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A Theory of Gains from Trade in Multilaterally Linked ETSs

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Abstract

Linkages between emissions trading systems (ETSs) have an important role to play in the successful, cost-effective implementation of the Paris Agreement. While the theory of bilateral linkages is well established, we know relatively little about the gains from trade in a multilaterally linked system, and less still about how they are shared among jurisdictions participating in the system. We characterize these gains for an arbitrary linkage coalition, show how they can be decomposed into gains in the coalition's internal bilateral linkages, and prove that linkage is superadditive. Our theoretical results imply the global market may not emerge endogenously and a quantitative exercise shows that this concern may have some validity in practice.

Keywords: International emissions trading systems, climate change policy, bilateral linking, multilateral linking, global carbon market.

JEL classification codes: Q58, H23.

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1 Introduction

Markets for emission permits have long been an important climate policy tool in regulating greenhouse gas emissions. A patchwork of emissions trading systems (ETSs) tailored to local circumstances and specific constraints are now operational in jurisdictions including Europe, Switzerland, South Korea, seven Chinese provinces and cities, and several US states and Canadian provinces among other places (ICAP, 2017). More are in the pipeline with China, the world's largest emitter, planning to start a national market in 2017. ETSs are expected to grow in number as focus gradually shifts from country promises in the Nationally Determined Contributions of the Paris Agreement to practical questions of delivering on them.

Against this backdrop, system integration will be a significant element of the global climate change policy framework in the future (Bodansky et al., 2016). Indeed, the Paris Agreement encourages the voluntary integration of emission reduction efforts. Linkages between ETSs is one way this can be done and would yield economic benefits by spreading abatement efforts cost-effectively among the participating systems, ultimately generating a uniform price signal. In fact, some jurisdictions are already linked (California and Québec), will link in the near future having completed the required negotiations (Europe and Switzerland), or are contemplating a link with an existing system (Ontario with California and Québec).

Current research examining the determinants of the benefits of linking ETSs has primarily focused on bilateral linkages.[1] By comparison, a formal study of multilateral linkages poses numerous challenges, as discussed in Mehling & Görlach (2016), who propose different options for a successful management of these linkages. This paper makes two contributions to the intensifying debate on system integration in the post-Paris Agreement era.

First, from a theory perspective, we develop a general model that allows us to describe and study multilateral linkages among ETSs. A novel and extremely useful tool that emerges from our analysis is the formula we derive for decomposing the gains in a multilateral linkage into gains in its internal bilateral linkages. The decomposition enables us to characterize and quantify the individual and aggregate gains for an arbitrary number of linked jurisdictions and to demonstrate that linkage is a superadditive mechanism. This makes our model apt for policy analysis shedding light on questions like which regions gain the most by linking their existing or planned ETSs.

[1] Economic and political aspects of linking two ETSs are explored in Flachsland et al. (2009), Jaffe et al. (2009), Mehling & Haites (2009), Tuerk et al. (2009), Burtraw et al. (2013), Ranson & Stavins (2016), Doda & Taschini (2017) and Quemin & de Perthuis (2017) among others.

Second, we take the model to the data and illustrate the quantitative implications of our general theory using a real-world example. We show that despite generating the largest aggregate savings, the global market may not be everyone's most preferred option. We further extend the policy application by introducing linkage costs and comparing the ability of alternative inter-jurisdictional cost-sharing arrangements in obtaining the socially efficient structure of linkage coalitions.

Our model explores the gains from linking under uncertainty by introducing idiosyncratic shocks à la Weitzman (1974) and Yohe (1976, 1978). To isolate those gains that are directly attributable to linkage, we assume that jurisdictional emissions caps are exogenous and fixed permanently. Therefore, there is no strategic interaction between jurisdictions' linking decisions and no anticipation of linkage when emissions caps are selected. The assumption is deliberate. Our aim is to understand the determinants of the economic gains from multilateral linkage and to be able to characterize them analytically.

