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April 2016

Centre for Climate Change Economics and Policy Working Paper No. 264

Grantham Research Institute on Climate Change and the Environment

Working Paper No. 233

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Losing the environment: the endowment effect and environmental discounting

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January 29, 2016

Abstract

It has recently been shown that, when discounting future improvements in the environment, relative prices matter. However, we argue relative prices are not the whole story. Not only is the environment a consumption good in its own right, with a corresponding environmental discount rate that depends on relative scarcity, it also matters that we tend to be *losing* it. That is, there is a considerable body of evidence from behavioural economics and stated-preference valuation showing that we are loss-averse, even in riskless choice settings. Therefore in this paper we introduce reference dependence and loss aversion – the endowment effect – to a model where welfare depends on consumption of a produced good and of environmental quality. We show that the endowment effect modifies the discount rate by introducing (i) an instantaneous endowment effect and (ii) a reference-level effect. Moreover we show that, when environmental quality is strictly decreasing, these two effects mostly combine to dampen our usual preference to smooth consumption over time – perhaps surprisingly, the endowment effect *increases* the environmental discount rate on these paths. In addition, on non-monotonic paths the endowment effect can give rise to substantial discontinuities in the discount rate.

Keywords: endowment effect, environmental discount rate, loss aversion, reference dependence, relative prices, social discount rate

JEL codes: D03, D61, H43, Q51

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"There is substantial evidence that initial entitlements do matter and that the rate of exchange between goods can be quite different depending on which is acquired and which is given up" (Tversky and Kahneman, 1991, p1039)

1 Introduction

Recent contributions have reminded us that, when discounting future improvements in environmental quality, relative prices matter (Weikard and Zhu, 2005; Hoel and Sterner, 2007; Sterner and Persson, 2008; Traeger, 2011).[1] The starting point is that, in a model where utility is obtained from the consumption of at least two goods, there is a discount rate for each of these goods, and each discount rate depends not only on consumption of the good in question, it also depends on the consumption of other goods, i.e. on relative scarcity, by virtue of the cross elasticity of marginal utility.

Typically there are just two goods in these models, one of which is environmental quality and the other of which is a composite of the remaining produced goods.[2] Improvements in environmental quality can be discounted at their good-specific rate, and compared with the opportunity cost of foregone 'material' consumption, also discounted at its good-specific rate, by imputing the initial exchange rate between the two goods. An equivalent approach is to discount future improvements in environmental quality at the rate pertaining to the produced good, *as long as* the change in the relative price of environmental quality is also factored in. That is, future gains in environmental quality would need to be converted into future gains in material consumption, using future relative prices, as well as being discounted (Weikard and Zhu, 2005).

Hoel and Sterner (2007) and Traeger (2011) explore the combined effect of discounting and relative price changes. Assuming material consumption is growing faster than environmental quality, Traeger's Proposition 1 (p218) is a particularly clear expression of the result that the wedge between the discount rates on the composite produced good and on environmental quality respectively is increasing in the difference between the consumption growth rates of the two goods, and the more limited is the degree of substitutability between the two goods. Moreover the larger is this wedge, the greater is the relative weight given to environmental quality. Sterner and Persson (2008) relate these ideas to the present value of reductions in greenhouse gas emissions, showing using the DICE Integrated Assessment Model (Nordhaus, 2008) that, when environmental quality is treated as a consumption good that is negatively impacted by climate change, much greater reductions in emissions are optimal than would otherwise be the case.

In our paper, we connect this line of thought, which is surely an important contribution to the literature on discounting investments in the future environment, with an entirely separate line of thought, which has been fundamental to our understanding of preferences as a whole. In one of their classic experiments, Kahneman et al. (1990) endowed half of their subjects with a coffee mug, asking them for the lowest price at which they would sell it, while the other half were asked how much they would pay for the same mug. Although conventional consumer theory would have it that the selling and buying prices should be

[1]The role of changing relative prices in the appraisal and optimal design of development projects with environmental costs was originally explored by Fisher et al. (1972) and Fisher and Krutilla (1975).

[2]This is in fact a familiar model in the literature on optimal growth with environmental pollution (see Xepapadeas, 2005, for a summary).

approximately the same, subjects endowed with the mug – those who would stand to lose it – were prepared to sell for a median price of \$5.79, more than twice as much as the remaining subjects – those who would stand to gain it – were willing to pay (see also Knetsch, 1989, 1992).[3] What Kahneman et al. therefore showed was that the reference point matters, and in particular that losses are ascribed more value than equivalent gains, which has been termed the 'endowment effect' (Thaler, 1980). As well as experiments, the endowment effect is consistent with a ubiquitous feature of contingent valuation studies into non-market goods, whereby there is a spread between stated willingness to accept compensation and willingness to pay (Horowitz and McConnell, 2002). It is also consistent with studies of various sorts into status quo bias (e.g. Samuelson and Zeckhauser, 1988; Knetsch, 1989),[4] and has been demonstrated in field studies (e.g. Genesove and Mayer, 2001).

So what is the connection between the role of relative price changes in discounting on the one hand, and the endowment effect on the other hand? The connection is that the scenario motivating interest in the former is one in which material consumption is increasing, while environmental quality is decreasing (Hoel and Sterner, 2007; Traeger, 2011). However, from the standpoint of the endowment effect, the fact that environmental quality is becoming relatively more scarce in this scenario is not the most interesting feature. Rather, the most interesting feature is that gains in material consumption are being weighed against losses in environmental quality. The purpose of this paper is hence to integrate the endowment effect in a two-good discounting model, where the two goods are a composite produced good and environmental quality. In particular, we would like to know how the endowment effect modifies the discount rate on each good, and how it modifies the discount rate for an 'environmental project', which requires current material consumption to be sacrificed in return for future improvements in environmental quality. Previous papers have established that this is the environmental discount rate (Weikard and Zhu, 2005; Hoel and Sterner, 2007).

In the next section we introduce the preferences on which our analysis rests, and explore some basic implications of them for the valuation of consumption paths in a highly simplified setting. We employ a utility function that combines in a tractable, additive framework both the traditional model of consumer theory, in which instantaneous utility is derived from the level of consumption of each of the two goods, and the 'behavioural' model, in which utility derives from gains and losses relative to a reference point. This function, and the assumptions we impose on it, builds on contributions by Bowman et al. (1999) and subsequently Kőszegi and Rabin (2006) in a single-good setting. In Section 3 we derive expressions for the discount rate from this model of preferences, and distinguish the role of the endowment effect from that of relative price changes. Doing so requires us to characterise how past consumption influences the appraisal of current gains and losses, and for this our work is related to the seminal paper by Ryder and Heal (1973), which explores reference dependence (but not loss aversion) in a single-good

[3] To allay concerns that the disparity could have been due to differences in wealth between subjects, Kahneman et al. conducted a further experiment, in which, rather than being asked how much they would be willing to pay to buy the mug, subjects were given the option of being gifted the mug or a sum of money, and asked at what value they would choose money over the mug. Those endowed with the mug were prepared to sell for around \$7, whereas those invited to choose valued the mug at under \$3.50.

[4] Status quo bias can, of course, also be explained in other ways, such as the existence of search and transaction costs.

optimal growth model. We show that the endowment effect modifies the discount rate via (i) an instantaneous endowment effect and (ii) a reference-level effect, and that the overall effect on the discount rate depends on the combination of these. However, we show that this combined effect can be related to stylised patterns of growth and decline in consumption, which means that the theoretical implications of the endowment effect for discounting emerge more clearly. Section 4 goes on to offer a numerical analysis of our model, with a view to establishing whether the endowment effect is quantitatively important in comparison with relative price change, and standard features of discounting, i.e. pure time preference and consumption smoothing. Section 5 offers some concluding comments.

2 Preferences and their basic implications

2.1 The utility function

We begin by setting out the assumptions we require our utility function to satisfy. We assume there exists a real-valued, instantaneous utility function, which depends on the consumption of two goods, a composite produced good $C \in [0, \infty)$ and the quality of the natural environment $E \in \left[0, \overline{E}\right]$.[5] But it does not just depend on the level of consumption of each of these, it also depends on the difference, for each good, between the level of consumption and a reference level. In particular, we further introduce gain-loss functions of the two goods (Kahneman and Tversky, 1979).

Our instantaneous utility function $U: \mathcal{R}^2 \times \mathcal{R}_+^2 \rightarrow \mathcal{R}$ in period t is

$$U_t(C_t, \underline{C}_t, E_t, \underline{E}_t) = v\left(C_t, E_t\right) + g\left(C_t - \underline{C}_t\right) + h\left(E_t - \underline{E}_t\right), \tag{1}$$

where $\underline{C}$ and $\underline{E}$ are the reference levels of the produced good and environmental quality respectively. We shall explain later how these are formed. In general, reference dependence allows for preferences to be inter-temporally dependent, in which case U_t still represents a flow of utility in period t, albeit it may not only depend on consumption in period t.

Instantaneous utility in (1) represents a mixed objective. The function $v(.)$ corresponds with the standard theory of preferences, in that individual utility remains directly responsive to the absolute level of consumption. Hence we shall refer to this element of the instantaneous utility function as *consumption-level utility*. We assume $v(.)$ is continuous and twice continuously differentiable in its arguments. Specifically we assume that ${}^{\partial v(.)}/_{\partial C_t} > 0$, ${}^{\partial^2 v(.)}/_{\partial C_t^2} < 0$, ${}^{\partial v(.)}/_{\partial E_t} > 0$ and ${}^{\partial^2 v(.)}/_{\partial E_t^2} < 0$. By contrast, the *gain-loss* functions $g(.)$ and $h(.)$ capture the endowment effect. They are assumed to be continuous, and twice continuously differentiable except when $C_t - \underline{C}_t = 0$ and $E_t - \underline{E}_t = 0$ respectively. A feature of (1) is that it does not place a restriction on ${}^{\partial^2 v(.)}/_{\partial C_t \partial E_t}$, whereas, by virtue of the additive way in which reference dependence enters the utility function, g' is assumed independent of the level or change in E, and h' is likewise assumed independent of the level or change in C. The analysis is considerably simplified as a consequence, without compromising the basic insights it yields.

[5] It would be obvious to interpret $\overline{E}$ as a pristine natural environment, for instance a primary rainforest, or the pre-industrial concentration of greenhouse gases in the atmosphere. But it can in principle stand for a human-modified natural environment instead, for instance an ancient agricultural landscape.

Let us now place some further behavioural restrictions on the gain-loss functions $g(.)$ and $h(.)$. Bowman et al. (1999) and later Kőszegi and Rabin (2006) proposed similar restrictions, in a single-good setting, as a formal representation of the value function in Kahneman and Tversky (1979).

Assumption 1. *[Bigger gains and smaller losses are weakly preferred]* $g' \geq 0$ *and* $h' \geq 0$.

Assumption 1 is analogous to weak non-satiation in the standard theory of preferences.

Assumption 2. *[Loss aversion for small changes] If* $x > 0$*, then* $\lim_{x \to 0} \frac{g'(-x)}{g'(x)} \equiv L_C > 1$ *and* $\lim_{x \to 0} \frac{h'(-x)}{h'(x)} \equiv L_E > 1$.

Assumption 2 represents loss aversion as made famous by Kahneman and Tversky (1979). To fix ideas formally, it will sometimes prove helpful to compare it with the assumption that $\lim_{x \to 0} \frac{g'(-x)}{g'(x)} = 1$ and $\lim_{x \to 0} \frac{h'(-x)}{h'(x)} = 1$, which we will refer to as 'loss neutrality for small changes'.

Assumption 3. *[Non-increasing sensitivity]* $g''(x) \leq 0$ *and* $h''(x) \leq 0$ *for all* $x > 0$*, and* $g''(x) \geq 0$ *and* $h''(x) \geq 0$ *for all* $x < 0$.

Non-increasing sensitivity in a riskless choice setting was invoked by Tversky and Kahneman (1991) to condition preferences, where the reference level does not coincide with the prospective consumption level for any good in the individual's bundle (they worked in a setting with two goods). The stronger assumption of diminishing sensitivity is required to represent a preference such as: "the difference between a yearly salary of \$60,000 and a yearly salary of \$70,000 has a bigger impact when current salary is \$50,000 than when it is \$40,000" (Tversky and Kahneman, 1991, p1048).

