific functional form for the cost function $v(\cdot)$ and provide equilibrium results in closed form.

⁷The duration of the banking period is computed as part of the equilibrium derived in Appendix A. Similar to [Schennach, 2000], we can represent τ by means of an implicit function. Under uncertainty, the banking period needs not be unique. However, this does not limit the generality of our results because, in expectations, a second banking period will not occur.

⁸The instant τ of full depletion of the aggregate bank (in equilibrium) transfers directly to the individual firm's optimisation problem in form of the constraint $B_{\tau}^i = 0$, since after τ the price increases at a lower rate than the rate of interest. Thus, there is no individual incentive to bank permits beyond τ .

2.1 Required abatement under uncertainty

To capture the impact of uncertainty on banking in a cap-and-trade programme under the MSR, we identify two key state variables of the system: the time-t expectations of (i) the instant τ when the aggregate bank is completely depleted and (ii) the corresponding required abatement, that is counterfactual emissions over $[0, \tau)$ minus the total number of allowances allocated in the same period (including the initial bank of allowances). When new information becomes available, firms update their expectations and adjust their strategies. That is, abatement and trading strategies are adapted at each time t, taking into account the current bank of allowances and the change in the required abatement.

We express the time-t expectation of the instant when the aggregate bank is completely depleted as $\mathbb{E}_t[\tau]$.⁹ The aggregate abatement required over the period $[0, \tau)$ is represented by $Y = Y(0, \tau)$; we refer to its expected value as $\mathbb{E}_t[Y]$. Finally, $d\mathbb{E}_t[Y]$ represents changes in expectations about the required abatement. These three expressions are key to understanding how abatement and allowance prices change when firms' expectations change during the banking period. They make it possible to readily describe the impact of various events that are particularly relevant to the EU ETS market: the substantial decrease in allowance demand observed in the recent economic recession and, most crucially, the proposed structural reform of the EU ETS via the implementation of the MSR. Intuitively, a permanent negative shock to counterfactual emissions would result in a downward shift in $\mathbb{E}_t[Y]$ and a longer (expected) banking period.¹⁰ Conversely, a permanent positive shock to emissions would increase $\mathbb{E}_t[Y]$ and shorten the expected period of a positive bank.

2.2 Equilibrium solution for risk-neutral firms

We now consider the optimisation problem in (1) and characterise the market equilibrium under riskneutrality. In order to make the analysis analytically tractable, we assume a linear functional form for the marginal abatement cost curve, $AC'(\alpha) = \Pi_t + 2\rho\alpha$, where Π_t and ρ represent the intercept¹¹ and the slope of the marginal cost curve, respectively. Firms can sell and buy allowances $|\beta_t^i|$ at a price P_t ; they face costs $TC(\beta)$ for each trade.¹²

⁹By convention, $\mathbb{E}_t[\cdot] = \mathbb{E}[\cdot|\mathcal{F}_t]$ represents the conditional expectation where \mathcal{F}_t indicates the information available at time t. We refer to the Appendix for an analytical specification of \mathcal{F}_t .

 $^{^{10}}$ [Schennach, 2000] investigates the effect of a permanent costless reduction in SO₂ emissions in a deterministic framework and provides approximate solution when the model is solved under uncertainty. In this context, our contribution extends the efforts of [Schennach, 2000] by deriving an exact analytical solution of the market equilibrium under uncertainty. This allows us to derive the equilibrium under risk-aversion, as described in Appendix B.

¹¹The intercept Π_t is assumed to increase at the risk-free rate r.

¹²In addition to the cost βP_t when buying (negative cost when selling) $|\beta|$ allowances, firms might face non-negligible transaction costs per trade. Among others, [Frino et al., 2010] and [Medina et al., 2014] document non-negligible transaction costs in the EU ETS. In our framework, we assume linear marginal trading costs, $TC'(\beta) = P_t - 2\nu\beta$. This ensures uniqueness of the equilibrium and allows us to derive the equilibrium in closed form. In aggregate terms, however, the equilibrium results are not affected by the level of ν and prevail for $\nu = 0$. We thereby consider negligible transaction costs, as is typically assumed in the environmental economics literature. Note that the impact of a specific distribution of firms' characteristics across their continuum can be studied by the individual strategies provided in our model results. However, this is not the focus of the present paper and is left for future research.

The instantaneous costs of reducing emissions via abatement and trading are thus given by

$$v_t^i(\alpha_t^i, \beta_t^i) = \Pi_t \alpha_t^i + \varrho \cdot (\alpha_t^i)^2 + TC(\beta_t^i).$$

In Appendix A we solve the optimisation problem in (1) and obtain the market equilibrium as a set $(\{\alpha^i, \beta^i\}_{i \in I}, P, \tau)$, where $P = (P_t)_{0 \le t \le \tau}$ is the equilibrium price process and τ denotes the length of the banking period in equilibrium. In what follows, we present all relevant results of our analytical analysis in aggregate terms.

In equilibrium, aggregate abatement at time t is given by

$$\alpha_t = re^{rt} \frac{\mathbb{E}_0[Y]}{e^{r\tau(0)} - 1} + re^{rt} \int_0^t \frac{d\mathbb{E}_s[Y]}{e^{r\tau(s)} - e^{rs}},\tag{2}$$

where for legibility we replace $\mathbb{E}_t[\tau]$ with $\tau(t)$ and $\mathbb{E}_t[Y] = \mathbb{E}_t[Y(\tau)]$.