In practice, emissions caps result from complex negotiation processes involving a host of domestic stakeholders with vested interests that must accommodate jurisdiction-specific constraints of different sorts (Flachsland et al., 2009). In addition, jurisdictions may also implement policies that are supplemental to ETSs, which can further reflect different priorities on both the appropriate level of domestic abatement efforts and the desirable level of the underlying price signal. As a consequence, it seems unlikely that jurisdictions select their domestic caps with an eye on linkage in the future. If, however, they do factor in linkage, it can be argued that this will be in a bid to align ambition levels across partnering jurisdictions and thereby render linkage politically feasible, rather than as a way to strategically inflate their gains from linkage. Therefore, we take cap selection as a decision of fundamentally domestic and political nature, and place it beyond the scope of this work.

With invariant jurisdictional emission caps and the abstraction they afford from strategic interactions, we evaluate multilateral linkage options using a combinatorial approach exploring all possible linkages. When linkage costs are negligible, linkage is always mutually beneficial, i.e. jurisdictions are always better off in any linkage coalition than under autarky. However, we show jurisdictions gain more when participating in some linkage coalitions than in others. In this respect, our analysis allows us to rank alternative coalitions from a jurisdictional perspective and thereby characterize jurisdictional linkage preferences.

There are potential gains from linkage whenever there exists a price differential between jurisdictions under autarky. In turn, the autarkic price wedge can be the result of varying jurisdiction-specific ambition levels or jurisdiction-specific shocks. Most existing analyses of linkage concentrate on the former as the sole source of gains. In this paper, we argue that these two sources of gains can exist independently of each other, and devote most of our attention to characterizing the latter. We show that regardless of the magnitude of the first source, i.e. even when jurisdictions have identical ambition, the second source reinforces the Pareto-argument in favor of linking relative to autarky. Analytically, these two sources show up as two non-negative components of the jurisdictional expected linkage gains: the first component is proportional to the square of the difference between the autarky and linking expected permit prices and the second component is proportional to the variance of the difference between the autarky and linking permit prices.

The theoretical foundations for gains in bilateral linkages are well-understood and intuitive: linkages with larger systems are more beneficial, all else constant. In addition, a jurisdiction prefers the permit demand in its partner's system to be variable and weakly correlated with its own. We show that these insights do not translate easily to multilateral linkages. To see why, consider a special case with three jurisdictions which have independent shocks with identical variances. Let also two jurisdictions have same size and the third one be larger. When evaluating possible linkages, the larger jurisdiction has little incentive to link exclusively with a single smaller jurisdiction and prefers to be part of the trilaterally linked market instead. Conversely, smaller jurisdictions prefer a bilateral linkage with the larger jurisdiction. Intuitively, this is because in all possible linking arrangements the linking price will settle closer to the autarky price of the large jurisdiction. Therefore, its gains from trade will be greater in the trilateral link as its own impact on the linking price is attenuated relative to bilateral linkages. This argument reverses for the two smaller jurisdictions.

In general, however, the identification of multilateral linkage outcomes and jurisdictional preferences in these linkages are not clear when one moves away from the special cases. Moreover, the number of possible linkage coalitions and coalition structures, i.e. partitions of the set of jurisdictions, increases exponentially with the number of jurisdictions. For instance, with four jurisdictions there are six possible bilateral linkages (with two jurisdictions in autarky), three groups of two bilateral linkages, four trilateral linkages (with one jurisdiction in autarky), one quadrilateral linkage, and complete autarky where each system operates in isolation; i.e. 15 coalition structures in total. With 10 jurisdictions, there are already 1,013 and 115,975 possible linkage coalitions and coalition structures, respectively.

Despite this apparent complexity, our model can in principle handle an arbitrary number of jurisdictions and complex coalition structures. It clarifies the mechanisms that govern multilateral linkage by deriving an analytical expression for the magnitude of jurisdiction-specific gains in multilateral linkages. The ability to decompose any multilateral linkage into its internal bilateral linkages permits an alternative and more practical formulation of these gains as a size-weighted function of aggregate gains from all bilateral links that can be formed within this coalition. Moreover, we show that linkage is superadditive, i.e. the aggregate expected gains from the union of disjoint coalitions of linked ETSs is no less than the sum of separate coalitions' expected gains. A formula for the quantification of the gains attributed to superadditivity is provided.