2.2 Some basic implications

Before we commence with our main analysis of discounting, it is helpful to show some of the basic effects of these assumptions on the valuation of consumption paths, which we shall do in a highly simplified setting.

The simple endowment effect with environmental degradation Starting with a model comprising two periods, the context is one in which environmental quality is decreasing, $E_2 < E_1$, and the change is small. The reference level is simply given by the previous period's consumption, $\underline{C}_2 = C_1$ and $\underline{E}_2 = E_1$.[6] That is, gains and losses are computed with respect to the level of consumption last enjoyed. Welfare is characterised in the standard fashion as the discounted sum of per-period utility,

$$J = U_1 + \beta U_2, \tag{2}$$

where $0 < \beta \leq 1$ is the pure time discount factor. Then it is straightforward to show that Assumption 2 gives a lower valuation to this development path than the alternative

[6] The reference level in the first period must then be exogenously specified. It is unimportant for the thought experiment here.

assumption of 'loss neutrality for small changes' – since the change in environmental quality is assumed to be small, we can substitute $h(x) = h'(x) \cdot x$ in (1) and (2), and Assumption 2 ensures $h'(-x) = h'(x) \cdot L_E$, where $L_E > 1$.

This is a simplified version of the more general observation that, in an (arbitrary) economy in which environmental quality deteriorates, the endowment effect – formalised here through loss aversion for small changes – leads individuals to obtain less instantaneous utility all along such a growth path than under loss neutrality for small changes. A utilitarian welfare criterion with $\beta > 0$ is sufficient to ensure the whole path is correspondingly valued lower.

Diminishing sensitivity to environmental degradation What of the implications of the strong form of Assumption 3, diminishing sensitivity? Define $E_2 - E_1 = \varepsilon$, $\varepsilon < 0$, and assume in this case that $C_2 = C_1$. Then

$$\frac{\partial^2 J}{\partial \varepsilon^2} = \beta \cdot \frac{\partial^2 v}{\partial E^2} (C_2, E_1 + \varepsilon) + \beta \cdot h''(\varepsilon).$$

As the loss in environmental quality increases, welfare obviously decreases, but whether it does so at an increasing, decreasing or constant rate depends on the opposing effects of concave consumption-level utility, which tends to raise the rate at which welfare decreases, and convex loss utility, which tends to depress the rate at which it decreases. In particular, welfare decreases in the size of loss of environmental quality ε at a diminishing rate if and only if

$$h''(\varepsilon) > \frac{\partial^2 v}{\partial E^2} (C_2, E_1 + \varepsilon) .$$

By contrast, in an economy where environmental quality *grows* between periods by ε, then

$$\frac{\partial^2 J}{\partial \varepsilon^2} < 0,$$

because diminishing sensitivity to gains in consumption *reinforces* the tendency for marginal consumption-level utility to diminish. The same clearly goes for increasing and decreasing consumption of the produced good respectively.

The effect of a longer memory Lastly, let us consider a richer model of reference-level formation, which necessitates adding an additional period, so that we now have three in total. In particular, consider a reference level in period t, which depends on previous reference levels, as well as changes in past consumption. Looking at any two successive periods,

$$\underline{C}_t = (1 - \alpha)\underline{C}_{t-1} + \alpha C_{t-1},$$
$$\underline{E}_t = (1 - \alpha)\underline{E}_{t-1} + \alpha E_{t-1}.$$

The parameter $\alpha \in [0, 1]$ characterises the responsiveness of the reference level to changes over time in consumption of the goods (Bowman et al., 1999). It represents an individual's memory for past consumption; the smaller is α, the longer that memory is. If $\alpha = 1$, then the current reference level is just the last period's consumption as above. At the other extreme, if $\alpha = 0$ then the current reference level is the same as the initial, exogenous reference level. It is worth noting that there is empirical support for the idea that a long history of consumption levels determines the reference level (Strahilevitz and Loewenstein, 1998). We could of course easily assign different memory parameters to each of the goods, however it is an unnecessary complication.

Again we assume an economy in which environmental quality is decreasing, $E_t < E_{t-1}$ for all t, and consumption of the produced good is constant, $C_t = C_{t-1}$ for all t. On this occasion it matters what the initial reference level $\underline{E}_1$ is, and we only require, in keeping with the assumption that environmental quality is on a strictly decreasing path, that $\underline{E}_1 > E_1$. Then the effect of the memory parameter α on welfare is

$$\begin{aligned}\frac{\partial J}{\partial \alpha} &= \beta \cdot (\underline{E}_1 - E_1) \cdot h' (E_2 - \underline{E}_2) + \\ &\quad \beta^2 \cdot (2(1-\alpha)\underline{E}_1 + (2\alpha - 1)E_1 - E_2) \cdot h' (E_3 - \underline{E}_3) > 0,\end{aligned}$$

in other words welfare decreases, the more weight is placed on the initial reference level, i.e. the lower is α.[7] The intuition behind this result is that, on a strictly decreasing path of environmental quality, a longer memory for past consumption implies larger losses in environmental quality and this has a larger negative effect on instantaneous utility, and therefore on welfare. The opposite holds for increasing paths. Clearly if consumption of the produced good is allowed to grow, then the effect of a longer memory is ambiguous and depends on the comparative growth of each good.

With these basic insights in hand, let us now move on to our main analysis of discounting. That is, we move from a setting in which the task is to value consumption paths, to one in which the task is to appraise marginal investments on a given path. The switch in focus turns out to be important, because the endowment effect has more complex implications.

3 Discounting with the endowment effect

The purpose of this section is to examine how environmental projects are discounted in the presence of the endowment effect. By an environmental project we mean a small investment with an opportunity cost that is paid in units of the produced good, typically at the beginning of the time horizon, and a future benefit that is received in extra units of environmental quality. As we explained in the Introduction, the evaluation of such projects raises two connected issues: the first is the good-specific discount rate on future consumption; the second is the relative scarcity of environmental quality and how that might change in the future (Weikard and Zhu, 2005; Hoel and Sterner, 2007; Traeger, 2011).

It is convenient in thinking about discounting to switch to continuous time. The welfare functional is then

$$J = \int_0^\infty U_t e^{-\delta t} dt, \tag{3}$$

where we specialise to exponential pure time discounting at rate $\delta > 0$. There is also an equivalent in continuous time to the evolution of reference consumption levels in the

[7] Note that $2(1-\alpha)\underline{E}_1 + (2\alpha-1)E_1 - E_2 = -2\alpha(\underline{E}_1 - E_1) + 2\underline{E}_1 - E_1 - E_2 > -2\alpha(\underline{E}_1 - E_1) + 2(\underline{E}_1 - E_1) > 0$.

previous section, whereby $\dot{\underline{C}}_t = \alpha(C_t - \underline{C}_t)$ and $\dot{\underline{E}}_t = \alpha(E_t - \underline{E}_t)$ (Ryder and Heal, 1973), so that the reference level at time t, as a function of the history of consumption levels, is

$$\begin{array}{l} \underline{C}_t = \alpha \int_{-\infty}^{t} e^{-\alpha(t-\tau)} C_\tau d\tau, \\ \underline{E}_t = \alpha \int_{-\infty}^{t} e^{-\alpha(t-\tau)} E_\tau d\tau. \end{array} \tag{4}$$

3.1 The produced-good consumption discount rate

For an individual with preferences given by Equations (1), (3) and (4), Appendix 1 shows that the marginal rate of substitution between consumption of the produced good at date 0 and date t, the produced-good discount factor, is

$$\begin{aligned} D^C(t,0) &\equiv \frac{\partial J/\partial C_t}{\partial J/\partial C_0} \\ &= e^{-\delta t} \frac{\partial v/\partial C_t + g'\left(C_t - \underline{C}_t\right) - \alpha \int_{\tau=t}^{\infty} e^{-(\alpha+\delta)(\tau-t)} g'\left(C_\tau - \underline{C}_\tau\right) d\tau}{\partial v/\partial C_0 + g'\left(C_0 - \underline{C}_0\right) - \alpha \int_{\tau=0}^{\infty} e^{-(\alpha+\delta)\tau} g'\left(C_\tau - \underline{C}_\tau\right) d\tau}, \end{aligned} \tag{5}$$

where the important feature is that $\partial J/\partial C_t$ is a functional derivative. By contrast, under a standard model of preferences for consumption of the two goods, where instead of (1) we simply have

$$U_t(C_t, E_t) = v(C_t, E_t), \tag{6}$$

i.e. gain-loss utility is omitted, the discount factor is just

$$\widehat{D^C}(t,0) \equiv \frac{\partial J/\partial C_t}{\partial J/\partial C_0} = e^{-\delta t} \frac{\partial v/\partial C_t}{\partial v/\partial C_0},$$

so the additional considerations raised by the endowment effect are immediately clear. As well as providing consumption-level utility $\partial v/\partial C_t$, a unit of consumption of the produced good at time t provides a contemporaneous gain, which contributes to utility via the gain-loss function, i.e. via g'. We will refer to this as the *instantaneous endowment effect*, which is the second element of $\partial J/\partial C_t$ in (5). In addition, a unit of consumption at time t affects the reference level from which gains are evaluated after time t. This we will describe as the *reference-level effect*, which is the third element of $\partial J/\partial C_t$ in (5), i.e. the integral. In evaluating investment projects, forward-looking individuals will anticipate the effect that changes in consumption have on reference levels thereafter. The reference-level effect is negative, because an increase in consumption today raises future reference levels, and thereby reduces future gains in consumption, or increases future losses. By how much an increase in consumption today raises future reference levels depends on the memory parameter α, and what effect this in turn has on welfare depends on the pure time discount rate δ. Another way to think of the reference-level effect is in relation to the literature on habit formation (e.g. Constantinides, 1990): a unit of consumption at time t contributes to our becoming habituated to higher consumption, which in turn reduces the marginal contribution to welfare of future increments in consumption.

It is important to note the role of loss aversion is implicit so far. That is, (5) could just as well describe a model with reference dependence, but without loss aversion, or in other words a model without Assumption 2. Loss aversion will have a quantitative effect on g' and therefore on the instantaneous endowment effect and the reference-level effect.

Appendix 1 goes on to show that the produced-good discount rate in the presence of the endowment effect can be expressed as

$$\begin{aligned} r^C &= \frac{d}{dt}\ln D^C(t,0), \\ &= \delta - \frac{v_{CC}\dot{C} + v_{CE}\dot{E} + g''(\dot{C} - \alpha C + \alpha \underline{C}) + \alpha g' - \alpha(\alpha+\delta)\int_{\tau=t}^{\infty} e^{-(\alpha+\delta)(\tau-t)} g' d\tau}{v_C + g' - \alpha \int_t^{\infty} e^{-(\alpha+\delta)(\tau-t)} g' d\tau}, \end{aligned} \tag{7}$$

where we drop the time subscripts for convenience's sake, use dots to indicate total derivatives with respect to time, and switch to using subscripts to indicate partial derivatives. Since this is still a rather complex expression, it is helpful to clean it up. To do so, we define the shadow price of reference consumption as

$$\mu^{\underline{C}} = \int_{\tau=t}^{\infty} e^{-(\alpha+\delta)(\tau-t)} g' d\tau. \tag{8}$$

This is the marginal effect on welfare at time t of reducing the reference level, without changing the consumption path. In Appendix 2, we show that if our discounting model is intergrated in a characteristic problem of optimal management of environmental quality, then $\mu^{\underline{C}}$ is also the negative of the costate variable on reference consumption. Substituting $\mu^{\underline{C}}$ into (7), we obtain

$$r^C = \delta - \frac{\dot{v_C} + \dot{g_C} - \alpha \dot{\mu^{\underline{C}}}}{v_C + g_C - \alpha \mu^{\underline{C}}}. \tag{9}$$

If we define the absolute value of the elasticity of consumption-level marginal utility of the produced good with respect to consumption of the produced good as

$$\eta^{CC} \equiv \frac{-v_{CC}C}{v_C},$$

and the elasticity of consumption-level marginal utility of the produced good with respect to environmental quality as

$$\eta^{CE} \equiv \frac{v_{CE}E}{v_C},$$

then we finally obtain a more convenient and recognisable expression for r^C:

Definition 1. In the presence of the endowment effect as characterised by Eq. (1), the produced-good consumption discount rate is

$$r^C = \delta + \theta^C \left(\eta^{CC} \frac{\dot{C}}{C} - \eta^{CE} \frac{\dot{E}}{E} \right), \tag{10}$$

where the 'produced-good endowment factor' is

$$\theta^C = \frac{1 + \frac{\dot{g'}}{\dot{v_C}} + \frac{\alpha \dot{\mu^{\underline{C}}}}{\dot{v_C}}}{1 + \frac{g'}{v_C} + \frac{\alpha \mu^{\underline{C}}}{v_C}}. \tag{11}$$

Equation (10) shows that the endowment effect modifies the consumption discount rate in a model with produced goods and environmental quality through the factor θ^C. In the absence of the endowment effect $\theta^C = 1$.