The first term on the right hand side of Equation (2) is the expected abatement given the information available at time 0 (we compute this below). The expected required abatement $\mathbb{E}_0[Y(\tau)]$ is spread over the banking period and increases at the rate r. At each time t, new information about the future required abatement becomes available and adjustments in the equilibrium abatement may occur. This is represented by the second term on the right hand side of Equation (2). When the expectation about the future required abatement changes, the corresponding adjustment $d\mathbb{E}_s[Y]$ is spread over the remainder of the banking period.¹³

In the following discussion we investigate the impact of changes in the expected required abatement and, ultimately, how the MSR affects the abatement and allowance price paths. We begin by considering the expected abatement path, computed at time t = 0 which provides a *static view* of the model results. The time-0 expectations of $d\mathbb{E}_t[Y]$ are all zero; hence the second summand of Equation (2) vanishes when considering the time-0 expectation.¹⁴ Thus, we obtain

$$\mathbb{E}_0[\alpha_t] = re^{rt} \frac{\mathbb{E}_0[Y(\tau)]}{e^{r\tau} - 1}.$$
(3)

From this expression we can see that if neither the time-0 expectation of τ nor the total number of allowances allocated until τ change, the time-0 expected required abatement $\mathbb{E}_t[Y(\tau)]$ is the same. The MSR changes the timing of the allocation of allowances, but not the total number of allocated allowances. Thus, the MSR leaves the equilibrium aggregate abatement unaltered. This result reproduces Proposition 4 in [Perino and Willner, 2015]: in time-0 expectation (corresponding to no uncertainty), the MSR has no

 $^{^{13}}$ [Ellerman and Montero, 2007] investigate how the level of reversibility in abatement decisions affect these adjustments (abatement corrections) in current abatement.

¹⁴This is a direct consequence of the tower property of the conditional expectation.

effect on abatement and allowance prices when firms' expectations about the future total demand do not change.

We now consider the impact of *previously unexpected* changes to the required abatement and provide a *dynamic view* of our results. We investigate the impact of previously unexpected changes by looking at allowance prices (marginal abatement costs). Equation (2) immediately yield the equilibrium price process

$$P_t = \Pi_t + 2\varrho\alpha_t = \Pi_t + 2\varrho r e^{rt} \frac{\mathbb{E}_0[Y]}{e^{r\tau(0)} - 1} + 2\varrho r e^{rt} \int_0^t \frac{d\mathbb{E}_s[Y]}{e^{r\tau(s)} - e^{rs}},$$

where price variability is generated by unanticipated changes to the required abatement $d\mathbb{E}_s[Y]$. Changes in the expected required abatement change the expected duration of the banking period too. Thus, the joint effect of possible changes $d\mathbb{E}_s[Y]$ and $d\mathbb{E}_s[\tau]$ determines the volatility of prices. We will see that this joint effect is in fact subject to changes in the programme of the allowance supply, such as the one introduced by the MSR. Such an effect has been explored in terms of a single random shock by [Perino and Willner, 2015]. They conclude that the MSR *increases* price volatility when the shock occurs in the period when the reserve is building up.

The following analysis extends the efforts of [Perino and Willner, 2015] by studying how the MSR affects price volatility and – under risk aversion – firms' discount rates. The rationale is the following: under risk aversion the impact of the MSR on price volatility is reflected in the risk-premium and, consequently, in firms' discount rate. The latter signals whether returns from allowance-related investments should promise higher or lower returns with consequent effects on allowance banking.

2.3 Changes in expectations: a dynamic view and risk-aversion

We now investigate how abatement and allowance prices respond to changes in time-t expectations. Recall that the required abatement Y represents counterfactual emissions over $[0, \tau)$ minus the total number of allowances allocated in the same period (including the initial bank of allowances). With an MSR, changes in time-t expectations about the required abatement, $d\mathbb{E}_t[Y]$, will yield one of two scenarios. First, if the aggregate bank is sufficient to cushion the shock, then the time-t bank remains positive and $t < \mathbb{E}_t[\tau]$. If, however, the change in $d\mathbb{E}_t[Y]$ leads to a situation where the bank is completely depleted, then $t = \tau$. We term this scenario *instantaneous breakdown*. Below we explore how the MSR influences the likelihood of this scenario and what conclusions we can draw in terms of policy implications.

We model time-t changes in expectations about the required abatement as $d\mathbb{E}_t[Y] = \sigma_t \sqrt{(\tau - t)} z_t$ where z_t are independent standard Gaussian shocks and $\sigma_t^2 \cdot (\tau - t)$ is the variance of $d\mathbb{E}_t[Y]$. The term $(\tau - t)$ captures a natural assumption: uncertainty about *cumulated* emissions diminishes as time goes by and we approach τ . The term σ_t , on the other hand, represents the variance of unexpected changes to τ , which

may be subject to the allocation programme as described further below.

In the short run, the MSR withholds allowances from auctions and places them in a dedicated allowance reserve – as long as the bank stays above the upper threshold. Therefore, the bank decreases in the short term. The smaller the aggregate bank, the larger the likelihood of an instantaneous breakdown. Later (but before τ), allowances from the reserve are made available, adding to the aggregate bank and reducing the likelihood of an instantaneous breakdown. This is illustrated in Figure 1, where we present the aggregate bank with and without the MSR (red and black line, respectively). We capture these changes to the likelihood of an instantaneous breakdown by modelling σ_t as a function of the current bank, $\sigma_t = \sigma(B_t) > 0$, where $\frac{\partial \sigma}{\partial B_t} < 0$. In effect, without an MSR, the likelihood of a breakdown would be smaller in the short run and larger in the long run. As such, the MSR 'flattens out' the probability of an instantaneous breakdown, increasing the risk associated to the abatement requirement in the short run and decreasing it in the long run.