A natural consequence of superadditivity is that the global market generates the highest aggregate gains. Absent inter-jurisdictional transfers, however, there is no guarantee that the global market is the most preferred linkage coalition from the perspective of an individual jurisdiction. In fact, the conditions for the global market to be the most preferred coalition universally are unlikely to be satisfied in practice. The combination of superadditivity and impracticality of transfers, say due to the prevalence of political-economy constraints, implies that the jurisdictional linkage preferences within an arbitrary coalition may not be aligned. This could be one reason why linkages are rarely observed in practice.

In our model the volatility of the permit price in any linkage coalition is a well-defined object whose properties we study. When the shocks affecting individual jurisdictions are correlated, there is no reason to expect the variance of the linkage price to decline as the number of jurisdictions participating in the coalition increases. However, we show that the most volatile jurisdictions will always experience reduced price volatility under linking relative to autarky as their domestic shocks are spread over a larger market and thus better buffered. Conversely, jurisdictions with the least volatile shocks may face an increase in price volatility under linking because links create exposure to shocks occurring abroad. This is more likely to be the case when these jurisdictions are small as the influence of larger jurisdictions on the link outcome will be relatively more pronounced.

We use a quantitative illustration, loosely calibrated to five real-world jurisdictions, to demonstrate the potential shortcomings of the existing theory of bilateral linkages. In this exercise, the global market turns out *not* to be the most preferred coalition for every jurisdiction. We explore further policy implications by introducing linkage costs to account for political-economy frictions and thus bring in minimal realism. We assume linkage costs have two components: implementation costs that are higher the larger the jurisdictions involved, and negotiation costs that are higher the larger the number of participants. The magnitude of linkage costs is thus endogenous to linkage coalition formation. In the literature these considerations have given rise to concepts such as minilateralism (Falkner, 2016) or polycentrism

(Ostrom, 2009; Dorsch & Flachsland, 2017).

With linkage costs and alternative assumptions about how they are shared, we show how non-degenerate coalition structures may yield higher aggregate net gains than the global market. We observe that such structures may feature some jurisdictions that remain in autarky, unlinked, as well as coexisting linkage coalitions. We find noticeable differences across cost-sharing rules. In a world where outright permit or cash transfers can run into significant political-economy hurdles, these differences can have far-reaching policy implications for initiatives aiming to steer jurisdictions towards efficient policy architectures, such as the World Bank's Partnership for Market Readiness and the G7 Carbon Market Platform.

The remainder is organized as follows. Section 2 reviews the related literature. Section 3 introduces the assumptions and building blocks of the model, offers a primer on bilateral linkages, and discusses complexities that arise in multilateral settings using a simple three-jurisdiction world. This section also presents the general theory of multilateral linkages and states our main analytical results. The quantitative illustration and the corresponding policy implications are in Section 4. Section 5 concludes. Appendices A to D provide the analytical derivations and proofs, discuss various generalizations of the model and provide additional technical details and context. All numbered tables and figures are provided at the end.

2 Related literature

Our model is similar in spirit to the multinational production-location decision studied in de Meza & van der Ploeg (1987) and the desirable degree of decentralization in permit markets analyzed by Yates (2002). First, de Meza & van der Ploeg (1987) consider a multinational firm whose objective is to maximize its expected profits by relocating production across plants situated in different countries with plant-specific shocks but crucially, the sizes of plants are irrelevant. There is thus a conceptual difference with our analysis of jurisdictional economic gains from linkage. Second, Yates (2002) develops a similar framework where a single regulator decides whether to allow trading across firms within a given jurisdiction in the presence of asymmetric information on abatement costs between firms and the regulator. Yates finds that full decentralization is socially optimal in the case of uniformly-mixed pollutants, anticipating our result that the global market is the most desirable outcome from an aggregate perspective.[2] However, he does not analyze the effects of decentralization at the firm level.

[2] In fact, Yates (2002) analyzes trading across 'natural divisions' of the considered permit market, e.g. compliance periods, firms, regions, etc. With the interpretation that divisions correspond to time periods, Yates

Regarding linkage, a recent theoretical study of the optimal scope of price and quantity policies by Caillaud & Demange (2017) analyzes the effect of merging ETSs on a global scale. Caillaud & Demange obtain the analog of Proposition 1 but do not decompose gains from merging ETSs any further nor do they study the mechanisms governing multilateral linkages and how the benefits are shared among participating jurisdictions. Analogously, Hennessy & Roosen (1999) study merger incentives among firms subject to a permit market when emissions are stochastic. Hennessy & Roosen find merging firms is beneficial almost surely for both risk-neutral and risk-averse firms. However, in their model the incentive to merge arises from the non-linearity in firms' objective functions induced by penalties charged above a predetermined emission threshold. Interestingly, they also find that merging is superadditive (i.e., total expected profits for the merged firms can be no less than the sum of expected profits for separate firms) but stop short of the description of the properties of this mechanism (as we do in Proposition 4) and its implications for merging firms.

