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Discounting and the Representative Median Agent*

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Abstract

Under assumptions typically used in applied work, we derive an 'inequality-adjusted' social discount rate (SDR) that reflects the dynamics of income inequality. The inequality-adjustment alters the wealth effect in the standard Ramsey rule to reflect the distributional consequences of consumption growth in the presence of inequality aversion. The adjustment is proportional to the difference between the growth of the mean and median income, where the constant of proportionality is the Atkinson index of inequality aversion. A special case leads to agents with median incomes being representative agents. With a degree of inequality aversion of 1 (1.5, 2), the UK and U.S. SDRs would be approximately 0.25% (0.5%, 1%) lower than the standard Ramsey rule, reflecting the slower growth of the median incomes compared with income per capita, and hence rising inequality over the past two decades. Where inequality is on the decline, higher SDRs are recommended. Our inequality-adjusted SDR accounts for the welfare effects of secular changes in inequality in the appraisal of public projects.

JEL Classification: H43, D61, Q58.

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1 Introduction

In this paper we derive a simple 'inequality-adjusted' social discount rate (SDR) for Cost Benefit Analysis (CBA) that takes into account the dynamics of income inequality. Under assumptions typically used in applied social discounting, the inequality-adjustment stems from an adjusted consumption growth rate, which then reflects the distributional consequences of secular growth on social welfare. This leads to an inequality-adjusted wealth effect in the standard Ramsey rule, which then better measures the future well-being of society when inequality is evolving over time. We argue that our inequality-adjusted SDR is easy to implement and could accompany the standard Ramsey rule, which is neutral to intra-temporal inequality since it uses the average (per capita) growth rate to reflect society's future well-being. The discounting formula we propose is a practical and appealing special case of previous work on discounting and intra-generational inequality by Gollier (2015) and Emmerling (2010). The purpose of this paper is to illustrate how previous theoretical contributions can be easily applied and form part of policy guidance on CBA.

Our approach brings recent concerns about economic inequality into the formal analysis of public projects. Our inequality-adjusted growth rate is simply the growth of Atkinson's Equally Distributed Equivalent (EDE) level of consumption. Coupled with the assumption that the income distribution is log-normal, our inequality-adjusted growth rate modifies the growth rate of per capita consumption by adding a term proportional to the difference between the growth rate of median and average incomes. The proportionality factor is the Atkinson measure of income inequality aversion. Under our simplifying, yet defensible, assumptions the difference between the growth rate of the mean and the median per capita income is a sufficient statistic for the evolution of inequality over time. Multiplication by the inequality-aversion parameter then yields an 'inequality premium' that society experiences in terms of growth due to aversion to secular changes in the distribution of income. The inequality premium can be positive or negative depending on whether growth is inequality reducing or inequality increasing.

The appeal of the approach we propose stems from its simplicity and its close correspondence to current discounting practices. However, the focus that our proposal places on the fortunes of people at the median level of income also means that there is a close correspondence with recent calls to count inequality among the measures used to evaluate economic performance. A key proposal has been to report growth of median incomes alongside the typical measures of average (per capita) income growth (Piketty, 2014; Aghion et al., 2013; OECD, 2017). Consequently this information is now routinely collected by government statistical agencies and multi-lateral bodies such as the World Bank and OECD (Nolan et al., 2016; ONS, 2016; OECD, 2017). Given that the only additional piece of information that one needs in order to implement our proposal is the growth rate of the median per capita income, the inequality adjusted discount rate will be straightforward to implement. Moreover, while the arguments for focussing on the median income are typically pragmatic and intuitive in nature, our work shows how standard economic welfare theory provides the conditions under which this focus has a conceptual welfare justification. Indeed, in one case of our model, the agent with the median income becomes the representative agent in CBA, rather than the ubiquitous representative agent characterized by average (per-capita) consumption.

The distinction between the prospects for average and median incomes is also quantitatively significant: at the global level, for instance, while per-capita income had reached almost \$10,000 by 2013, the median household income amounted only to \$2,920 (Gallup, 2013). Furthermore, in recent years, it has become clear that while average growth has been positive in many OECD countries, income growth at lower quantiles of the income distribution has been considerably lower. For instance, in the U.S. between 1984 and 2014, on average incomes grew at 1% per year, while growth for the median household was barely positive at 0.3%, with lower quantiles performing even worse. Higher average growth was driven largely by higher rates of growth among the richer quantiles of the income distribution, which increased income inequality. Similar patterns have been witnessed within individual countries (OECD, 2017; Nolan et al., 2016) and discussed at length in policy circles (Piketty, 2014).