3.2 Discounting an environmental project

As discussed, in evaluating an environmental project we must take into account not only the good-specific marginal rate of inter-temporal substitution, we must also take into account any relative price change. In particular, an environmental project paid for in units of the produced good at date 0, which increases environmental quality at a future date t, is welfare-preserving if and only if $J_{C_0} dC_0 = -J_{E_t} dE_t$. Define the accounting price of environmental quality as $p_t \equiv J_{E_t}/J_{C_t}$. Then the project is welfare-preserving if and only if

$$J_{C_0} dC_0 = -J_{E_t} \frac{dC_t}{p_t}.$$

The appropriate discount factor in a trade-off between consumption of the produced good at date 0 and environmental quality at date t depends on how environmental quality is priced. If environmental quality at date t is converted into units of the produced good using the relative price p_t, the appropriate comparison is between consumption of the produced good today and in the future using the discount factor D^C from (5). If, by contrast, environmental quality is converted into units of the produced good at p_0, the appropriate comparison is between environmental quality today and in the future using an environmental discount factor D^E. This is

$$D^E(t,0) = \frac{J_{E_t}}{J_{E_0}} = \frac{J_{C_t}}{J_{C_0}} \frac{p_t}{p_0},$$

which leads to the following equivalence between environmental and consumption discount rates (Weikard and Zhu, 2005; Hoel and Sterner, 2007):

$$r = r^E = r^C - \frac{\dot{p}}{p}, \tag{12}$$

where $\dot{p}/p$ is the relative price change . Expanding the term $\dot{p}/p$ we get:

$$\begin{aligned}
\frac{\dot{p}}{p} &\equiv \frac{\frac{d}{dt}\left(J_E/J_C\right)}{J_E/J_C} \\
&= \frac{v_{EE}\dot{E} + v_{CE}\dot{C} + \dot{h}' + \alpha h' - \alpha(\alpha+\delta)\int_t^\infty e^{-(\alpha+\delta)(\tau-t)} h' d\tau}{v_E + h' - \alpha \int_t^\infty e^{-(\alpha+\delta)(\tau-t)} h' d\tau} \\
&\quad - \frac{v_{CC}\dot{C} + v_{CE}\dot{E} + \dot{g}' + \alpha g' - \alpha(\alpha+\delta)\int_t^\infty e^{-(\alpha+\delta)(\tau-t)} g' d\tau}{v_C + g' - \alpha \int_t^\infty e^{-(\alpha+\delta)(\tau-t)} g' d\tau} \\
&= \theta^E \left(\eta^{EC} \frac{\dot{C}}{C} - \eta^{EE} \frac{\dot{E}}{E}\right) - \theta^C \left(\eta^{CE} \frac{\dot{E}}{E} - \eta^{CC} \frac{\dot{C}}{C}\right),
\end{aligned} \tag{13}$$

where η^{EC} is the elasticity of consumption-level marginal utility of environmental quality with respect to consumption of the produced good,

$$\eta^{EC} \equiv \frac{v_{CE} C}{v_E},$$

and η^{EE} is the elasticity of consumption-level marginal utility of environmental quality with respect to environmental quality,

$$\eta^{EE} \equiv \frac{-v_{EE} E}{v_E}.$$

Substituting (9) and (13) into (12), and defining the shadow price of reference environmental quality as

$$\mu^{\underline{E}} = \int_{\tau=t}^{\infty} e^{-(\alpha+\delta)(\tau-t)} h' d\tau, \tag{14}$$

we obtain the IRR of the project or equivalently the discount rate on environmental quality:

Definition 2. In the presence of the endowment effect as characterised by Eq. (1), the discount rate for an environmental project is

$$r^E = \delta + \theta^E \left(\eta^{EE} \frac{\dot{E}}{E} - \eta^{EC} \frac{\dot{C}}{C} \right) \tag{15}$$

where the 'environmental endowment factor' is

$$\theta^E = \frac{1 + \frac{\dot{h'}}{\dot{v_E}} + \frac{\alpha \dot{\mu^{\underline{E}}}}{\dot{v_E}}}{1 + \frac{h'}{v_E} + \frac{\alpha \mu^{\underline{E}}}{v_E}}. \tag{16}$$

Equation (15) shows the distinct roles played by the endowment effect and changes in the relative scarcity of environmental quality in modifying the produced-good consumption discount rate for the purposes of evaluating an environmental project. When accounting for the change in relative prices (12), the individual incorporates his/her desire to smooth consumption of environmental quality between dates, while growth (or decline) in consumption of the produced good affects the IRR through the cross-elasticity η^{EC}. If we are in a setting where environmental quality is declining while consumption of the produced good is increasing, $\eta^{EE} \cdot \dot{E}/E < 0$, which will reduce the IRR. The sign of $\eta^{EC} \cdot \dot{C}/C$ is ambiguous. Parallel to the previous analysis, the endowment effect enters via the environmental endowment factor θ^E.

3.3 The endowment factor

The obvious question that flows from Definitions 1 and 2 is, what is the sign and size of the endowment factor θ^i, $i \in \{C, E\}$? In the spirit of the main emphasis of the paper, we will focus in what follows on the environmental discount rate r^E and the corresponding environmental endowment factor θ^E, but all the analysis and results carry over to r^C and θ^C with appropriate substitution of C for E and $g(.)$ for $h(.)$.

First, note that in the particular case within Assumption 3 of constant sensitivity, i.e. $h''(x) = 0$ for all $x = E_t - \underline{E}_t$ (except $x = 0$), the environmental endowment factor on a strictly increasing/decreasing path simplifies to

$$\theta^E = \frac{1}{\left[1 + \left(\frac{\delta}{\alpha + \delta} \right) \frac{h'}{v_E} \right]}. \tag{17}$$

In this case, $0 < \theta^E < 1$ if and only if $\delta > 0$. Positive pure time preference, however small, is a fairly uncontroversial assumption. Even if the 'prescriptive' view (Arrow et al., 1996) is taken that the discount rate is derived from a social welfare functional and it should be impartial to the date at which utility is enjoyed, (very) small positive utility discounting still follows from taking into account the probability of extinction of society (e.g. Stern, 2007; Llavador et al., 2015).[8] But that is not all. Equality (17) will also hold even if preferences obey diminishing sensitivity, i.e. $h''(x) < 0$ for all $x > 0$ and $h''(x) > 0$ for all $x < 0$, along a linear increasing/decreasing consumption path. Let us prove this, and capture both of these results, in the following Proposition:

Proposition 1. *[The endowment effect dampens consumption smoothing on a linear path] On a linear increasing or decreasing consumption path, or on any strictly increasing/decreasing consumption path with constant sensitivity,* $0 < \theta^E < 1$ *if and only if* $\delta > 0$*.*

Proof. We begin by proving the environmental endowment factor is given by (17). In the case of diminishing sensitivity, but where the consumption path is linear decreasing, for any arbitrary date in the past $T \in (-\infty, t]$ we have $E_t = E_T + k(t-T),\ k < 0$ and we can write Eq. (4) as

$$\begin{aligned}
\underline{E}_t &= \alpha \int_{-\infty}^{T} e^{-\alpha(t-\tau)} E_\tau d\tau + \alpha \int_{T}^{t} e^{-\alpha(t-\tau)} [E_T + k(\tau - T)] d\tau, \\
&= \alpha \int_{-\infty}^{T} e^{-\alpha(t-\tau)} E_\tau d\tau + \alpha e^{-\alpha t} \int_{T}^{t} e^{\alpha\tau} E_T d\tau + \alpha k e^{-\alpha t} \int_{T}^{t} e^{\alpha\tau} (\tau - T) d\tau, \\
&= \alpha \int_{-\infty}^{T} e^{-\alpha(t-\tau)} E_\tau d\tau + E_T(1 - e^{-\alpha(t-T)}) + k(t-T) - \frac{k}{\alpha} + \frac{1}{\alpha} e^{-\alpha(t-T)}. \quad (18)
\end{aligned}$$

Taking the limit as T goes to minus infinity we obtain

$$\lim{}_{T\to\infty} \underline{E}_t = E_T + k(t-T) - \frac{k}{\alpha} = E_t - \frac{k}{\alpha}.$$

Therefore $h\left(E_t - \underline{E}_t\right) = h\left(\frac{k}{\alpha}\right)$, which is constant over time. If consumption follows a linear increasing path instead, T is taken to be the time when consumption was zero, $E_T = 0$. This eliminates the first two terms in Eq. (18). Since we cannot take the limit as T goes to minus infinity, we approximate the same result if T is sufficiently far in the past: $\underline{E}_t \approx E_T + k(t-T) - \frac{k}{\alpha}$. In this case too $h\left(E_t - \underline{E}_t\right) = h\left(\frac{k}{\alpha}\right)$. Therefore on a linear path the instantaneous endowment effect h' is constant, which also means that $\dot{h}_E = 0$. This is self-evidently true if preferences obey constant sensitivity, as long as consumption is strictly increasing or decreasing, in other words the increase/decrease need not be linear. Either way, since h' is constant over time, the reference-level effect

$$\alpha \int_{t}^{\infty} e^{-(\alpha+\delta)(\tau-t)} h' d\tau = \alpha h' \left[\frac{e^{-(\alpha+\delta)(\tau-t)}}{-\alpha-\delta} \right]_t^\infty = \frac{\alpha}{\alpha+\delta} h'.$$

Substituting this result into (16) results in Eq. (17). From (17), it is clear that $\delta > 0$ is a necessary and sufficient condition for $0 < \theta^E < 1$. $\square$

[8]In performing global-scale CBA this would be the extinction of the human species, but for national investment problems it would be the country in question.(Stern, 2008)

Proposition 1 considers paths along which marginal gain-loss utility is constant. Either preferences are characterised by constant sensitivity, or consumption follows a linear path. One way of expressing this is to say that

$$h'_t = h'_0 e^{kt},\ k = 0. \tag{19}$$

Put in this way, we can then see that, under *diminishing* sensitivity, $k < 0$ corresponds with a consumption path that is either convex increasing or concave decreasing (i.e. decreasing at an increasing rate), while $k > 0$ corresponds with a consumption path that is either concave increasing or convex decreasing (i.e. decreasing at a decreasing rate). This opens up further insights into the sign and size of the endowment factor on non-linear consumption paths, because we can combine (16) and (19) to describe a functional relationship between θ^E and k as follows:

$$\theta^E = \frac{1 + \frac{\dot{h}'_t}{\dot{v_{E_t}}}\left(\frac{\delta-k}{\alpha+\delta-k}\right)}{1 + \frac{h'_t}{v_{E_t}}\left(\frac{\delta-k}{\alpha+\delta-k}\right)}, \tag{20}$$

where $k = \dot{h}'/h'$.