Figure 1: The aggregate bank under risk-neutrality without the MSR (black line) and with the MSR (red dotted line). The MSR decreases the bank in the short run and adds to the bank in the long run, when the reserve is re-injected. As a result, shocks ε in the short run, that could be absorbed by the bank, may lead to an unexpected breakdown when the MSR transfers permits to the reserve. This effect is inverted in the long run, where shocks that would lead to a breakdown can now be absorbed by the increased bank. Overall, the likelihood of a breakdown is increased in the short run and decreased in the long run.

Although changes in the allowance allocation programme determined by the cap-preserving MSR leave $\mathbb{E}_t[\tau]$ unchanged, the time-t variance of τ differs. In order to understand the implications of these changes, the next natural step is to consider an extension of the modelling framework where changes in the variance of τ are properly reflected in firms' abatement strategies. We thus expand our analysis to study how risk-averse firms react to changes to the allowance allocation schedule. With risk-aversion, the firm's dynamic cost minimisation problem is

$$\min_{\alpha_t^i,\beta_t^i} \mathbb{E}\left[\int_0^\tau e^{-\mu_t \cdot t} v^i(\alpha_t^i,\beta_t^i) dt\right],\tag{4}$$

where the discount rate $\mu_t = r + q_t$ includes the risk-free rate r and a (time-dependent) risk-premium q_t .¹⁵ Allowances (and related low-carbon investments) are perceived as risky investments and are discounted accordingly, at the rate $\mu_t = r + q_t$. If alternative investments promise higher returns (discounted according to their respective riskiness), firms would prefer to postpone abatement and use their bank of allowances to offset emissions. In turn, lower abatement levels will be reflected in lower prices. Intuitively, a larger discount rate due to a positive risk premium should thus imply a lower level of aggregate abatement and, consequently, a lower aggregate bank compared to the case where $\mu = r$. This has been observed by [Fell, 2015] and [Ellerman et al., 2015] in their sensitivity analysis.

As described in more detail in Appendix B, we use our equilibrium results to analytically characterise the level of aggregate abatement under risk-aversion α_t^A and compare it to abatement under risk-neutrality α_t^N . We obtain the following identities:

$$\alpha_t^N - \alpha_t^A = \frac{q_t}{2\varrho} \, \frac{e^{r\tau^N} - e^{rt}}{r} \, e^{-rt} P_t^N > 0 \quad \text{for} \quad t < \tau^A(t) \tag{5}$$

and

$$P_t^N - P_t^A = q_t \; \frac{e^{r\tau^N} - e^{rt}}{r} \; e^{-rt} P_t^N > 0 \quad \text{for} \quad t < \tau^A(t),$$

where $\tau^{A}(t)$ denotes the expected instant when the aggregate bank is completely depleted under riskaversion; and P_{t}^{N} and P_{t}^{A} denote the allowance price under risk-neutrality and risk-aversion, respectively. As expected, aggregate abatement under risk-aversion is strictly smaller than under risk-neutrality for $t < \tau^{A}$ and, consequently, the bank is depleted more quickly, $\tau^{A} < \tau^{N}$.



Figure 2: Abatement curves for risk-neutral (α^N) and risk-averse firms (α^A) . Total abatement under risk-aversion is smaller over the period $[0, \tau^A)$, where τ^A represents the end of the banking period under risk-aversion. The two abatement curves converge as time goes by and would intersect at τ^N . This event, however, will never be observed since under risk-aversion, the market breaks down at $\tau^A < \tau^N$.

 $^{^{15}}$ There is growing empirical evidence of time-dependent risk premia, see [Gagliardini et al., 2015] and references therein. In the financial econometric literature, moves in risk premia are often ascribed to changes in volatility or risk aversion.

We now explore what drives the difference $\alpha_t^N - \alpha_t^A$ and examine how the MSR affects abatement under risk-aversion. During the banking period, when the bank is drawn down, the risk-premium q_t typically decreases. As time goes by, uncertainty about the required abatement is gradually reduced, making the holding of allowances and the associated investments less risky. The risk premium enters linearly in the expression of the difference and is multiplied by the term $(e^{r\tau^N} - e^{rt})$. This last term determines the influence of q_t on abatement and decreases in time. In case of an unexpected instantaneous breakdown, firms' past abatement is seen as excessive. That is, ex-post it would have been optimal to deplete the bank faster. The instant the unexpected breakdown occurs, the value of the bank is almost completely lost. The more time was left until the expected τ , the the larger was the value of the bank. Similarly, the closer the expected τ when the instantaneous breakdown occurs, the smaller the bank loss. The bank value that is at risk hence decreases in time and consequently, the impact of risk-aversion on firms' strategies diminishes when approaching the expected end of the banking period. Since $e^{-rt}P_t$ is constant in expectation, the two expected abatement paths converge exponentially, when approaching the end of the banking period, as illustrated in Figure 2.