This paper illustrates is that the disconnection between growth of the average and the median income, and the dynamics of growth and the income distribution more generally, are also important for the welfare analysis of public investments. The welfare significance of the average level of consumption in CBA is shown to be questionable since it typically under or over-states the true wealth effect that society experiences. A much better approximation for discounting purposes comes from focusing on the fortunes of the median income. These claims are formalised in a manner that policy makers will find intuitive and amenable, two factors that are key to ensuring policy change and application (Groom and Hepburn, 2017).

2 Discounting and Inequality Aversion

We start by deriving the economic framework for the consumption discount rate in a stylized economy. Suppose we have an economy with a continuum of agents of type θ with cumulative distribution function $H(\theta)$, whose (unique) instantaneous felicity function is given by $U(c_t(\theta))$, where $c_t(\theta)$ is consumption of type θ at time t. Assume that all agents have the same pure rate of time preference δ. Using the standard expected utility framework, inter-temporal well-being can be represented by the following social welfare function (SWF)(see also Gollier, 2011, 2015):

$$W_0 = \sum_{t=0} \exp(-\delta t) \int_\theta EU(c_t(\theta))\, dH(\theta) \tag{1}$$

Fubini's theorem allows us to reverse the order of the integration, and the symmetric treatment of risk and inequality in the Utilitarian framework can be interpreted in terms of the veil of ignorance. The SWF can therefore be written equivalently as

$$W_0 = \sum_{t=0} \exp(-\delta t)\, E \int_\theta U(c_t(\theta))\, dH(\theta). \tag{2}$$

At a given point in time, we can use Atkinson (1970)'s concept of the equally distributed equivalent (EDE) level of consumption to rewrite the instantaneous welfare of this society. This consumption level at time t is defined as follows:

$$\begin{aligned} U(c_{ede,t}) &= \int_\theta U(c_t(\theta))\, dH(\theta) \\ &\Rightarrow \\ c_{ede,t} &= U^{-1}\left(\int_\theta U(c_t(\theta))\, dH(\theta)\right) \end{aligned} \tag{3}$$

Clearly, $c_{ede,t}$ depends on the characteristics of the felicity function $U(.)$, in particular the aversion to income inequality that it embodies. Given this definition of the EDE, the SWF can now be re-written equivalently as:

$$W_0 = \sum_{t=0} \exp(-\delta t)\, EU(c_{ede,t}) \tag{4}$$

We take the deterministic case (where future consumption is considered certain), and assume that the costs and benefits of the public project are shared equally among the individuals of the economy. This is a natural assumption for the derivation of the discount rate, which focusses on the marginal impact of a policy on the total population without changing its distribution.[1] The standard derivation of the social discount rate yields:

$$r^* = \delta + \eta g_{ede,t} \tag{5}$$

where $g_{ede,t} = \frac{1}{t} log(c_{ede,t}/c_{ede,0})$ represents the annualized growth rate of the EDE income between time 0 and t, and $\eta \equiv \frac{-c_{ede,t}U''(c_{ede,t})}{U'(c_{ede,t})}$ the elasticity of marginal utility with respect to consumption. This should be compared to the standard Ramsey rule given as $r' = \delta + \eta g_{pc,t}$, where $g_{pc,t}$ refers to the annualized growth of per capita consumption. In comparing discounting with and without inequality considerations, Gollier (2015) derived the conditions under which the presence of a constant level of inequality at each point in time will affect the social discount rate compared to the standard Ramsey rule. The general answer to this question is not straightforward and depends on the relationship between higher order derivatives of the utility function.[2] It is not immediately clear how the results can be broadly applied and placed in, say, government guidelines on CBA. In our framework, the analysis of inequality reduces to defining the conditions under which the growth of the EDE and per-capita consumption differ. The following section shows that this comparison can be made even more straightforward if two further assumptions are made.

3 Discounting with representative growth

In this section we derive a simple adjustment of the Ramsey rule, which takes into account aversion to income inequality in society. This can be done via the use of two commonplace assumptions, one theoretical and one empirical. Together these assumptions define a special case of Gollier (2015) and Emmerling (2010).