Figures 1 and 2 plot this functional relationship on strictly decreasing consumption paths. Therefore the setting is one of declining environmental quality, a typical backdrop for investments to improve the environment. Whether a convex or concave decreasing path better describes the situation is clearly an empirical question, and depends on, among other things, the environmental (dis)amenity in question, as well as the relevant time horizon. In their exploration of the concept of the 'Anthropocene', Steffen et al. (2011) plot the evolution of 12 global environmental indicators, ranging from the atmospheric stock of greenhouse gases and ozone, to the depletion of fisheries, forests and biological diversity, and show that in all of the aforementioned cases environmental quality has in effect been on a concave decreasing path since the beginning of the industrial revolution, branding the last 70 years in particular the 'Great Acceleration'.[9] Over a shorter time horizon of a few decades, some environmental pressures such as atmospheric carbon dioxide appear to remain on an exponentially increasing path (IPCC, 2013); in our context we would again think of a concave decreasing path for environmental quality. On the other hand, we know from the literature on Environmental Kuznets Curves, for instance, that other environmental amenities such as clean local air and water can follow a convex decreasing path (Grossman and Krueger, 1995; Shafik and Bandyopadhyay, 1992; Stern, 2004; Copeland and Taylor, 2004), even beginning to eventually increase again. And, irrespective of past trends, as $E \to 0^+$ we might expect that in many (but not all) cases the rate of deterioration of environmental quality starts to slow down.

The rate of decrease of environmental quality turns out to matter here, whether the path is convex or concave. Therefore Figure 1 depicts a setting of rapidly decreasing environmental quality, defined as $\dot{v_E}/v_E > \delta + \frac{\alpha}{1+h'/v_E}$. By contrast, Figure 2 depicts the

[9] Where environmental quality is the inverse of the stock of pollution (carbon dioxide and ozone), the stock of pollution has increased exponentially. The percentage of global fisheries fully exploited, and the percentage of global forest cover destroyed since 1700, have both increased exponentially. The rate of species extinctions has increased exponentially, with approximately no species additions. See Steffen et al. (2011), figure 1.

Figure 1: The environmental endowment factor as a function of k on strictly decreasing consumption paths, where $\dot{v_E}/v_E > \delta + \frac{\alpha}{1+h_E/v_E}$.

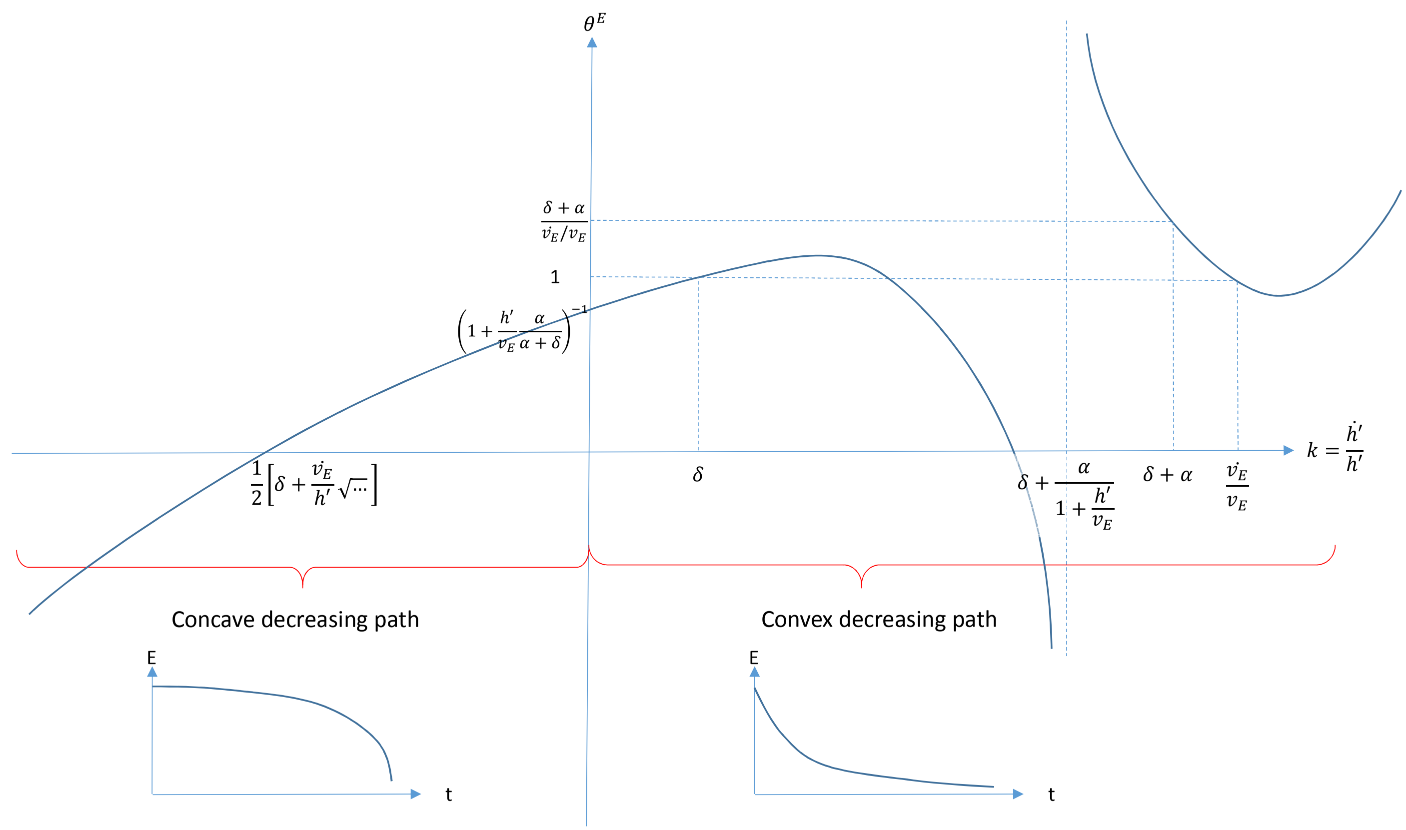
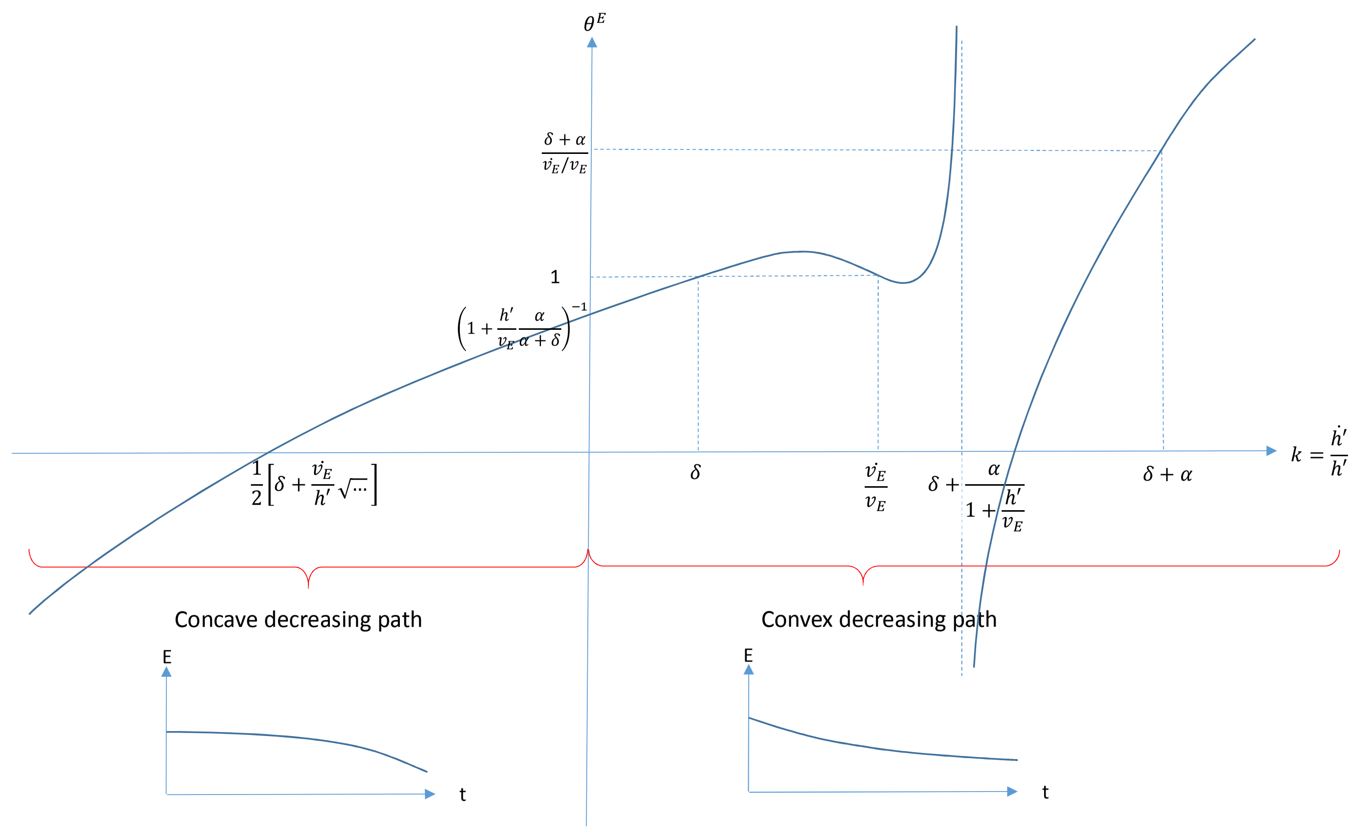


Figure 2: The environmental endowment factor as a function of k on strictly decreasing consumption paths, where $\dot{v_E}/v_E < \delta + \frac{\alpha}{1+{h_E}/{v_E}}$.



opposite setting of slowly decreasing environmental quality, where $\dot{v_E}/v_E < \delta + \frac{\alpha}{1+h'/v_E}$. Looking first at concave decreasing paths, the following Proposition is plain to see:

Proposition 2. *[The endowment factor is less than unity on concave decreasing consumption paths] When consumption is strictly decreasing and $k < 0$, it is always the case that $\theta^E < \left(1 + \frac{h'_0}{v_E}\right)^{-1} < 1$.*

When environmental quality is in decline, then – pure time preference notwithstanding – consumption-level utility most likely makes us want to postpone consumption of environmental quality until a later date, at which we enjoy less of it.[10] But Proposition 2 suggests that the endowment effect will dampen this desire to postpone consumption, when consumption is concave decreasing. On a concave decreasing path, it is always the case that $\theta^E < 1$, and when k is large negative, $\theta^E < 0$.[11] What is the intuition here? As a preliminary step, it is helpful to rewrite the environmental discount rate as

$$r^E = \delta - \frac{\dot{v_{E_t}} + \dot{h'_t} - \left(\frac{\alpha}{\alpha+\delta-k}\right)\dot{h'_t}}{v_{E_t} + h'_t - \left(\frac{\alpha}{\alpha+\delta-k}\right)h'_t}. \tag{21}$$

This makes it clear that the reference-level effect is a fixed proportion of the instantaneous endowment effect when k is constant. On a concave decreasing path, diminishing sensitivity means that marginal loss utility h' is relatively high today and relatively low in the future. This in turns means that the instantaneous endowment effect falls over time, and so does the reference-level effect, but, as Eq. (21) shows, the latter falls in proportion to the former. Moreover, the reference-level effect is always the smaller of the two effects, as long as $\delta \geq 0$. Therefore marginal welfare decreases at a faster rate than in the standard model of preferences, $\theta^E < 1$ and the discount rate increases.