We now turn to the effect of changes to the allowance allocation by the MSR. Figure 3 illustrates the aggregate bank under risk-aversion with and without the MSR (red and blue line, respectively). The solid black line represents the aggregate bank without the MSR, when firms are risk-neutral, $q_t = 0$. As discussed earlier, the MSR decreases the bank in the short run and adds to the bank in the long run, when allowances return to the market. Accordingly, we model the volatility parameter $\sigma_t(g_t)$ as a function of the allowance allocation g_t at time t. In line with our previous discussion, an increase in g_t has a negative effect on volatility, $\frac{\partial \sigma_t(g_t)}{\partial g_t} < 0$. In order to examine how the changes in allowance allocation of the MSR affect abatement decisions under risk-aversion, we first consider the risk-premium q_t . As shown in Appendix B, the rate of change of q_t with respect to g_t is:

$$\frac{\frac{\partial q_t(g_t)}{\partial g_t}}{q_t} = \frac{\frac{\partial \sigma_t(g_t)}{\partial g_t}}{\sigma_t} \tag{6}$$

During the building up of the allowance reserve, the change in allowance allocation increases the likelihood of an instantaneous breakdown. This is reflected in an adjustment of the risk-premium q_t . More precisely, Equation (6) reveals that a change in g_t generates a change in σ_t which equally transfers to a change in q_t . Consequently, risk-averse firms adjust their abatement behaviour, as quantified in Equation (5). As previously discussed, the impact of q_t is larger in the short term, when the expected end to the banking period lies in the distant future and the costs associated to an instantaneous breakdown are high. As time goes by and the expected time τ of complete depletion of the bank approaches, allowances from the reserve will eventually be released. This increases the bank and hence the likelihood of an immediate



Figure 3: The aggregate bank without an MSR under risk-neutrality (black line) and under riskaversion (blue line); aggregate bank with the MSR under risk-aversion (red dotted line). The MSR decreases the bank in the short run and adds to the bank in the long run, when the reserve is reinjected. As in the risk-neutral case, the likelihood of a breakdown is increased in the short run and decreased in the long run. However, the effect of risk-aversion on the slope in banking (dictated by α^A) decreases in time and hence short-term effects are amplified whereas long-term effects diminish.

breakdown decreases compared to the no-MSR case, modelled by a decrease in $\sigma_t(g_t)$. Accordingly, we would expect a lower q_t , higher aggregate abatement and higher allowance prices. However, during this period the potential loss associated to an instantaneous breakdown are lower than in the short run. As we can see from Equation (5), the extra allowances added to the bank in the long run reduce q_t and increase abatement and allowance prices, but to a lower extent than both were decreased earlier during the banking period. Overall, we find that the impact of a change in the likelihood of a breakdown affects prices and abatement significantly more in the short run than it does in the long run.

In conclusion, we find that under risk aversion, short- and long term effects do not even out. These findings corroborate concerns about rising price volatility in the short run, as raised by [Perino and Willner, 2015] and [Fell, 2015]. When firms are not indifferent towards the risk of an unexpected breakdown, the effect of a lower price volatility in the long-run at the expense of higher price volatility in the short run is by no means equivalent and this difference is significantly large when the expected end to the banking period lies in the distant future.

3 Conclusions

The supply of allowances in the European Union Emissions Trading System (EU ETS) has been inflexible and determined within a rigid allocation programme. As such, the system lacked provisions to address severe imbalances in demand and supply of allowances resulting from economic shocks. The European Commission has proposed a structural reform of the EU ETS, including the implementation of a Market Stability Reserve (MSR), scheduled to start operating from 2019. The MSR will adjust the allocation programme based on the aggregate bank of allowances: In times of a large surplus, allowances are transferred to a dedicated reserve to be released in times of scarcity. As such, the MSR preserves the total number of allowances issued over the regulatory phase.

We develop a stochastic equilibrium model of inter-temporal trading of emission allowances to investigate under which conditions an MSR like that proposed for the EU ETS can alter allowance price and emissions abatement paths. We find that the timing of allocation is largely irrelevant as long as changes in expected emissions can be dealt with the existing bank of allowances. In this context, we consider unexpected changes in firms' expectations that triggers an instantaneous depletion of the bank of allowances (unexpected breakdown). Risk neutral firms are indifferent to changes in the likelihood of this event and we show that the MSR neither affects equilibrium abatement nor the expected price paths.

However, when firms account for the risk in the change of their future required abatement – i.e. counterfactual emissions minus total number of allowances allocated over the same period – adjustments in the allowance allocation programme matter. In particular, an unexpected breakdown can generate substantial losses, depending on the time of its occurrence. Ex-post, firms' past abatement can be seen as excessive. We expand our analysis to study how risk-averse firms' strategies are affected by the MSR at different points in time of the banking period.

Our findings support the concerns raised by [Perino and Willner, 2015] and [Fell, 2015] that the MSR increases price volatility in the short run, when allowances are transferred to the reserve. We concur with this result but demonstrate that the MSR decreases price volatility in the long run, when allowances are released from the reserve.

On average, the allowance price volatility remains the same. Short- and long term effects on abatement even out when firms are risk-neutral. However, this is not the case for risk adverse firms. Abatement in equilibrium is significantly more affected by a rise in price volatility in the short run than by a corresponding decrease in the long run and this difference is particularly large when the expected end of the banking period lies in the distant future.

Appendix

In the following sections we provide the derivations of the key results.