Assumption 1: Constant Relative Risk Aversion (CRRA). When the Ramsey rule is presented in policy documents, e.g. H.M. Treasury (2003), CRRA is the typical assumption for the utility function: $U(c_t) = (1-\eta)^{-1} {c_t}^{1-\eta}$. It is also a widely used assumption in applied theory. With CRRA, the equally distributed equivalent level of consumption, $c_{ede,t}$, becomes

[1] In the risk domain the equivalent assumption would be that the project does not affect the probability distribution of potential outcomes.

[2] See Proposition 1 of Gollier (2015, p 57)

$$c_{ede,t} = \left[\int_{\theta} c_t(\theta)^{1-\eta} \, dH(\theta) \right]^{\frac{1}{1-\eta}} \tag{6}$$

where η is a measure of inequality aversion. In essence, $c_{ede,t}$ is the $(1-\eta)^{th}$ raw moment of the distribution of $c_t(\theta)$, raised to the power of $(1-\eta)^{-1}$. Note that, no matter how consumption is distributed in society, when $\eta = 0$, $c_{ede,t}$ equals the per-capita consumption, and when $\eta = \infty$, $c_{ede,t}$ is equal to the consumption of the poorest individual in society. Broadly speaking, social welfare is measured by the utility derived from the consumption of ever poorer people, the larger inequality aversion becomes. The observation that $c_{ede,t}$ is the $(1-\eta)^{th}$ raw moment of the distribution of consumption is useful when combined with the next assumption.

Assumption 2: Log-normal income distribution. Empirical evidence suggests that the log-normal distribution is a good approximation to country level income distributions within countries across the world. While different parametric distributions including Pareto, Beta, Gamma etc. have been used to describe income inequality, in many cases log-normality provides a superior fit to the data than distributions with more parameters. Even in cases where a long right tail exists, the log-normal distribution is a good approximation for the vast majority of the population (Sala-I-Martin, 2006). We make this assumption here in relation to $c_t(\theta)$ so that $c_t(\theta) \sim LN(\mu_t, \sigma_t^2)$.

3.1 The simple 'inequality-adjusted' SDR

Together, Assumption 1 and 2 lead to a convenient form for the equally distributed equivalent income. Note that in general, the k^{th} raw moment of the log-normally distributed variable x (with mean and variance parameters μ and σ^2) is given by $E\left[x^k\right] = \exp\left(k\mu + 0.5k^2\sigma^2\right)$. This means that when the distribution of income at time t is log-normally distributed, and preferences are CRRA, the equally distributed equivalent consumption, $c_{ede,t}$, is given by:

$$c_{ede,t} = \exp\left(\mu_t + 0.5(1-\eta)\sigma_t^2\right) \tag{7}$$

This can be compared to the average, per-capita income level, $c_{pc,t}$,

$$c_{pc,t} = \exp\left(\mu_t + 0.5\sigma_t^2\right) \tag{8}$$

and the median income level, $c_{med,t}$:

$$c_{med,t} = \exp(\mu_t) \tag{9}$$

Given the closed form expression for the EDE level of consumption, it is now possible to calibrate the social discount rate in equation (5) by calculating the annualised growth of EDE consumption in this log-normal and CRRA case, which yields

$$\begin{aligned} g_{ede,t} &= \frac{1}{t}\left(\frac{exp\left(\mu_t + 0.5\left(1-\eta\right)\sigma_t^2\right) - exp\left(\mu_0 + 0.5\left(1-\eta\right)\sigma_0^2\right)}{exp\left(\mu_0 + 0.5\left(1-\eta\right)\sigma_0^2\right)}\right) \\ &= \frac{1}{t}\left[exp\left(\mu_t - \mu_0\right) \cdot exp\left(0.5\left(1-\eta\right)\Delta\sigma_{0,t}^2\right) - 1\right], \end{aligned} \tag{10}$$

where $\Delta\sigma_{0,t}^2 = \sigma_t^2 - \sigma_0^2$ represents the change in the variance of log-consumption from time 0 to t. Equation (10) shows that $g_{ede,t}$ depends on three factors: 1) $\mu_t - \mu_0$: the growth in the log of median income; 2) η : inequality aversion; and, 3) $\Delta\sigma_{0,t}^2$: the change in the variance of log consumption over time. Given the definitions above, $g_{ede,t}$ can be compared with annualized average (per-capita) growth rate:

$$g_{pc,t} = \frac{1}{t}\left[exp\left(\mu_t - \mu_0\right) \cdot exp\left(0.5\Delta\sigma_{0,t}^2\right) - 1\right] \tag{11}$$

and the annualized growth rate of median consumption:

$$g_{med,t} = \frac{1}{t}\left[exp\left(\mu_t - \mu_0\right)\right]. \tag{12}$$

It is important to note that for a given level of inequality aversion, η, in the log-normal case the variance of log consumption, σ_t^2, is a sufficient statistic for the Atkinson index of inequality, $I(\eta)$. Hence, $\Delta\sigma_{0,t}^2$ is sufficient to measure changes in inequality over time, and the 'variance of log consumption' and 'inequality' are interchangeable terms.[3]