On a convex decreasing path, diminishing sensitivity instead results in an increase in marginal loss utility over time. Future losses of environmental quality are smaller, and therefore the instantaneous endowment and reference-level effects grow. Again, they grow in step with each other according to (21), but of course the relationship between $k > 0$ and δ is no longer assured. If $k < \delta$, a situation in which environmental quality decreases relatively slowly, the instantaneous endowment effect is always larger than the reference-level effect, which in itself puts comparatively more weight on the future. However, when $k < \delta$ it is also the case that $k < \dot{v_{E_t}}/v_{E_t}$. That is, the overall marginal endowment effect grows more slowly than marginal consumption-level utility. Consequently $\theta^E < 1$ and the discount rate increases. If $k > \delta$ – environmental quality is decreasing more rapidly – the reference-level effect is larger than the instantaneous endowment effect. As the Figures show, this can result in $\theta^E > 1$, but the picture is complicated. There is the special case of $k = \dot{v_{E_t}}/v_{E_t}$, when clearly $\theta^E = 1$. There is also the fact that the absolute value of θ^E becomes unbounded in the limit as $k \to \delta + \frac{\alpha}{1+\frac{h'}{v_E}}$, with the limit behaviour of θ^E in the region of the vertical asymptote varying between the two panels (see Appendix 3). In this

[10] Only if consumption of the produced good were falling rapidly might the relative-price effect $\eta^{EC}\dot{C}/C$ reverse this.

[11] Specifically when $k < \frac{1}{2}\left[\delta + \frac{\dot{v_E}}{h'}\sqrt{\left(\frac{\dot{v_E}}{h'} + \delta\right)^2 + 4(\alpha+\delta)\frac{\dot{v_E}}{h'}}\right]$.

situation, the reference-level effect fully cancels out the sum of marginal consumption-level utility and the instantaneous endowment effect. If environmental quality has no value today (tomorrow), the discount rate is infinitely large (small).

Figure 3 performs the same analysis, but this time considers increasing consumption paths. This time the asymptotic behaviour of θ^E does not depend on whether environmental quality is increasing slowly or quickly, as defined above. Looking first at convex increasing paths, the following Proposition is established:

Proposition 3. *[On convex increasing paths, the endowment factor is positive but depends on how fast marginal gain-loss utility falls] When consumption is strictly increasing and $k < 0$, $0 < \theta^E < 1$ if and only if $k > \frac{\dot{v_E}}{v_E}$, otherwise $\theta^E > 1$.*

In a setting of consumption growth, our preference to smooth consumption implies bringing it forward towards the present. On a convex increasing path, gains grow over time, hence h' falls over time, and so do both the instantaneous endowment effect and the reference-level effect. By now we know that the latter is a fixed fraction of the former, and since with convex increasing consumption $k < \delta \geq 0$, it is smaller. When $k > \dot{v_{E_t}}/v_{E_t}$, the endowment effect lowers the discount rate ($\theta^E < 1$). By contrast when $k < \dot{v_{E_t}}/v_{E_t}$, the endowment effect increases the discount rate ($\theta^E > 1$).

On a concave increasing path, the reference-level effect is increasing in k. For $k > \delta$, the reference-level effect is larger than the instantaneous endowment effect. Initially this has the result of increasing the discount rate ($\theta^E > 1$). Eventually θ^E becomes unbounded in the limit as $k \to \delta + \frac{\alpha}{1+\frac{h'}{v_E}}$. To the right of the asymptote, $\theta^E < 0$.

4 Numerical illustrations

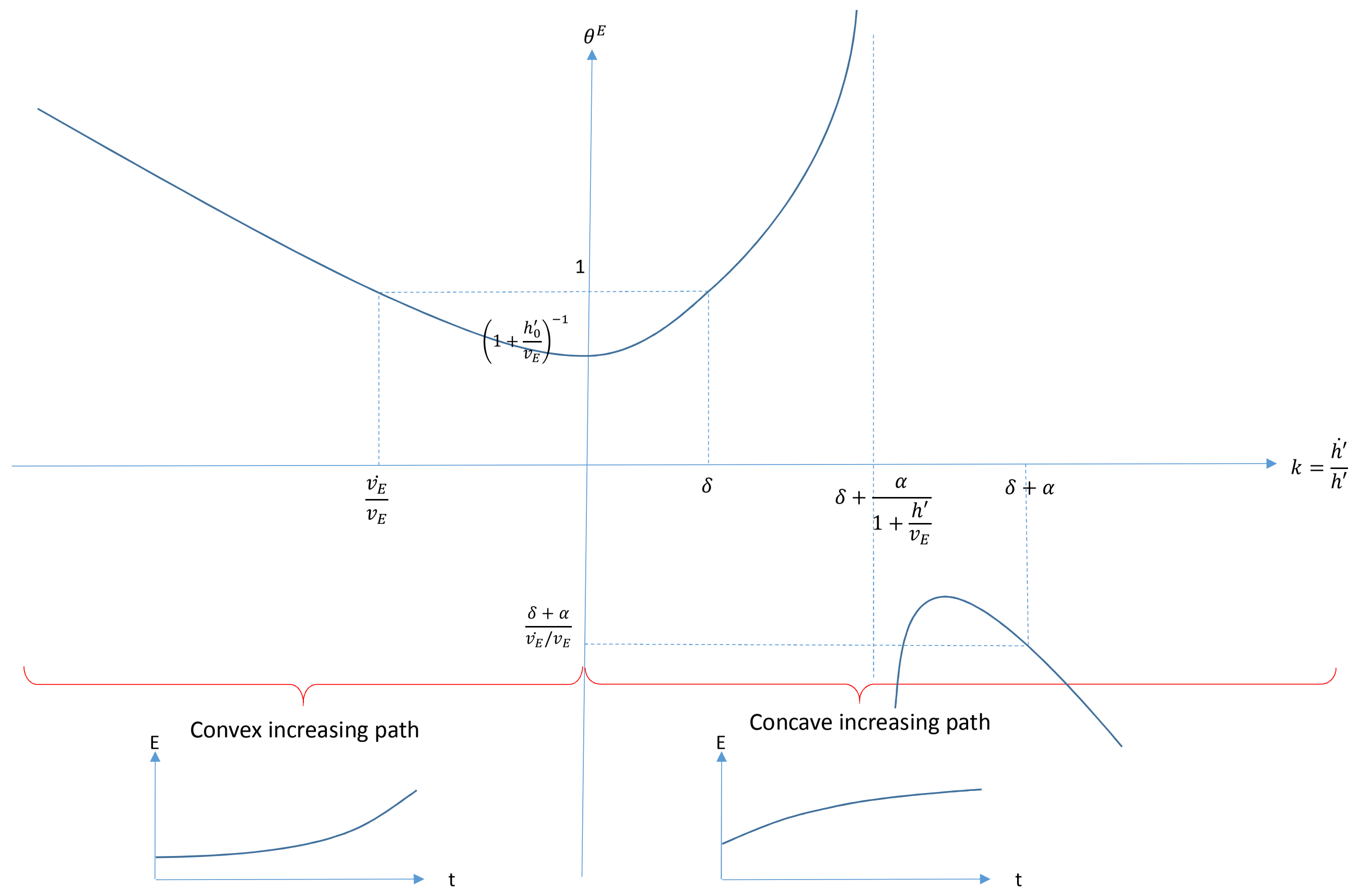
Some numerical examples will be helpful at this point. In particular, for plausible trajectories of growth in consumption of the two goods, we would like to quantify the endowment effect on discounting and compare it with the effect of introducing relative prices into the standard model of preferences, as explored in previous work by Hoel and Sterner (2007) and Traeger (2011). In addition, numerical analysis enables us to look at the endowment effect on discounting, when the consumption path is not strictly increasing or decreasing, whereas the previous section was confined to such paths. In the presence of loss aversion specifically, we might expect substantial effects to emerge around the point in time that consumption switches from growth to decline and *vice versa*.

Functional forms and parameter scheme We need a specific form for the instantaneous utility function (1). Like Hoel and Sterner (2007) and Traeger (2011), we specify a consumption-level utility function that exhibits both a constant elasticity of substitution between the produced good and environmental quality, and a constant elasticity of intertemporal substitution:

$$v(C_t, E_t) = \frac{1}{1-\phi}\left[\gamma C_t^\rho + (1-\gamma)\, E_t^\rho\right]^{\frac{1-\phi}{\rho}}, \tag{22}$$

where $\phi > 0$ is the inverse of the elasticity of intertemporal substitution and $\rho = 1 - 1/\sigma$, with σ being the elasticity of substitution between the produced good and environmental quality.

Figure 3: The environmental endowment factor as a function of k on strictly increasing consumption paths.



For gain-loss utility $g(x)$ and $h(x)$, we use a generalisation of the functional form proposed by Tversky and Kahneman (1992), which is consistent with Assumptions 1-3. Using $g(x)$ as the example,

$$g(x) = \begin{cases} (x+\psi)^{\beta} - \psi^{\beta}, & x \geq 0 \\ -\lambda\left[(-x+\psi)^{\beta} - \psi^{\beta}\right] & x < 0 \end{cases}, \tag{23}$$

where $\beta \in [0,1]$ and $\lambda \geq 1$. Equation (23) also characterises $h(x)$. Compared with Tversky and Kahneman, we introduce the parameter $\psi > 0$ to ensure marginal gain-loss utility is bounded from above as $x \to 0$, in a similar fashion to the bounding parameter in harmonic absolute risk aversion (HARA) functions (Gollier, 2001). This parameter enters twice, whatever value is taken by x, in order to also satisfy the property that $g(0) = h(0) = 0$. A weighted sum of (22) and (23) makes up the instantaneous utility function. Assuming consumption of the produced good is increasing while environmental quality is decreasing, this would be written as

$$\begin{aligned} U_t(C_t, \underline{C}_t, E_t, \underline{E}_t) \quad = \quad & \tfrac{\zeta}{1-\phi}\left[\gamma C_t^{\rho} + (1-\gamma)\, E_t^{\rho}\right]^{\frac{1-\phi}{\rho}} + (1-\zeta)\,\gamma\left[(C_t - \underline{C}_t + \psi)^{\beta} - \psi^{\beta}\right] \\ & - (1-\zeta)\,(1-\gamma)\,\lambda\left[(-E_t + \underline{E}_t + \psi)^{\beta} - \psi^{\beta}\right]. \end{aligned} \tag{24}$$

If consumption of the produced good is instead falling and/or if environmental quality is increasing, then the alternative specifications of (23) should be substituted into (24) as appropriate.

The value share of the produced good relative to environmental quality is determined by $\gamma \in [0,1]$. We initialise the model twenty years in the past, so that by the time our discounting analysis begins (at $t = 0$), reference consumption levels have formed, which are consistent with historical data. By normalising the initial level of environmental quality such that $E_{-20} = C_{-20}$, and the initial reference levels such that $\underline{E}_{-20} = \underline{C}_{-20} = C_{-20}$,

$$\gamma_{-20} \approx \frac{\frac{\partial U}{\partial C_{-20}} C_{-20}}{\frac{\partial U}{\partial C_{-20}} C_{-20} + \frac{\partial U}{\partial E_{-20}} E_{-20}}.$$

The parameter $\zeta \in [0,1]$ governs the value share of consumption-level utility relative to gain-loss utility. In order to calibrate ζ, we target the initial value share of consumption-level utility relative to overall instantaneous utility,

$$Z \approx \frac{\zeta\left(\frac{\partial v}{\partial C_{-20}} C_{-20} + \frac{\partial v}{\partial E_{-20}} E_{-20}\right)}{\frac{\partial U}{\partial C_{-20}} C_{-20} + \frac{\partial U}{\partial E_{-20}} E_{-20}}.$$

Table 1 lists the default parameter values chosen in order to populate (24), as well as the pure rate of time preference δ and the reference-level decay parameter α. We choose typical values from empirical studies of the elasticity of intertemporal substitution $\phi = 1.5$ (Groom and Maddison, 2013) and the parameters of the gain-loss functions $\beta = 0.9$ and $\lambda = 2.3$ (Barberis, 2013). Choosing the pure rate of time preference is particularly controversial, so we opt for a middle-of-the-road value of 1.5%. The remaining parameters are hard to pin down with empirical evidence. The elasticity of substitution $\sigma = 0.5$, so the setting we consider is one of partial substitutability of C and E.

Table 1: Default parameter values

Parameter	Value
Z	0.75
ϕ	1.5
γ	0.9
σ	0.5
β	0.9
λ	2.25
δ	1.5%
α	0.5
ψ	$1

Convex increasing consumption and convex decreasing environmental quality Figure 4 plots the mean produced-good and environmental discount rates in an illustration, in which annual consumption per capita of the produced good grows at a permanent rate of 1.5% (this is the global-average growth rate over the last 30 years),[12] and in which environmental quality falls at 0.5%. Discount rates with and without the endowment effect are shown.