Additionally, using a computable general equilibrium model, Carbone et al. (2009) generate estimates of economy-wide gains from linkage by considering the formation of a single coalition of linked ETSs with endogenous selection of non-cooperative emissions caps. Similarly, Heitzig (2013) numerically explores the dynamic process of formation of coalitions of linked ETSs where jurisdictions also have the possibility to coordinate on emissions cap selection. These last two contributions, however, do not characterize multilateral linkage analytically nor do they investigate the determinants of gains from linkage.

Flachsland et al. (2009), Jaffe et al. (2009), Fankhauser & Hepburn (2010) and Pizer & Yates (2015) argue that linkage ought to reduce overall permit price volatility by pointing out that domestic shocks are dispersed over a larger market.[3] Empirically, this claim is supported by Jacks et al. (2011) who find that gradual market integration has reduced overall commodity price volatility over time since 1700. At the same time, these studies note that this by no means imply that price volatility experienced by linked jurisdictions is actually lowered as some may face greater exposure to link-transmitted shocks. Empirically, similar results have been established by Caselli et al. (2015) in the international trade context. They show that openness to international trade has potential to lower GDP volatility when country-specific shocks are the most significant source of volatility (i.e., 'diversification through trade') but underline that this is not guaranteed in general.

& Cronshaw (2001), Feng & Zhao (2006) and Fell et al. (2012) show that providing for intertemporal trading of permits can be an optimal regulatory response to abatement cost shocks.

[3] Analogously, Colla et al. (2012) show that the presence of speculators with whom risk averse firms can trade permits augments the risk bearing capacity of the market and tends to reduce permit price volatility.

Finally, the assumption of invariant caps makes our model distinct from the literature on self-enforcing international environmental agreements (IEA) initiated by Carraro & Siniscalco (1993) and Barrett (1994). First, we rule out strategic interactions and spillovers associated with linkage. For instance, Helm (2003) shows how anticipation of linkage alters jurisdictional incentives in the determination of their domestic caps. In Habla & Winkler (2017) a principal-agent problem leads to an overissuance of permits and may undermine incentives to link. Second, most of this literature studies a Cartel game where only one single coalition can form and sets aside the question of multiple coalitions. It typically assumes that coalition members choose their emission caps cooperatively (the coalition is a metaplayer).[4] Third, we abstract from coalition stability considerations. In general, the literature finds somewhat pessimistic results regarding the size of stable coalitions and identifies a trade-off between efficiency and stability.[5] Moreover, varying coalition membership rules and equilibrium concepts in the literature lead to different predictions regarding stability.[6]

Transfers can increase participation in and stability of coalitions (Nagashima et al., 2009; Lessmann et al., 2015). We approach transfers indirectly via alternative linkage cost-sharing rules rather than via alternative permit allocation rules as is usually the case (Altamirano-Cabrera & Finus, 2006).[7] Finally note that a recent contribution by Caparrós & Péreau (2017) shows that a sequential negotiation process always leads to the grand coalition even when it is not stable in a multilateral (one-shot) negotiation stage.

[4]Absent uncertainty, however, inter-jurisdictional emissions trading does not alter total emissions as the effort sharing is already efficient from the coalition's perspective. Notable exceptions include Finus & Maus (2008) and Carbone et al. (2009), e.g. via non-cooperative cap-setting by coalition members.

[5]McGinty (2007) argues that larger coalitions can be stable when jurisdictional asymmetry is allowed.