If inequality is constant over time ($\Delta\sigma_{0,t}^2 = 0$), then growth of average and EDE consumption coincide: $g_{ede,t} = g_{pc,t}$, since both depend only on the growth of μ .[4] That a fixed level of inequality will not affect the SDR in the isoelastic case, even if the social planner is inequality averse, was first shown by Gollier (2015, p 57). Another result is that when the social planner is not inequality averse ($\eta = 0$), then $g_{ede,t} = g_{pc,t}$.

The most relevant case is when inequality is changing over time ($\Delta\sigma^2 \neq 0$), and

[3] The Atkinson index of inequality is given by: $I(\eta) = 1 - \frac{c_{ede,t}}{c_{pc,t}} = 1 - \exp\left(-\frac{\eta}{2}\sigma_t^2\right)$, which changes over time only according to changes in σ_t^2. Appendix B has a more complete proof.

[4] When $\Delta\sigma_{0,t}^2 = 0$, we have that $g_{ede,t} = \frac{1}{t}\left[\frac{\exp(\mu_t) - \exp(\mu_0)}{\exp(\mu_0)}\right] = g_{median,t} = g_{gc,t}$

$\eta > 0$. In this case $g_{ede,t}$ should be used to calibrate the social discount rate. The effect on the SDR compared to the standard Ramsey rule depends on the difference between $g_{pc,t}$ and $g_{ede,t}$. The basic intuition is rather simple and can be seen by inspection of (10) and (11): if secular growth increases inequality in the economy, EDE income will grow less than average income. This reflects a welfare penalty on secular growth since society is averse to increased inequality. From the perspective of CBA, well-being has not increased as much for the inequality averse representative agent as it would have for the typical representative agent who is only concerned with average growth. The true wealth effect is effectively smaller. Alternatively, if secular growth is inequality reducing, the opposite effect happens.

We can go even further by using properties of the log-normal distribution (Assumption 2) to derive analytically a very simple expression for the EDE growth rate. By combining the formulae for the growth rate of per-capita consumption (11), median consumption (12), and the EDE (10), one finds that the EDE growth rate can be written as a function of inequality aversion, and the the per-capita and median growth rates:

$$g_{ede,t} = g_{pc,t} + \eta \left(g_{med,t} - g_{pc,t} \right), \tag{13}$$

This expression can be directly inserted into the formula for the SDR in 5 to yield

$$r^* = \delta + \eta g_{pc,t} + \eta^2 \left(g_{med,t} - g_{pc,t} \right). \tag{14}$$

In this case $\eta^2 \left(g_{med,t} - g_{pc,t} \right)$ is the adjustment for intra-temporal inequality aversion. This simple adjustment exploits the fact that under Assumption 2, the difference between the growth of median and average income is a sufficient statistic for the evolution of inequality over time. The welfare effect of changes in inequality is reflected in the discount rate as an adjusted wealth effect. The correction to growth is proportional to difference between the growth of median and per capita income where, under Assumption 1, the constant of proportionality is the inequality aversion parameter, η. When applied to the social discount rate, this adjustment factor is multiplied by the inverse of the intertemporal elasticity of substitution, just like a standard wealth effect. This means that $\left(g_{med,t} - g_{pc,t} \right)$ scales in η^2, since both inter- and intra-temporal fluctuation/inequality aversion coincide.[5]

[5] Note that this dependence on η is special feature of the Utilitarian model in which the parameter η captures both inequality aversion and the (inverse) elasticity inter-temporal substitution. Emmerling (2010) shows that using Kreps-Porteus preferences with the assumption of isoelastic functional specifications, it is possible to separate these characteristics and obtain a Ramsey like formula for discounting: $r_t = \delta + \varepsilon g_{ede,t}$, where ε is the inverse of the elasticity of inter-temporal substitution. Appendix A explores this approach in more detail.