Without the endowment effect, the average produced-good discount rate 'r^C std.' begins at 3.94% and nudges upwards to 4.12% in 100 years. If we were to have specified the utility function as simply $U_t(C_t) = {}^1/_{1-\phi}C_t^{1-\phi}$, then the 'Ramsey rule' would yield a constant produced-good discount rate of $1.5 + 1.5 * 1.5 = 3.75\%$, so the effect of including consumption-level utility from environmental quality, $-\eta_{CE}\dot{E}/E$, is to increase the initial discount rate on produced goods by 0.19 percentage points, and the change in relative prices further increases this difference to 0.37 ppts. by the end of a century. With falling environmental quality, additional consumption of the produced good becomes less valuable and is discounted more.

The average environmental discount rate 'r^E std.' begins at -0.12% and nudges upwards to 0.06% over the same period of time. Remember this is the discount rate that should be used to evaluate an 'environmental project' implemented on these paths. If $U_t(E_t) = {}^1/_{1-\phi}E_t^{1-\phi}$, then according to the Ramsey rule the environmental discount rate would be $1.5 + 1.5 * -0.5 = 0.75\%$, so the effect of including consumption-level utility from the produced good, $-\eta^{EC}\dot{C}/C$, is to pull the discount rate on environmental quality significantly downwards à la Hoel and Sterner (2007) and Traeger (2011). Increases in environmental quality are more valuable when the produced good is relatively abundant. For this to be the case, it must be that $\eta^{EC} > 0$, which can be verified for our parameter scheme.[13]

When the endowment effect is present, the average produced-good discount rate

[12] The initial value, which is also the reference value, is $3467. All these data are on global household final consumption expenditure per capita, and are taken from the World Bank *World Development Indicators*.

[13] In particular, given (23),

$$\eta^{EC} = \frac{(\gamma - 1)(\phi + \rho - 1)E^\rho}{[\gamma C^\rho + (1 - \gamma)\, E^\rho]},$$

so $\eta^{EC} > 0 \iff (\gamma - 1)(\phi + \rho - 1) > 0$.

Figure 4: Discount rates with and without the endowment effect

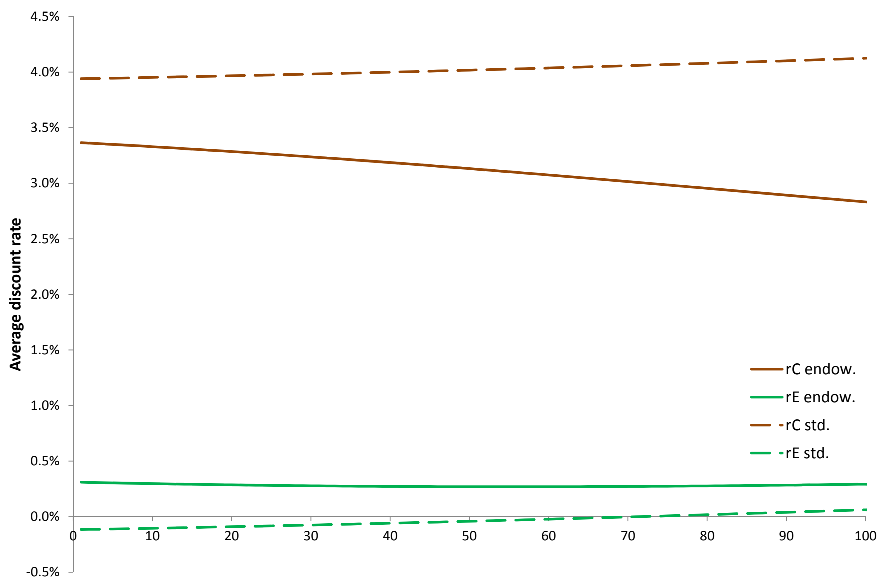


Table 2: Sensitivity of r^E to parameters at $t = 50$.

	min.	max.	range of r^E endow.	range of r^Estd.
α	0	1	0.26% to 0.30%	-0.04%
Z	≈ 0	1	-0.04% to 1.45%	-0.04%
γ	≈ 0	≈ 1	0.11% to 0.89%	-0.23% to 0.74%
β	≈ 0	1	-0.04% to 0.38%	-0.04%
λ	1	5	0.13% to 0.44%	-0.04%

'r^C endow.' is initially just 3.37%, and falls further to 2.24% in 100 years. So the endowment effect makes a big difference in this example, indeed it makes a bigger difference than relative prices do under standard consumption-level utility (the aforementioned comparison between r^C std. and the Ramsey rule). That r^C endow. is lower than, and falling relative to, r^C std. means the produced-good endowment factor $0 < \theta^C < 1$ and that $d\theta^C/dt < 0$. Since, with a constant growth rate, consumption of the produced good is on a convex increasing path, this in turn implies that ${}^{\dot{v_C}}/_{v_C} < {}^{\dot{g}'}/_{g'} < 0$ (by analogy with Figure 3 and Proposition 3). That is, in this empirical example marginal gain-loss utility falls slowly enough that the endowment effect dampens our preference to smooth consumption of the produced good by enjoying that consumption earlier.

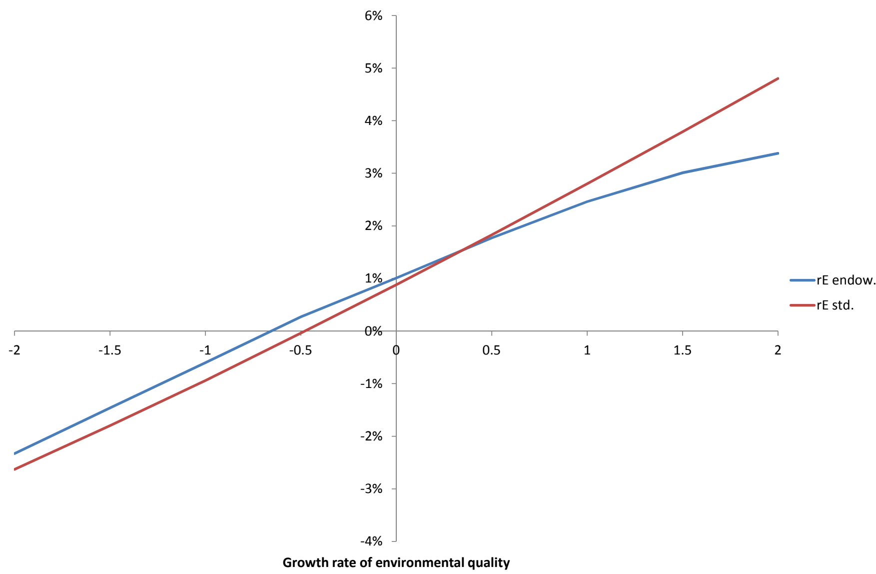
The average environmental discount rate 'r^E endow.' is around 0.3% throughout, so the endowment effect *increases* the rate at which we would discount an environmental project on these paths. This implies the environmental endowment factor $0 < \theta^E < 1$ too. Again, the endowment effect tempers our preference to smooth consumption, but in the case of falling environmental quality a preference to smooth consumption is driving us to postpone consumption to a future date, so gain-loss utility does the opposite.

Sensitivity analysis Table 2 analyses the sensitivity of r^E endow. to various preference parameters on this strictly decreasing path at a maturity of $t = 50$, a typical horizon for a long-run environmental project. The key finding is that, across the entire parameter space, r^E endow. lies between r^Estd. and δ, the pure rate of time preference. This means our finding that the endowment effect tends to dampen our preference to smooth consumption of environmental quality – $0 < \theta^E < 1$ – is robust.

We find that r^E endow. is relatively insensitive to α, the reference-level decay parameter. Otherwise r^E endow. depends more sensitively on parameter values. The range is greatest with respect to the domain of feasible values of Z, the initial value share of consumption-level utility relative to overall instantaneous utility. The smaller is Z, giving a larger weight to gain-loss utility, the greater is r^E endow. Indeed when $Z \approx 0$, r^E endow. is 1.45%, close to the rate of pure time preference, implying that the environmental endowment factor $\theta^E \approx 0$. The environmental discount rate is sensitive to γ, the value share of the produced good relative to environmental quality, with/without the endowment effect. It is somewhat sensitive to the curvature of $h(.)$, parameterised by β. It is also sensitive to λ over a domain that includes as its extremes, on the one hand, loss neutrality ($\lambda = 1$) and, on the other hand, loss aversion just over double our central case ($\lambda = 5$).

Lastly, Figure 5 displays the difference between r^E endow. and r^Estd. at a maturity

Figure 5: r^E endow. and r^Estd. at $t = 50$ as a function of $\dot{E}/E$.



of $t = 50$, as a function of the permanent growth rate of environmental quality. The key result is again that the endowment effect draws the discount rate closer to the pure rate of time preference, both when environmental quality is falling and when it is rising. Indeed, r^E endow. and r^Estd. coincide when environmental quality is growing at about 0.4%, which yields r^Estd. $= \delta$.

A non-monotonic path for environmental quality Figure 6 focuses on the average environmental discount rate on an alternative path for environmental quality, whereby the initial growth rate is 0.5%, but the growth rate falls by 0.01 ppts. per year. This has the result that environmental quality grows for the first thirty years, and then falls. Therefore we move out of the framework of strictly decreasing environmental quality. As well as the base-case parameterisation of r^E endow., and as well as r^E std., we aid interpretation of the results by providing plots of r^E endow., in which loss aversion is omitted ($\lambda = 1$), and/or constant sensitivity is assumed ($\beta = 1$).

While r^E std. decreases over time, along with the average growth rate of environmental quality, base-case r^E endow. ($\lambda = 2.25$; $\beta = 0.9$) exhibits striking, non-monotonic and discontinuous behaviour. As $t \to 30$, it increases sharply to over 7%, before suddenly dropping to about -0.2%, and then increasing again to become close to r^E std. at the end of the time horizon. Remember, this is the average discount rate: the instantaneous or marginal discount rate behind these results exhibits an even larger jump.

In order to appreciate how this comes about, it is worth referring back to the basic breakdown of marginal welfare at time t contained in Eq. (5). This showed that $\partial J/\partial E_t$ comprises (i) marginal consumption-level utility, (ii) the instantaneous endowment effect and (iii) the reference-level effect. Furthermore it showed that $\partial J/\partial E_t$ is increasing in the size of (i) and (ii), and decreasing in the size of (iii). Under loss aversion, the instantaneous endowment effect is discontinuous on a non-monotonic consumption path. In this case, it will jump upwards when consumption growth turns negative. This in turn has an influence on the reference-level effect, which is of course the discounted and memory-adjusted sum of future marginal gain-loss utility. Moreover this influence will start to be seen prior to the turning point in consumption.