A The model under risk-neutrality

We consider an inter-temporal optimisation problem where, at each point in time, firms have to decide how much they want to offset their emissions (either by abating or by trading allowances), considering the current and future costs of reducing emissions. We model the banking period $[0, \tau)$ of a secondary emissions allowances market in a partial equilibrium framework under perfect competition. Firms $i \in I$ are assumed to be atomistic, that is, individual quantities x^i are continuously distributed under a measure m^x such that aggregate quantities can be obtained by integration, $x = x^I = \int_I x^i dm^x(i)$. Each firm continuously minimises expected abatement and trading costs at each point in time $t \in [0, \tau^i)$, where τ^i denotes the first instance when the bank B^i is completely depleted,

$$\tau^i = \min\{t \ge 0, B^i_t = 0\}$$

and her instantaneous cost function is given by

$$v^i(\alpha^i_t, \beta^i_t) = \Pi_t \alpha^i_t + \varrho \cdot (\alpha^i_t)^2 - P_t \beta^i_t + \nu \cdot (\beta^i_t)^2.$$

That is, each firm has the same marginal abatement cost curve $\Pi_t + 2\rho \alpha_t^i$, where we assume that the intercept Π_t increases by the risk-free rate r and $\rho > 0$ is constant. Firms face non-negligible transaction costs per trade. More specifically, we assume marginal trading costs to be linear in the number of allowances sold ($\beta_t^i > 0$) or bought ($\beta_t^i < 0$). The parameter ν represents the magnitude of transaction costs and P_t denotes the time-t allowance price. We define $\tau = \tau^I$ as the first time when the aggregate bank of allowances is completely depleted,

$$\tau = \min\{t \ge 0, B = B^I = 0\}.$$

By definition of τ , there is no incentive for the aggregate market in equilibrium to hold allowances beyond the end of the banking period. Therefore, at times $t > \tau$ the incentive to hold allowances vanishes and and allowance prices increase at a rate less than the risk-free rate. Since firms are atomistic and face the same marginal abatement cost function, it is always suboptimal for each individual firm to hold allowances beyond the time τ . We represent this by the requirement

$$\tau^i = \tau$$
, i.e. $B^i_\tau = 0$.

For convenience, we write $\varpi_t^i = \mathbb{E}_t[Y^i(t,\tau)]$, where $Y^i(t,\tau)$ is the time-t residual offsetting requirement:

$$Y^{i}(t,\tau) = Y^{i}(0,\tau) - \int_{0}^{t} \alpha_{s}^{i} \, ds + \int_{0}^{t} \beta_{s}^{i} \, ds$$

We note that

$$dE_t[Y^i(t,\tau)] = (\beta_t^i - \alpha_t^i)dt + d\mathbb{E}_t[Y^i(0,\tau)].$$

Furthermore, for $t = \tau$ we have

$$B^i_\tau = -Y^i(\tau,\tau).$$

Hence, we can replace the requirement $B_{\tau} = B_{\tau^i} = 0$ with the constraint $Y_{\tau}^i = 0$.

The equilibrium consists of abatement- and trading strategies α_t^i and β_t^i for each firm *i*, the market clearing price process P_t and the equilibrium length (duration) τ of the banking period. In equilibrium, individual deviations from the equilibrium do not yield expected additional cost savings for any firm. The market is assumed to be free of arbitrage and complete. We can therefore postulate the existence of a martingale measure \mathbb{Q} that is equivalent to the real-world measure \mathbb{P} . We first assume that firms are risk-neutral. Accordingly, all expectations in this section are taken under the measure \mathbb{Q} . In Appendix B, we transfer our results to risk-averse firms by deriving the change of measure from \mathbb{Q} to \mathbb{P} .

We begin by assuming Markovian strategies $\alpha^j = \alpha(Z_t^j)$, $\beta^j = \beta(Z_t^j)$ for every firm $j \in I \setminus \{i\}$ except for *i*. These strategies are given as functions of each firms' individual state processes Z_t^j , which will be specified later. We show that it is optimal for firm *i* to replicate the other firms' strategies given below, as a function of her own state process Z_t^i . For convenience, we define

$$h_t = \frac{re^{rt}}{e^{r\tau(t)} - e^{rt}}, \quad \text{where} \quad \tau(t) = \mathbb{E}_t[\tau].$$

For each firm $j \in I \setminus \{i\}$, let her abatement and trading strategies be given by

$$\alpha_t^j = \frac{P_t - \Pi_t}{2(\nu + \varrho)} + \frac{\nu}{\nu + \varrho} \ h_t \varpi_t^j \quad \text{and} \quad \beta_t^j = \frac{P_t - \Pi_t}{2(\nu + \varrho)} - \frac{\varrho}{\nu + \varrho} \ h_t \varpi_t^j$$

The market clearing condition $\beta^{I} = 0$ yields

$$P_t = \Pi_t + 2\varrho h_t \varpi_t^I. \tag{7}$$

Substituting for the strategies α_t^j , β_t^j above, we obtain the dynamics for the process ϖ_t^j :

$$d\varpi_t^j = (\beta_t^j - \alpha_t^j) dt + d\mathbb{E}_t[Y^j(0,\tau)] = -\frac{re^{rt}}{e^{r\tau(t)} - e^{rt}} \varpi_t^j dt + d\mathbb{E}_t[Y^j(0,\tau)]$$

Solving the above, we obtain:

$$\varpi_t^j = \varpi_0^j \, \frac{e^{r\tau(0)} - e^{rt}}{e^{r\tau(0)} - 1} + (e^{r\tau(t)} - e^{rt}) \int_0^t \frac{d\mathbb{E}_t[Y^j(0,\tau)]}{e^{r\tau(s)} - e^{rs}}$$