3.2 The representative median income

Two special cases are worth noting here. As before, $g_{ede,t}$ equals the growth rate of per-capita consumption for $\eta = 0$. More interestingly $g_{ede,t}$ becomes $g_{med,t}$ when $\eta = 1$. Applying $\eta = 1$ to (10) we get:

$$g_{ede,t} = \frac{1}{t}\left[exp\left(\mu_t - \mu_0\right)\right] = g_{med,t}, \tag{15}$$

Here the appropriate growth rate for the SDR reflects the growth fortunes of the median income, not the average per capita income. Essentially, agents with the median income become representative agents.

It should be of no surprise that changing the social welfare function with which marginal projects are evaluated in CBA, changes the SDR. What is surprising is that introducing intra-generational inequality aversion into the SWF can be easily accommodated within the standard guidelines for the SDR via a simple adjustment to growth to reflect changes in inequality over time. Under commonly-used (Assumption 1) or empirically justified (Assumption 2) assumptions, the only additional information required to implement the SDR in (14) is an estimate of the growth rate of the median income $g_{med,t}$.

4 Inequality adjustment in practice

In this section we undertake a numerical exercise to show the practical implications of our proposed inequality adjustment to the SDR, and the focus on the median household that it relies on. With regard to inequality aversion by the social planner, there are numerous estimates of the extent of inequality aversion in society (Groom and Maddison, 2013; Stern, 1977; Cowell and Gardiner, 1999). The context in which these estimates are obtained varies from one study to the next. Most work with the assumption that utility is CRRA as in this paper. For instance, experiments undertaken with students tend to give estimates of η that range from 0.8 to 2. When socially revealed inequality aversion is used to estimate parameters of inequality aversion, as revealed in progressive income tax schedules or international transfers of aid, values of η between 0.4 and 1.5 are found (Tol, 2010). More broadly, estimates of the societal elasticity of marginal utility, which can be estimated based on inequality, inter-temporal or risk preferences, find values of η between 1 and 2 (Stern, 1977; Groom and Maddison, 2013). In practice, it is often argued that $\eta = 1$ is a reasonable reflection of societal inequality aversion. The Stern (2006) Review took exactly this

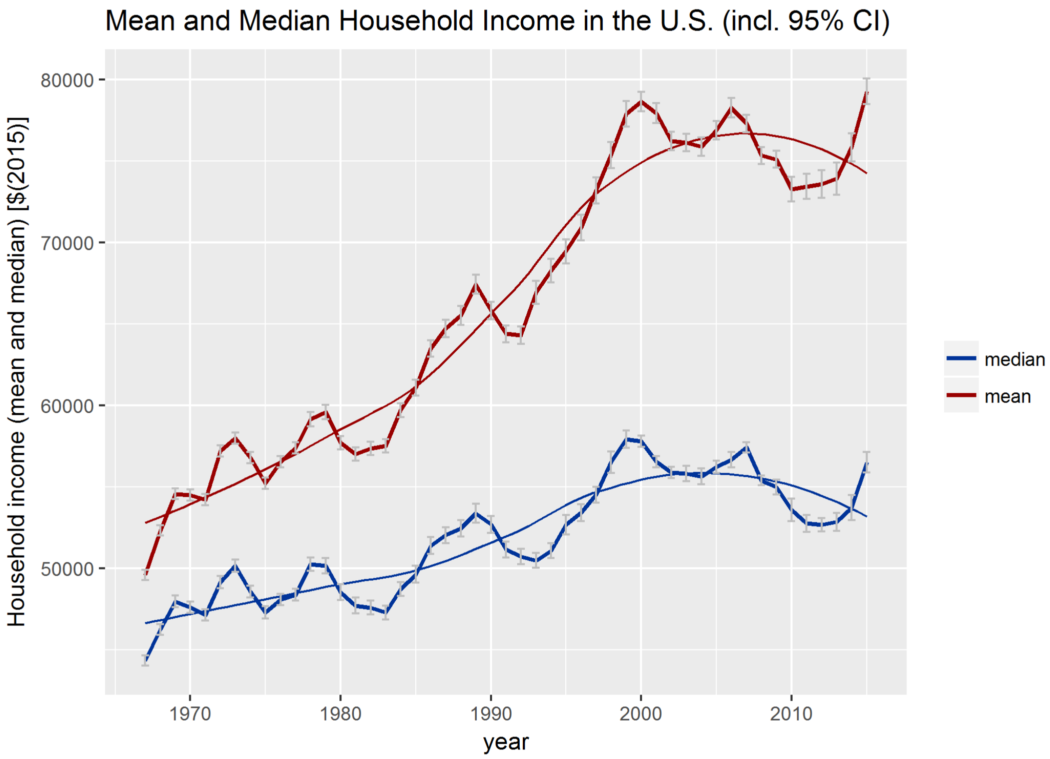

Figure 1: Household mean and median consumption growth in the U.S. (Source: Proctor et al. (2016))