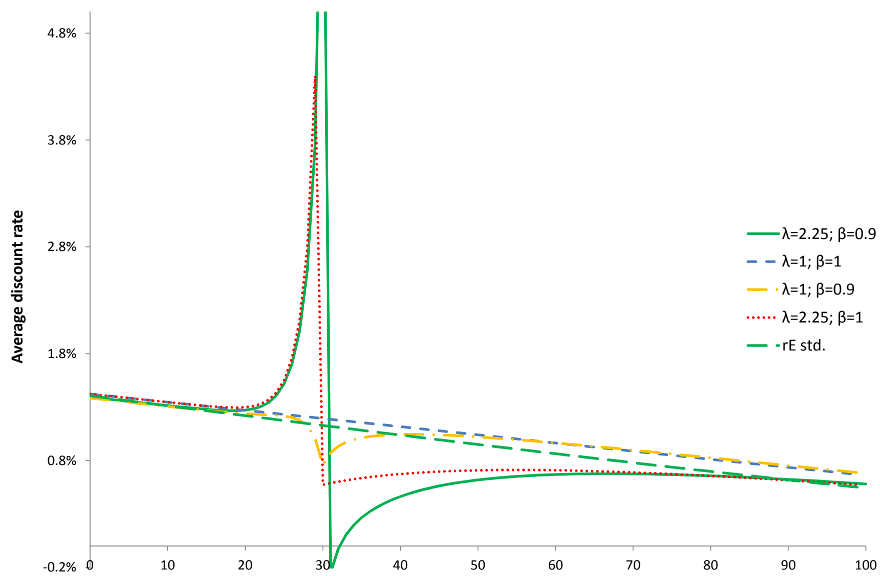
Bringing these together, what is happening to r^E endow. ($\lambda = 2.25$; $\beta = 0.9$) in Figure 6 is that the reference-level effect starts increasing rapidly in size as $t \to 30$. Since the reference-level effect reduces $\partial J/\partial E_t$, the instantaneous discount rate increases, as does the average discount rate. But at exactly $t = 30$, the reference-level effect ceases its ascent, while the instantaneous endowment effect suddenly jumps. Since the instantaneous endowment effect increases $\partial J/\partial E_t$, this accounts for the sudden fall in the discount rate. Notice that in the absence of loss aversion ($\lambda = 1$), there is no jump in the discount rate. Under diminishing sensitivity but without loss aversion – in other words when the marginal gain-loss utility function is a smooth sigmoid – there is a trough in the discount rate around the turning point in consumption, just because marginal gain-loss utility becomes large when the change in consumption is small. But the effects of gains and losses are symmetrical. Moreover the discount rate only appears discontinuous due to the effect of the bounding parameter ψ. As one would expect, when neither loss aversion nor diminishing sensitivity is present ($\lambda = 1$; $\beta = 1$), so that the marginal gain-loss function is linear, the discount rate does not deviate from its declining path.

5 Conclusions

Our analysis has shown that the endowment effect can make a substantial difference to the discount rate. Our focus has been on discounting environmental quality in a two-good model, where the other good is a composite of all produced goods. However, the model and analysis could equally be applied to any two goods of interest.

Often, the context for investments to improve the environment is one in which environmental quality is decreasing. When environmental quality is in decline, standard, concave utility with respect to the *level* of consumption of environmental quality should (depending on what is happening to consumption of the produced good) create a preference to smooth consumption over time by postponing it into the future, when environmental quality is more scarce. However, if (a) gain/loss utility conforms to constant sensitivity, (b) environmental quality is decreasing arithmetically (i.e. linearly), or (c) it is concave decreasing, then Section 3 showed that the endowment effect dampens this preference to smooth consumption. Conditions (a)-(c) are each sufficient for the endowment factor $\theta^E < 1$. None is necessary. If instead environmental quality is convex decreasing, the endowment effect can in principle dampen or amplify the preference to smooth consumption, but numerical analysis in Section 4 showed that, across a large space of growth scenarios and parameter values, $0 < \theta^E < 1$.

Figure 6: The environmental discount rate when E follows an inverse-U shaped path.



The key implication of all of this is that the environmental discount rate on a decreasing path for environmental quality is most likely higher in the presence of the endowment effect than it is without it. So, an investment to improve environmental quality in the future, at the expense of foregone material consumption today, is less likely to be welfare-improving. This is perhaps surprising. One might have thought that loss aversion would increase the value placed on an investment on a path where environmental quality is being lost. But it must be remembered that the exercise here is not to value the path itself (cf. Section 2), rather the discounting literature engages with valuation of a marginal investment along a path. In this setting, what matters is that on a strictly decreasing path, environmental quality is being lost not only in the future, it is being lost today. If the marginal utility of losses today weighs more heavily on our welfare than the marginal utility of losses tomorrow, the endowment effect makes us less willing to postpone consumption to the future. A rather different way to think about exactly the same phenomenon is to say that, because our model of the endowment effect embodies habit formation, we become accustomed – habituated – to losing environmental quality, such that future losses decrease our utility less.

For non-monotonic consumption paths, Section 4 illustrated that the endowment effect can have a much larger effect on the discount rate. This is because loss aversion introduces a discontinuity or kink in the gain/loss utility function when the change in consumption $x = 0$. On a non-monotonic path, which itself can be smooth as in our example, there will be a point in time when growth hits zero on its way from positive to negative territory, and *vice versa*. At this point, marginal gain/loss utility, the instantaneous endowment effect, jumps, and the reference-level effect changes rapidly in advance of this jump. The chief implication of this particular analysis is that valuation of environmental investments, which incur net benefits in the region of a turning point in consumption growth, is likely to be substantially modified by the endowment effect. It is clear, however, that the effect on valuations is context-specific and, in a sense, rather unpredictable.

Moving beyond a summary of our results to broader issues, there is naturally the question of whether the endowment effect ought to be considered in evaluating public environmental investments in the first place. There are at least two dimensions to this. First, there is the question of how strong the evidence behind the endowment effect is. Second, there is the question of whether preferences that represent the endowment effect should be afforded normative status, insofar as they are included in public/social decision-making.

On the first question, there is indeed much empirical evidence that demonstrates the endowment effect both in laboratory and field settings (e.g. Camerer and Loewenstein, 2004; DellaVigna, 2009). This is not to deny the existence of dissenting evidence. Most famously, List (2003) showed that experienced traders of a good do not exhibit the endowment effect with respect to that good, a result that is consistent either with those traders not being loss averse, or with those traders forming different reference points to inexperienced traders (DellaVigna, 2009). However, the preferences of people who trade baseball cards at least half a dozen times a month (i.e. an experienced trader) seem a poor analogy for those preferences of interest here, which are over changes in environmental quality that are not in general traded in markets.

The second question drags us into a much broader debate about the implications of deviations from standard economic preferences for public decision-making. At one extreme, a simple application of the doctrine of consumer sovereignty would have it that, if the endowment effect generally characterises people's preferences, then the preferences of a policy-maker or social planner should include it too. At the other extreme, it might be concluded that deviations from standard economic preferences are irrational by some yardstick and should therefore not be reflected in social planning. Since the requirements of preferences are usually axioms or primitives, this yardstick is not obvious. Nonetheless, it is certainly possible to find examples of this objection in discussions about the normative status of related phenomena, such as hyperbolic discounting (e.g. Hepburn et al., 2010) and ambiguity aversion (Al-Najjar and Weinstein, 2009; Gilboa et al., 2009). With hyperbolic discounting, the concern is that preferences are time-inconsistent and therefore explain patterns of behaviour, such as addiction and procrastination, that are fairly obviously not in the best interests of those who hold these preferences. However, it is important to highlight that models of habit formation such as ours do not lead to time-inconsistency, even though the utility function is not time-separable (Végh, 2013). We feel that a proposed resolution to this debate is clearly beyond the scope of the present paper, even in the particular case of the endowment effect. At the very least, our results indicate how average consumers exhibiting the endowment effect, especially in relation to goods whose trade they are unfamiliar with, value environmental projects.

Lastly, there are at least three extensions to the present work, which are worthwhile considering. First, Appendix 2 points the way towards an analysis of optimal control of pollution under the endowment effect. This will not be simple, however, given the large number of state variables implied by having two goods, both of which are evaluated in part against reference points. Second, our results assume perfect foresight, a natural consequence of minimally extending standard preferences. In fact, this likely has important implications for our results, because the strength of the reference-level effect rests on our anticipating the effect on future gain/loss utility of increments in consumption today. But what if we don't fully anticipate habituation to higher or lower consumption levels? This would be worth looking into. Third, we have only examined the endowment effect in a riskless choice setting, in the tradition of Tversky and Kahneman (1991), even though reference dependence and loss aversion were first invoked to explain risky choices (Kahneman and Tversky, 1979). Therefore we could allow consumption of the two goods to follow a stochastic process. Again, this will not be wholly trivial, because the state space of future consumption levels could span the kink in marginal gain/loss utility that is implied by loss aversion. Under such circumstances, not only will there be familiar-looking results about the expectation of marginal gain/loss utility that derive from application of Jensen's inequality, there will also be a 'kink effect', so to speak.

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Appendix 1

A functional or Frechet derivative describes the change in the welfare functional J with respect to a change in the consumption *path*. We follow Karp and Traeger (2009) and define the functional derivative with respect to a change in the consumption path $\tilde{C}$ as

$$\begin{aligned}
\hat{J}[C;E;\tilde{C}] &= \frac{d}{d\epsilon} J\left[C+\epsilon\tilde{C};E\right]|_{\epsilon=0}, \\
&= \frac{d}{d\epsilon}\int_0^\infty e^{-\delta t} U\left(C(t)+\epsilon\tilde{C(t)}, \underline{C}\left(C(t)+\epsilon\tilde{C(t)}\right), E(t), \underline{E}\left(E(t)\right)\right) dt, \\
&= \frac{d}{d\epsilon}\int_0^\infty e^{-\delta t}\left[U\left(C(t), \underline{C}\left(C(t)\right), E(t), \underline{E}\left(E(t)\right)\right)\right. \\
& \quad + U_C\left(C(t), \underline{C}\left(C(t)\right), E(t), \underline{E}\left(E(t)\right)\right)\epsilon\tilde{C(t)} \\
& \quad \left. + U_{\underline{C}}\left(C(t), \underline{C}\left(C(t)\right), E(t), \underline{E}\left(E(t)\right)\right)\frac{d}{d\epsilon}\underline{C}\left(C(t)+\epsilon\tilde{C(t)}\right)\right] dt|_{\epsilon=0}, \\
&= \frac{d}{d\epsilon}\int_0^\infty e^{-\delta t}\left(v_C(C,E)+g_C\left(C-\underline{C}\right)\right)\epsilon\tilde{C(t)}dt|_{\epsilon=0} \\
& \quad + \int_0^\infty e^{-\delta t} g_{\underline{C}}\left(C-\underline{C}\right)\frac{d}{d\epsilon}\left[\alpha\int_{-\infty}^t e^{-\alpha(t-\tau)}\epsilon\tilde{C(\tau)}d\tau\right]|_{\epsilon=0}.
\end{aligned}$$

Using $\frac{d}{d\epsilon}\left[\alpha\int_{-\infty}^t e^{-\alpha(t-\tau)}C(\tau)d\tau\right]|_{\epsilon=0} = 0$ we have

$$\begin{aligned}
\hat{J}[C;E;\tilde{C}] &= \int_0^\infty e^{-\delta t}\left(v_C(C,E)+g_C\left(C-\underline{C}\right)\right)\tilde{C(t)}dt \\
& \quad + \int_0^\infty e^{-\delta t} g_{\underline{C}}\left(C-\underline{C}\right)\left[\alpha\int_{-\infty}^\infty 1_{\tau\leq t} e^{-\alpha(t-\tau)}\tilde{C(\tau)}d\tau\right] dt.
\end{aligned}$$

In the second term, $e^{-\delta t} g_{\underline{C}}\left(C-\underline{C}\right)$ is independent of τ, therefore

$$\begin{aligned}
\hat{J}[C;E;\tilde{C}] &= \int_{-\infty}^\infty 1_{t>0} e^{-\delta t}\left(v_C(C,E)+g_C\left(C-\underline{C}\right)\right)\tilde{C(t)}dt \\
& \quad + \left[\alpha\int_{-\infty}^\infty\int_0^\infty e^{-\delta t} g_{\underline{C}}\left(C-\underline{C}\right)1_{\tau\leq t} e^{-\alpha(t-\tau)}\tilde{C(\tau)}dtd\tau\right], \\
&= \int_{-\infty}^\infty \left[1_{t>0} e^{-\delta t}\left(v_C(C,E)+g_C\left(C-\underline{C}\right)\right)\right. \\
& \quad \left. + \alpha\int_0^\infty e^{-\delta\tau} g_{\underline{C}}\left(C-\underline{C}\right)1_{\tau\geq t} e^{\alpha(t-\tau)}d\tau\right]\tilde{C(t)}dt. \qquad (25)
\end{aligned}$$

Being a linear operator, the Frechet derivative can also be written as the inner product of the consumption perturbation $\tilde{C}$ and a density function $\frac{\partial J}{\partial C}[C;E;t]$, which is defined by the relationship

$$\hat{J}[C;E;\tilde{C}] = \int_{-\infty}^\infty \frac{\partial J}{\partial C}[C;E;t] * \tilde{C(t)}dt. \qquad (26)$$