Integrating over I yields, together with Equation (7) that

$$P_{t} = \Pi_{t} + 2\rho h_{t} \varpi_{t}^{I} = \Pi_{t} + 2\rho \frac{re^{rt}}{e^{r\tau(0)} - 1} \ \varpi_{0}^{I} + 2\rho re^{rt} \int_{0}^{t} \frac{d\mathbb{E}_{t}[Y^{I}(0,\tau)]}{e^{r\tau(s)} - e^{rs}}.$$

In particular, we observe that P has the following dynamics

$$dP_t = rP_t dt + 2\rho h_t d\mathbb{E}_t [Y^I(0,\tau)].$$

Let the random shocks to $\mathbb{E}_t[Y^I(0,\tau)]$ be governed by a driftless diffusion

$$d\mathbb{E}_t[Y^I(0,\tau)] = \sigma_t^I dW_t^{\mathbb{Q}},$$

where σ_t^I is deterministic and $W_t^{\mathbb{Q}}$ is a Brownian motion under the measure \mathbb{Q} .

The process σ_t^i describes how changes in the expected future net-supply of allowances are distributed across the set of firms *I*. We abstract from specific assumptions about the form of σ_t^i . However, we note that it is reasonable to assume different σ_t^i for different firms, since pre-abatement emissions levels and allowances allocations can vary depending on the type of industry in consideration.

We consider changes in pre-abatement allowances demand and in the (possibly contingent) supply of allowances. Their degree of impact on firms can vary. However, all firms are subject to systemic shocks. Hence, we consider the same Brownian motion $W_t^{\mathbb{Q}}$ for each $i \in I$, whereas differences in size, technology etc. are represented by distribution of σ_t^i across I. Accordingly, shocks to $\mathbb{E}_t[Y^i(0,\tau)]$ are represented by

$$d\mathbb{E}_t[Y^i(0,\tau)] = \sigma_t^i dW_t^{\mathbb{Q}}.$$

We now consider the problem of optimal pollution control and allowance trading for firm *i*. Let *p* denote an observed allowance price and let $P^{t,p}$ denote the price process with time-*t* value $P_t^{t,p} = p$. Analogously, let $\Pi_t^{t,\pi} = \pi$. At time *t*, the firm *i* has to bear costs v_t^i given by

$$v_t^i(\alpha_t^i,\beta_t^i) = \Pi_t^{t,\pi} \ \alpha_t^i + \varrho \cdot (\alpha_t^i)^2 - P_t^{t,p} \beta_t^i + \nu \cdot (\beta_t^i)^2.$$

Firm *i*'s problem is to find (Markovian) abatement- and trading strategies α^i and β^i respectively, such that, for all $t \in [0, \tau)$, the cost function J, given by

$$J(t, \varpi^i, p, \pi, \alpha^i, \beta^i) = \mathbb{E}\left[\int_t^\tau e^{-rs} v_s^i(\alpha_s^i, \beta_s^i) \ ds\right],$$

is minimised by α^i , β^i for all $\pi > 0$, $p \ge 0$, and such that the constraint $\mathbb{E}_t[\varpi_{\tau}^i] = 0$ is satisfied for all $t \in [0, \tau)$. Let $w(t, \varpi^i, p, \pi) = \inf_{(\alpha^i, \beta^i)} J(t, \varpi^i, p, \pi, \alpha^i, \beta^i)$ denote the value function for firm *i*. The firm observes the state process $Z_t^i = (\varpi_t^i, P_t, \Pi_t)$, where

$$d\varpi_t^i = (\beta_t^i - \alpha_t^i) dt + d\mathbb{E}_t[Y^i(0,\tau)] = (\beta_t^i - \alpha_t^i) dt + \sigma_t^i dW_t^{\mathbb{Q}},$$

$$dP_t = rP_t dt + 2\varrho h_t d\mathbb{E}_t[Y^I(0,\tau)] = rP_t dt + 2\varrho h_t \sigma_t^I dW_t^{\mathbb{Q}},$$

$$d\Pi_t = r\Pi_t dt.$$

Let the firm's filtration $(\mathcal{F}_t^i)_{t\geq 0}$ be generated by the process Z^i and accordingly, let (\mathcal{F}_t^I) , generated by Z^I , denote the aggregate filtration. The Hamilton-Jacobi-Bellman (HJB) equation associated to the minimisation problem above is given by

$$\begin{split} 0 &= D_t w + \inf_{a,b} \left[\mathcal{L}^{(a,b)} w + e^{-rt} v_t^i(a,b) \right] \\ &= D_t w + \inf_{a,b} \left[(b-a) D_{\varpi^i} w + rp D_p w + r\pi D_\pi w + \frac{1}{2} \text{tr}(\Sigma \Sigma' D_z^2 w) + e^{-rt} (\pi a + \varrho a^2 - pb + \nu b^2) \right] \\ &= D_t w + rp D_p w + r\pi D_\pi w + \frac{1}{2} \text{tr}(\Sigma \Sigma' D_z^2 w) + \inf_{a,b} \left[(b-a) D_{\varpi^i} w + e^{-rt} (\pi a + \varrho a^2 - pb + \nu b^2) \right], \end{split}$$