position in its analysis of climate change mitigation. The UK Treasury guidelines also take this position and are unlikely to change it in the ongoing refresh of the Government guidelines on CBA (H.M. Treasury, 2003). That is, using a logarithmic utility function as one of the assumptions seems defensible based on these grounds.

The choice of the individual/household in the middle of the income distribution to represent the society has intuitive appeal. Indeed, taking the distribution into account when measuring national income has been a recommendation that goes back decades (Sen, 1976). More recently, the choice of the median household income growth rate as a measurement of growth and welfare has been argued for prominently. The LSE growth commission (Aghion et al., 2013) argued that it would be preferable to measure growth of median household income, since "the median is better than the mean since it is reflective of progress in the middle of the income distribution." They believe that the median income would focus more on distributional issues than the mean. The European Commission recommends the use of the median income as it "better reflects progress in the middle of the income distribution" (European Commission, 2014), as does the Stiglitz report (Stiglitz, 2012).

Note that the economic justification for the wealth effect in the discount rate is that the future generation will be richer than the present justifying a lower welfare weight for future generations. Since the median growth rate can be considered a better measure of welfare than the average per-capita growth rate, its use in the Ramsey rule can be justified from a normative perspective. Hence, even if the median growth rate will underestimate the impact of inequality, it would still preferable to the per-capita growth rate, which would ignore it completely. For example, the LSE growth commission acknowledges that the median is not perfect but "better than ignoring distribution entirely" (Aghion et al., 2013).

Finally, we apply our obtained formula for the SDR to a set of 25[6] countries and quantify the inequality effect due to the difference between median and average growth. We use a recently created dataset by Nolan et al. (2016) and Max Roser et al. (2016), who collected data on median income and compare it with per-capita values arguing that indeed a significant difference between the two measures is often found. In Figure 1, we computed the growth rates of the average and median household income per capita, using the longest available time period available. Note that Nolan et al. (2016) decompose the difference between GDP per capita and median household income into five factors, notably price adjustments (CPI vs. GDP deflator), domestic product vs. national income, macroeconomic data vs. household surveys, variation in the household size, and inequality (mean vs. median). For the purpose of the discount rate, it seems reasonable to consider all of these factors in, apart from considering households and rather consider per-capita levels of income or consumption, given that for public projects one would consider public projects affecting all individuals in the economy. Taking into account household size would implicitly lead to consideration of different population sizes over time. Hence, we use the per-capita household income (mean and median) of the dataset of Max Roser et al. (2016). Based on these growth rates, we compute the discount rates based on these growth rates thus comparing the standard Ramsey rule with the one we derived in this paper. For this sample of 25 countries, in 15 countries median growth fell short of average income, while in 10 (mostly middle income) countries median growth was actually higher. Thus, the effect of the discount rate can go in both directions. For high income countries such as the UK and U.S., the growth rate adjustment factor is in the order of magnitude of -0.25% so that it leads to a reduction in the SDR by the same amount for $\eta = 1$ and by 1 percentage point ($2^2 \cdot -0.25\% = -1\%$) for $\eta = 2$.

[6] We include all countries of the original dataset of Max Roser et al. (2016) for which at least a time series of 10 years could be used to estimated the average growth rates.