The density function is also known as the Volterra derivative. While the Frechet derivative $\hat{J}[C;E;\tilde{C}]$ is a functional that takes in three time paths as arguments (C, E and $\tilde{C}$), the Volterra derivative $\frac{\partial J}{\partial C}[C;E;t]$ has time paths C and E and a moment in time t as arguments. The value of the Volterra derivative $\frac{\partial J}{\partial C}[C;E;t]$ at a given date t can also be understood as the effect of a marginal increase of the consumption path at t. Therefore it can be written as $\frac{\partial J}{\partial C}[C;E;t] = \hat{J}[C;E;\delta_t]$, where δ_t is the delta distribution, i.e. a density function that concentrates full weight on a point of time t.[14]

Combining Eq. (25) and Eq. (26) and considering only consumption perturbations after $t = 0$,

$$\begin{aligned}\frac{\partial J}{\partial C}[C;E;t] &= e^{-\delta t}\left(v_C(C,E) + g_C\left(C - \underline{C}\right)\right) + \alpha \int_0^\infty e^{-\delta\tau} g_{\underline{C}}\left(C - \underline{C}\right) 1_{\tau \geq t} e^{\alpha(t-\tau)} d\tau, \\ &= e^{-\delta t}\left(v_C(C,E) + g_C\left(C - \underline{C}\right) + \alpha \int_t^\infty e^{-(\alpha+\delta)(\tau - t)} g_{\underline{C}}\left(C - \underline{C}\right) d\tau\right) \quad (27)\end{aligned}$$

The discount factor for a transfer of consumption from time 0 to t is

$$D^C = \frac{\frac{\partial J}{\partial C}[C;E;t]}{\frac{\partial J}{\partial C}[C;E;0]}.$$

The corresponding discount rate at a given point in time is defined as

$$r^C = \frac{d}{dt} \ln D^C(t,0).$$

Writing the logarithm of a product as the sum of the logarithms of the seperate factors, we have

$$\begin{aligned}r^C &= \frac{d}{dt}\delta t + \frac{d}{dt}\ln\left[\partial v/\partial C_t + g'\left(C_t - \underline{C}_t\right) - \alpha\int_{\tau=t}^\infty e^{-(\alpha+\delta)(\tau-t)} g'\left(C_\tau - \underline{C}_\tau\right) d\tau\right] \\ &\quad - \frac{d}{dt}\left[\partial v/\partial C_0 + g'\left(C_0 - \underline{C}_0\right) - \alpha\int_{\tau=0}^\infty e^{-(\alpha+\delta)(\tau-0)} g'\left(C_\tau - \underline{C}_\tau\right) d\tau\right]. \quad (28)\end{aligned}$$

The third term is independent of t. Using the chain rule to take the derivative of the second term we find that

$$r_t^C = \delta - \frac{\dot{v_C} + \dot{g_C} - \alpha\dot{\mu^{\underline{C}}}}{v_C + g_C - \alpha\mu^{\underline{C}}}.$$

Finally, by again applying the chain rule we find that

$$\begin{aligned}\alpha\dot{\mu^{\underline{C}}} &= \frac{d}{dt}\alpha e^{(\alpha+\delta)t}\int_{\tau=t}^\infty e^{-(\alpha+\delta)\tau} g'\left(C_\tau - \underline{C}_\tau\right) d\tau \\ &= \alpha(\alpha+\delta)e^{(\alpha+\delta)t}\int_{\tau=t}^\infty e^{-(\alpha+\delta)\tau} g'\left(C_\tau - \underline{C}_\tau\right) d\tau + \alpha e^{(\alpha+\delta)t} e^{-(\alpha+\delta)t} g'\left(C_t - \underline{C}_t\right),\end{aligned}$$

so that we obtain Eq. (7), i.e.

$$r^C = \delta - \frac{v_{CC}\dot{C} + v_{CE}\dot{E} + g''(\dot{C} - \alpha C + \alpha\underline{C}) + \alpha g' - \alpha(\alpha+\delta)\int_{\tau=t}^\infty e^{-(\alpha+\delta)(\tau-t)} g' d\tau}{v_C + g' - \alpha\int_t^\infty e^{-(\alpha+\delta)(\tau-t)} g' d\tau}.$$

[14]The delta distributionδ_t is characterized by the relation $\int_{-\infty}^\infty \tilde{C(\tau)}\delta_t d\tau = \tilde{C(t)} \forall \tilde{C} \in C^\infty$

Appendix 2

In this Appendix we derive the environmental discount rate from a problem of optimal management of environmental quality, in particular when environmental quality is negatively impacted by a pollutant. We begin with flow pollution $S = -E$. Following Brock (1973), we characterise the relationship between production of the composite material good and pollution by writing production as a positive function of the flow of pollution. The production function is

$$Y = F(K, S),$$

where K is capital. We assume that $F_K > 0$ and $F_{KK} < 0$, and that the Inada conditions hold. For a given capital stock, production is an increasing and strictly concave function of the pollution intensity of the capital stock, i.e. $F_S > 0$ and $F_{SS} < 0$. Production is either consumed or re-invested, leading to the following expression for capital accumulation,

$$\dot{K} = F(K, S) - C.$$

Population and the production technology are assumed to be constant for simplicity, and for the same reason we omit capital depreciation.

The social planning problem corresponding with this setting is

$$\max_{\{C,S\}} J = \int_0^\infty e^{-\delta t} \left[v(C_t, E_t) + g(C_t - \underline{C}_t) + h(E_t - \underline{E}_t) \right] dt \tag{29}$$

$$\text{s.t.} \qquad \dot{K} = F(K, S) - C, \tag{30}$$

$$\dot{\underline{C}} = \alpha (C - \underline{C}), \tag{31}$$

$$\dot{\underline{E}} = \alpha (E - \underline{E}), \tag{32}$$

and initial K, E, $\underline{C}$ and $\underline{E}$. The current value Hamiltonian is defined as

$$\begin{aligned} \mathcal{H} &= v(C, E) + g(C - \underline{C}) + h(E - \underline{E}) + \\ &\quad \mu^K [F(K, S) - C] + \check{\mu}^{\underline{C}} [\alpha (C - \underline{C})] + \check{\mu}^{\underline{E}} [\alpha (E - \underline{E})]. \end{aligned}$$

As mentioned in the main text, $\check{\mu}^{\underline{C}} = -\mu^{\underline{C}}$ is the costate variable on reference consumption of the produced good, and it has a counterpart $\check{\mu}^{\underline{E}} = -\mu^{\underline{E}}$ on reference consumption of environmental quality. This would also be true in a model with a stock pollutant.

Necessary conditions for a maximum include that

$$\mu^K = v_C + g' + \check{\mu}^{\underline{C}} \alpha, \tag{33}$$

$$\mu^K F_S = v_E + h' + \alpha \check{\mu}^{\underline{E}}, \tag{34}$$

$$\frac{\dot{\mu}^K}{\mu^K} = \delta - F_K, \tag{35}$$

$$\frac{\dot{\check{\mu}}^{\underline{C}}}{\check{\mu}^{\underline{C}}} = \delta + \alpha - \frac{g'}{\check{\mu}^{\underline{C}}}, \tag{36}$$

$$\frac{\dot{\check{\mu}}^{\underline{E}}}{\check{\mu}^{\underline{E}}} = \delta + \alpha - \frac{h'}{\check{\mu}^{\underline{E}}}. \tag{37}$$

As an aside, combining (35) and (33) leads to an extended version of the standard Euler equation:

$$r^C = F_K = \delta - \frac{\dot{v_C} + \dot{g'} + \alpha \dot{\check{\mu}}^{\underline{C}}}{v_C + g' + \alpha \dot{\check{\mu}}^{\underline{C}}}.$$

Since we are dealing with a flow pollutant, the current-valued shadow price of environmental quality is just $-\mu^K F_S$. Therefore the environmental discount rate is defined as

$$r^E = \delta - \frac{\dot{\mu^K F_S}}{\mu^K F_S}.$$

Combined with Eq. (34), this gives the discount rate we established in Section 3:

$$r^E = \delta - \frac{\dot{v_E} + \dot{h'} + \alpha \dot{\check{\mu}}^{\underline{E}}}{v_E + h' + \alpha \check{\mu}^{\underline{E}}} = \delta - \frac{\dot{v_E} + \dot{h'} - \alpha \dot{\mu}^{\underline{E}}}{v_E + h' - \alpha \mu^{\underline{E}}}. \tag{38}$$

In the case of a stock pollutant, where $\dot{E} = -S - \omega E$ and ω is the decay rate of the pollutant in the environment, the Hamiltonian becomes

$$\begin{aligned} \mathcal{H} \quad = \quad & v\,(C, E) + g\,(C - \underline{C}) + h\,(E - \underline{E}) + \\ & \mu^K \left[F(K, S) - C\right] + \mu^E \left[-S - \omega E\right] + \check{\mu}^{\underline{C}} \left[\alpha\,(C - \underline{C})\right] + \check{\mu}^{\underline{E}} \left[\alpha\,(E - \underline{E})\right]. \end{aligned}$$

The first order conditions will be the same as in the flow pollutant problem, except that stock pollution requires an additional costate equation,

$$\frac{\dot{\check{\mu}}^{\underline{E}}}{\check{\mu}^{\underline{E}}} = \delta + \omega - \frac{v_E + h' + \check{\mu}^{\underline{E}} \alpha}{\mu^E}, \tag{39}$$

and Eq. (34) becomes just

$$\mu^E \quad = \mu^K F_S. \tag{40}$$

The appropriate discount rate to trade off a marginal unit of stock pollution over time is therefore defined as

$$r^E = \delta - \frac{\dot{\mu}^E}{\mu^E}.$$

Combined with Eq. (40), this gives the following environmental discount rate:

$$r^E = -\omega + \frac{v_E + h' + \check{\mu}^{\underline{E}} \alpha}{\mu^E},$$

which differs from Eq. (38), because it includes the fact that adding a unit of pollution at a given date will affect the quality of the environment at future dates.

Appendix 3

Equation (20) can also be written in the following way:

$$\theta^E = \frac{\alpha + \gamma - k + \frac{h}{\dot{v}_E} k(\delta - k)}{\alpha + \gamma - k + \frac{h'}{v_E}(\delta - k)}.$$

To understand the sign of the denominator, consider θ^E in the neighbourhood of the vertical asymptote at $k = \delta + \frac{\alpha}{1+g_C/v_C} + \epsilon$ with, arbitrarily small ϵ. Substituting this value of k in the denominator gives $\alpha + \gamma - k + \frac{h'}{v_E}(\delta - k) - \epsilon - \frac{h'}{v_E}\epsilon = -\epsilon - \frac{h'}{v_E}\epsilon$. The denominator will therefore be positive to the left of the asymptote and negative to the right of it.

The denominator is positive in the neighbourhood of the asymptote at $k = \delta + \frac{\alpha}{1+h'/v_E}$ if

$$\alpha - \frac{\alpha}{1 + \frac{h'}{v_E}} + \frac{h'}{\dot{v}_E}\left(\delta + \frac{\alpha}{1 + \frac{h'}{v_E}}\right)\left(\frac{-\alpha}{1 + \frac{h'}{v_E}}\right) > 0$$

$$\Leftrightarrow \frac{v_E}{\dot{v}_E}\left(\delta + \frac{\alpha}{1 + \frac{h'}{v_E}}\right) < 1.$$

Therefore the numerator is positive if $\frac{\dot{v}_E}{v_E} < 0$ or $\frac{\dot{v}_E}{v_E} > \delta + \frac{\alpha}{1+\frac{h'}{v_E}}$. As a result, θ^E jumps from infinity to minus infinity as k increases beyond the asymptote. On the contrary, the numerator is negative if $0 < \frac{\dot{v}_E}{v_E} < \delta + \frac{\alpha}{1+\frac{h'}{v_E}}$, with the result that θ^E jumps from minus infinity to infinity as k passes the asymptote.