where Σ is the vector

$$\Sigma = \begin{pmatrix} \sigma_t^i \\ 2\varrho h_t \sigma_t^I \end{pmatrix}$$

which implies that

$$\operatorname{tr}(\Sigma\Sigma'D_z^2w) = (\sigma_t^i)^2 D_{\varpi^i}^2 w + 2\varrho h_t \sigma_t^i \sigma_t^I D_p D_{\varpi^i} w + 2\varrho h_t \sigma_t^i \sigma_t^I D_{\varpi^i} D_p w + 4\varrho^2 h_t^2 (\sigma_t^I)^2 D_p^2 w + 2\varrho h_t \sigma_t^i \sigma_t^I D_p D_{\varpi^i} w + 2\varrho h_t \sigma_t^i \sigma_t^I D_{\varphi^i} w + 2\varrho h_t \sigma_t^i \sigma_t^i \sigma_t^I D_{\varphi^i} w + 2\varrho h_t \sigma_t^i \sigma_t^i \sigma_t^i \sigma_t^i \sigma_t^i w + 2\varrho h_t \sigma_t^i \sigma_t^i \sigma_t^i \sigma_t^i \sigma_t^i \sigma_t^i w + 2\varrho h_t \sigma_t^i \sigma_t^i \sigma_t^i \sigma_t^i \sigma_t^i \sigma_t^i \sigma_t^i \sigma_t^i \sigma_t^i w + 2\varrho h_t \sigma_t^i \sigma_t^i$$

We notice that the minimisers a, b in the above equation have to satisfy

$$a = \frac{1}{2\varrho} \left(e^{rt} D_{\varpi^i} w - \pi \right) \quad \text{and} \quad b = \frac{1}{2\nu} \left(p - e^{rt} D_{\varpi^i} w \right).$$
(8)

Furthermore, we notice that the second-order condition is satisfied for all *a*, *b*. This yields the following **Lemma 1.** The HJB equation can be rewritten as

$$0 = e^{rt} \left(D_t w + rp D_p w + r\pi D_\pi w \right) + \frac{e^{rt}}{2} tr(\Sigma \Sigma' D_z^2 w) - \frac{1}{4\varrho} \left(e^{rt} D_{\varpi^i} w - \pi \right)^2 - \frac{1}{4\nu} \left(p - e^{rt} D_{\varpi^i} w \right)^2.$$
(9)

In order to enforce the constraint $\mathbb{E}_t[\varpi_{\tau}^i] = 0$ for all t, we impose the singular terminal condition

$$\lim_{t \nearrow \tau} w(t, \varpi^{i}, p, \pi) = \begin{cases} 0 & : \varpi^{i} = 0, \\ \infty & : \varpi^{i} \neq 0. \end{cases}$$
(10)

Theorem 2. The HJB equation (9), together with the terminal conditon (10) is solved by

$$w(t, \varpi^{i}, p, \pi) = \frac{r\nu\varrho(\varpi^{i})^{2}}{(e^{r\tau} - e^{rt})(\nu + \varrho)} + e^{-rt}\left(\pi + \frac{\varrho(p - \pi)}{\nu + \varrho}\right)\varpi^{i} + \frac{(1 - e^{r(\tau - t)})p^{2}}{4re^{rt}(\nu + \varrho)} + \int_{t}^{\tau} C_{s} \, ds$$

where

$$C_s = \frac{r\nu\varrho(\sigma_s^i)^2}{(e^{r\tau} - e^{rs})(\nu + \varrho)} + \frac{2\varrho^2 h_s \sigma_s^i \sigma_s^I e^{-rs}}{\nu + \varrho} + \frac{\varrho^2 h_s^2 (\sigma_s^I)^2 (1 - e^{r(\tau - s)})}{re^{rs}(\nu + \varrho)} \quad for \quad t \le s < \tau.$$

The above theorem can be proved by simple differentiation. The verification argument for w is straightforward but lengthy. Thus, we omit the full proof. We note that standard arguments of verification confirm α^i , β^i as the firm's optimal strategies. Substituting $D_{\varpi^i} w$ in Equation (8) yields

$$\alpha_t^i = \frac{P_t - \Pi_t}{2(\nu + \varrho)} + \frac{\nu}{\nu + \varrho} \ h_t \varpi_t^i \quad \text{and} \quad \beta_t^i = \frac{P_t - \Pi_t}{2(\nu + \varrho)} - \frac{\varrho}{\nu + \varrho} \ h_t \varpi_t^i$$

This proves the equilibrium strategies α^i , β^i to be given as above for all $i \in I$. Furthermore, the aggregate abatement is given by

$$\alpha_t = re^{rt} \frac{\overline{\omega}_0^I}{e^{r\tau(0)} - 1} + re^{rt} \int_0^t \frac{d\mathbb{E}_s[Y^I]}{e^{r\tau(s)} - e^{rs}}$$

and, accordingly, the market-clearing price process is given by

$$P_t = \Pi_t + 2\varrho \frac{re^{rt}}{e^{r\tau(0)} - 1} \ \varpi_0^I + 2\varrho re^{rt} \int_0^t \frac{d\mathbb{E}_s[Y^I]}{e^{r\tau(s)} - e^{rs}}.$$

Let ϵ_t^I and g_t^I denote time-*t* aggregate emissions before abatement and aggregate allocations, respectively. At τ , the inter-temporal optimisation problem breaks down and thus $\alpha_{\tau}^I = \epsilon_{\tau}^I - g_{\tau}^I$. Also, by the definition of τ and the zero-borrowing constraint, we have that $\min_t B_t^I = B_{\tau}^I = 0$, which yields the first order condition $dB_t^I = (g_t^I - \epsilon_t^I + \alpha_t^I)dt = 0$ for $t = \tau$. Both conditions coincide and yield

$$\mathbb{E}_t[\alpha_\tau^I] = re^{r\tau(t)} \left(\frac{\varpi_0^I}{e^{r\tau(0)} - 1} + \int_0^t \frac{d\mathbb{E}_s[Y^I]}{e^{r\tau(s)} - e^{rs}} \right) = \mathbb{E}_t[\epsilon_\tau^I - g_\tau^I].$$