Table 1: Mean/Median growth rates and impact on SDR $\eta^2(g_{med} - g_{pc})$

country	period	g_pc	g_med	$\eta = 1$	$\eta = 2$
Australia	1981-2010	1.61	1.45	-0.16	-0.62
Austria	1994-2004	1.07	1.20	0.13	0.52
Belgium	1985-2000	3.18	2.41	-0.77	-3.07
Canada	1981-2010	1.01	0.96	-0.04	-0.18
Czech Republic	1992-2010	3.28	3.10	-0.17	-0.69
Denmark	1987-2010	0.97	0.94	-0.03	-0.13
Estonia	2000-2010	5.69	6.38	0.69	2.76
Finland	1987-2010	1.95	1.70	-0.25	-1.02
France	1978-2010	1.13	1.28	0.14	0.57
Germany	1984-2010	0.89	0.83	-0.06	-0.25
Greece	1995-2010	2.07	2.32	0.25	1.00
Hungary	1991-2012	0.19	0.25	0.06	0.25
Ireland	1987-2010	3.61	3.73	0.12	0.48
Israel	1986-2010	1.93	1.79	-0.14	-0.57
Italy	1986-2010	1.27	1.26	-0.01	-0.03
Luxembourg	1985-2010	3.08	2.98	-0.11	-0.42
Netherlands	1993-2010	1.62	1.78	0.16	0.62
Norway	1979-2010	2.60	2.72	0.11	0.45
Poland	1992-2010	2.07	1.83	-0.23	-0.94
Slovak Republic	1992-2010	2.61	2.41	-0.20	-0.79
Slovenia	1997-2010	2.53	2.58	0.05	0.19
Spain	1980-2010	2.21	2.36	0.15	0.61
Sweden	1981-2005	1.89	1.68	-0.21	-0.85
United Kingdom	1979-2010	2.37	2.15	-0.22	-0.88
United States	1979-2013	0.77	0.49	-0.28	-1.11

5 Conclusion

The wealth effect in the standard Ramsey rule for the social discount rate stems from per capita consumption growth and the curvature of the representative agent's utility function. We discount the future because we anticipate future agents will be richer, and value less a marginal contribution to their consumption. Yet, even if we assume that per-capita growth will follow historical trends and continue to be positive in the foreseeable future, it is worth asking whether the growth of average income is representative of the growth experience of the majority of people in society? If average consumption growth is being driven by growth among the rich, inequality will increase over time and most people in society will not experience the average growth rate. In this case, discounting marginal changes in consumption at the average growth rate would put too little weight on changes in well-being for the majority of the population. The opposite would be true if growth is inequality reducing. In both cases the average wealth effect is not representative of the fortunes of most people in society. Consequently, public investment portfolios may not be welfare enhancing for most people.

In this paper, we develop a simple policy rule that would allow CBA guidelines on discounting to take account of changes in income inequality in the evaluation of public projects. Under two simple and defensible assumptions (isoelastic utility and log-normal income distribution) we derive an 'inequality-adjusted' growth rate that corrects the wealth effect for changes in inequality. This leads to a simple inequality adjusted SDR. Under our assumptions the adjustment to average (per capita) growth is proportional to the difference between the growth of median and average incomes. The inequality adjusted SDR is reduced if median income growth lags behind average growth, and vice versa. The approach provides a welfare basis for focussing on median incomes as a measure of economic performance and in the appraisal of public policy. In one plausible case, agents with median incomes become the representative agents.

Our approach has the significant virtue of being easily implementable, since it requires only data on the growth of median incomes, which are routinely collected by national statistical agencies. Empirical evidence shows that growth has been inequality increasing in some countries, and inequality reducing in others. In the U.S., for instance, average (per capita) growth has been driven by growth in the upper tail of the income distribution. The mean grew at a rate of 1% p.a. since 1970, while the median income increased by only 0.3% p.a. (see Figure 1). Similar figures are true for the UK, while in Spain inequality has decreased. The inequality adjustments to the SDR are therefore of consequence. With an inequality aversion parameter of 1 (1.5, 2), the UK and U.S. discount rates would be approximately 0.25% (0.5%, 1%) lower than the standard Ramsey rule. In other countries, e.g. Spain, the adjustment works in the opposite direction since inequality has decreased. Such adjustments could have a large influence on the portfolio of public investments.

Overall, our approach to discounting ensures that our view of future prospects is not skewed by the fortunes of the average income, and our evaluation of welfare changes in public policy appraisal is more representative and fair. But one possible criticism of the approach concerns the implicit commitments that a public policy maker has to make when calibrating the inequality-adjusted SDR. The inequality adjustment requires a prediction concerning how inequality is expected to evolve in the future. At first glance it would be unusual for a government to build-in to CBA a position on the expected changes in inequality. If increasing inequality is built-in, this might be seen as a failure for an incumbent left-wing government, or provide political leverage for a left wing opposition. Similar political machinations would occur if declining inequalities were built-in. So our proposal appears to introduce some unwanted political dimensions to the SDR.