This implies that $\tau(t)$ is given by

$$\tau(t) = \frac{1}{r} \left(\ln \left(\mathbb{E}_t [\epsilon_\tau^I - g_\tau^I] \right) - \ln \left(r \frac{\varpi_0^I}{e^{r\tau(0)} - 1} + r \int_0^t \frac{d\mathbb{E}_s[Y^I]}{e^{r\tau(s)} - e^{rs}} \right) \right).$$

In particular, $\tau(0)$ is given in terms of the implicit function

$$\tau(0) = \frac{1}{r} \ln \left(\frac{\mathbb{E}_0[\epsilon_\tau^I - g_\tau^I](e^{r\tau(0)} - 1)}{r\varpi_0^I} \right),$$

where ϖ_0^I depends on $\tau(0)$.

B The model under risk-aversion

Recall that

$$h_t = \frac{re^{rt}}{e^{r\tau(t)} - e^{rt}}, \quad \text{where} \quad \tau(t) = \mathbb{E}_t^{\mathbb{Q}}[\tau].$$

Under risk-aversion, q_t represents the (possibly time-dependent) risk-premium and the allowance price is

$$dP_t = (r+q_t)P_t dt + 2\rho h_t \sigma_t dW_t^{\mathbb{P}}.$$

Recalling that under the risk-neutral measure $\mathbb Q$ we have

$$dP_t = rP_t dt + 2\rho h_t \sigma_t dW_t^{\mathbb{Q}},$$

we obtain the change of measure by requiring

$$dW_t^{\mathbb{Q}} = dW_t^{\mathbb{P}} + \underbrace{\frac{q_t}{2\varrho h_t \sigma_t} P_t}_{=:\zeta_t} dt.$$
(11)

We then obtain the Radon-Nikodým density of \mathbb{Q} with respect to \mathbb{P} , restricted on \mathcal{F}_t as

$$\frac{d\mathbb{Q}}{d\mathbb{P}}\Big|_{\mathcal{F}_t} = L_t = \exp\left(-\int_0^t \zeta_s \ dW_s^{\mathbb{P}} - \frac{1}{2}\int_0^t \zeta_s^2 \ ds\right),$$

since $L = (L_t)_{t \ge 0}$ is in fact a martingale. Recall that the expected residual abatement requirement for the aggregate market follows the dynamics

$$d\varpi_t = -\alpha_t dt + \sigma_t dW_t^{\mathbb{Q}}.$$

Let α_t^A denote the aggregate abatement under risk-aversion. By definition of the aggregate abatement requirement, we obtain that

$$d\varpi_t = -\alpha_t^A dt + \sigma_t dW_t^{\mathbb{P}}.$$

The two equations above imply

$$dW_t^{\mathbb{Q}} = dW_t^{\mathbb{P}} + \frac{\alpha_t - \alpha_t^A}{\sigma_t} dt.$$

Comparing this to Equation (11) reveals that the following must hold:

$$\alpha_t - \alpha_t^A = \frac{q_t}{2\varrho h_t} P_t. \tag{12}$$

This shows that aggregate abatement under risk-aversion is strictly smaller than under risk-neutrality, whenever $P_t > 0$ and $t < E_t^{\mathbb{Q}}[\tau]$. However, notice that

$$\alpha_t^A \to \alpha_t \quad \text{for} \quad t \to \mathbb{E}_t^{\mathbb{Q}}[\tau].$$

Furthermore, notice that since $P_t = \Pi_t + 2\rho\alpha_t$ we can directly relate α_t to α_t^A :

$$\alpha_t^A = \alpha_t - \frac{q_t}{2\varrho h_t} \left(\Pi_t + 2\varrho \alpha_t \right) = \frac{h_t - q_t}{h_t} \alpha_t - \frac{q_t \Pi_t}{2\varrho h_t}.$$

We now want to examine how changes to the allocation affect how the market processes risk. First notice that, given the real-world measure \mathbb{P} , the risk-neutral measure is parameterised by the risk-premium q_t . And, conversely, q_t becomes an implicit function of \mathbb{Q} . Therefore, we can fix \mathbb{Q} in order to see how q_t is affected by the time-t allocation of allowances, denoted by g_t . Since the timing of allocation does not affect the equilibrium in \mathbb{Q} -expectation, σ_t and q_t are the only parameter that are then affected by g_t . Fixing \mathbb{Q} in Equation (11) then yields

$$\frac{\partial}{\partial g_t} \mathbb{E}^{\mathbb{Q}} \left[\frac{q_t}{2\varrho h_t \sigma_t} P_t \right] = \frac{\mathbb{E}^{\mathbb{Q}} P_t}{2\varrho h_t \sigma_t} \frac{\partial q_t(g_t)}{\partial g_t} - \frac{q_t \mathbb{E}^{\mathbb{Q}} P_t}{2\varrho h_t \sigma_t^2} \frac{\partial \sigma(g_t)}{\partial g_t} = 0,$$

from which we obtain

$$\frac{\frac{\partial q_t(g_t)}{\partial g_t}}{q_t} = \frac{\frac{\partial \sigma(g_t)}{\partial g_t}}{\sigma_t},$$

that is, changes to σ through adjustments to the allowance allocation programme are equally reflected in changes to the risk-premium q_t .

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