Yet, in the UK for instance, a positive (2%) growth is built-in to the SDR, despite this being the one of the main political issues over which political parties compete. Growth expectations for the purposes of CBA are reasonably based on historical data to avoid this kind of political manipulation. A similarly positive (descriptive) line could be taken in relation to income distribution, with growth of the median and mean income being treated in the same, descriptive way. In fact the dependence of the inequality adjusted SDR on these simple growth statistics is helpful precisely because it circumvents direct estimation of changes in the income distribution. This is just another way of saying that secular changes in inequality can be treated as exogenous in CBA. Of course, academics must remain mindful of the prospect of political constraints to policy. However, the current approach to the SDR ignores secular changes to inequality altogether. The benefits of correcting this existing error in the welfare analysis of public projects needs to be weighed against the, in our view limited, prospect of politicising the discount rate in the dimension of inequality. A reasonable compromise on this front would be to use the inequality adjusted SDR for sensitivity analysis. This would at least bring the issue of whose welfare is being evaluated in CBA into the analysis of public projects.

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Appendix

A Disentangling inequality aversion and the elasticity of inter-temporal substitution

Using Kreps-Porteus preferences, it is possible to first model the preferences for inequality aversion and hence the EDE income using the function $U(c_{it})$, and then model the inter-temporal dimension separately as a function of the EDE income. Following Epstein and Zin (1989), we use isoelastic functions. The EDE income is then defined as in equation (6) as $c_t^{ede} = U^{-1}E\left[U(c_{it})\right] = E\left[c_{it}^{1-\eta}\right]^{\frac{1}{1-\eta}}$. Then inter-temporal social welfare at time t is defined as:

$$
\begin{aligned}
W_t &= \sum\nolimits_t v\left(c_t^{ede}\right)\exp\left(-\delta t\right) = \\
&= \sum\nolimits_t \left(c_t^{ede}\right)^{1-\varepsilon}\exp\left(-\delta t\right) \\
&= \sum\nolimits_t \left(E\left[c_{it}^{1-\eta}\right]\right)^{\frac{1-\varepsilon}{1-\eta}}\exp\left(-\delta t\right)
\end{aligned}
$$

where the expectations operator is taken over the distribution of consumption across individuals at time t. With such a social welfare function, the social discount rate becomes:

$$SDR = \delta + \varepsilon g_{ede,t}$$

where ε is the inverse of the elasticity of inter-temporal substitution. As discussed in Section 2, in the case where $\eta = 0$, that is without inequality aversion, this collapses to the standard Ramsey model in which ε as before represents inter-temporal preferences. Emmerling (2010) undertakes a more extensive analysis which extends this framework to also include uncertainty about future consumption c_{it}.

B Atkinson's inequality measure and the variance of log consumption.

Proposition B1. *Changes in Atkinson inequality index, $I(\eta) = 1 - \frac{c_{ede,t}}{c_{pc,t}}$, with inequality aversion parameter η, depend only on the change in the variance of log consumption over time,$\Delta\sigma^2_{0,t}$, when the distribution of consumption is log-normal, $c_t \sim LN(\mu_t, \sigma_t^2)$.*

Proof. Define $\Delta I = I_t - I_0$ is the difference between the Atkinson index at period t and 0. Inequality increases (decreases) if $\Delta I > 0$ ($\Delta I < 0$) and is constant if $\Delta I = 0$. With isoelastic utility, and using Equations (7) and (8) we get:

$$\begin{aligned}\Delta I &= \frac{c_{ede,0}}{c_{pc,0}} - \frac{c_{ede,t}}{c_{pc,t}} \\ &= \frac{exp(\mu_0 + \frac{1-\eta}{2}\sigma_0^2)}{exp(\mu_0 + \frac{1}{2}\sigma_0^2)} - \frac{exp(\mu_t + \frac{1-\eta}{2}\sigma_t^2)}{exp(\mu_t + \frac{1}{2}\sigma_t^2)} \\ &= exp\left(-\frac{\eta}{2}\sigma_0^2\right) - exp\left(-\frac{\eta}{2}\sigma_t^2\right)\end{aligned}$$

So, for an inequality averse planner($\eta > 0$), $\Delta I > 0$ ($\Delta I < 0$) if and only if $\sigma_t^2 > \sigma_0^2$ ($\sigma_t^2 < \sigma_0^2$) . Or, if $\eta > 0$, $\Delta I > 0$ if $\Delta\sigma^2_{0,t} > 0$, vice versa for $\Delta\sigma^2_{0,t} < 0$. QED.