[6]For instance, Ray & Vohra (1997) study equilibrium binding agreements where coalitions can break up into smaller sub-coalitions, but not vice versa. Ray & Vohra (1999) consider a type of Rubinstein bargaining game for coalition formation. Bloch (1995) and Bloch (1996) analyze an alternating-offers bargaining game and an infinite-horizon coalition formation game, respectively, both requiring unanimity for a coalition to form. Yi (1997) considers alternative coalition membership rules, e.g. open membership, unanimity and equilibrium bindingness. In the climate context, Osmani & Tol (2009) analyze farsightedly stable linkage coalitions in the sense of Chwe (1994). Finally, Konishi & Ray (2003) consider a dynamic coalition formation process with farsighted players, a concept applied more directly to linking by Heitzig (2013).

[7]Using alternative approaches Gersbach & Winkler (2011) and Holtsmark & Midttømme (2015) investigate different mechanisms that incentivize tighter emissions caps.

3 Theory

In this section we first describe the assumptions and building blocks of the economic environment to study linkages. We then characterize the autarky equilibrium which serves as a reference throughout. Next, we introduce linkages of increasing complexity starting with a primer on bilateral linkages and then moving to the simplest multilateral setting with three jurisdictions to illustrate the complexities that arise with the arrival of a third jurisdiction. Finally, we set out the general theory of multilateral linkages with n jurisdictions. Our main analytical results are in section 3.4.2. They relate to the magnitude of individual gains in linkages (Prop. 1); the variance of prices in linkage equilibria (Prop. 2); the bilateral decomposition of gains in multilateral linkages (Prop. 3); and the superadditivity of linkage (Prop. 4). Readers who are familiar with the topic or those more comfortable with theory can skip Sections 3.2 and 3.3 without loss of continuity.

3.1 Assumptions and building blocks

We consider a standard static model of competitive markets for emission permits designed to regulate uniformly-mixed pollution in several jurisdictions with independent regulatory authorities. We make four key assumptions. First, markets for permits and markets for other goods and services are separable. That is, we conduct a partial-equilibrium analysis focusing exclusively on the jurisdictions' regulated emissions and abstract from interactions with the rest of the economy. Second, the only uncertainty is in the form of additive shocks affecting the jurisdictions' unregulated emissions levels. These two assumptions are somewhat restrictive but standard (Weitzman, 1974; Yohe, 1976). Third, jurisdictions' benefits from emissions are quadratic functional forms, which facilitate the derivation of analytical results and can be viewed as a local approximation of more general functional specifications (Newell & Pizer, 2003). Fourth, the international political economy dimension is omitted. Each jurisdiction has a regulatory authority who can design policies independently of authorities in other jurisdictions with no anticipation of linkage.

Jurisdictions. There are n jurisdictions and $\mathcal{I} = \{1, \ldots, n\}$ denotes the set of jurisdictions. Benefits from emissions in jurisdiction $i \in \mathcal{I}$ are a function of its level of emissions $q_i \geq 0$ and are subject to the jurisdiction-specific shock θ_i such that

$$B_i(q_i; \theta_i) = (b_1 + \theta_i)q_i - \frac{b_2}{2\psi_i} q_i^2. \tag{1}$$

The parameters $b_1, b_2 > 0$ are identical across jurisdictions and $\psi_i > 0$ is a parameter specific to jurisdiction i. The ratio b_2/ψ_i controls the slope of i's linear marginal benefit schedule. We adopt the interpretation that the common b_2 characterizes the abatement technology and that ψ_i is a measure of the volume of emissions to be regulated in jurisdiction i.[8] To see this, note that jurisdiction i's optimal emissions in response to an arbitrary permit price $p > 0$ are given by $q_i^*(p) = \psi_i(b_1 + \theta_i - p)/b_2$. Then jurisdiction i's laissez-faire emissions (i.e., when $p = 0$) amount to

$$\tilde{q}_i = \psi_i(b_1 + \theta_i)/b_2, \tag{2}$$

which are proportional to ψ_i. Below we refer to ψ_i as the size of jurisdiction i and emphasize it is imperfectly correlated with the jurisdiction's other relevant economic dimensions such as GDP or population. Business-as-usual emissions in i are defined by $\bar{q}_i \doteq \mathbb{E}\{\tilde{q}_i\} = \psi_i b_1/b_2$. For analytical convenience and without loss of generality, we assume that jurisdictional shocks are mean-zero with constant variance and may be correlated across jurisdictions, that is

$$\mathbb{E}\{\theta_i\} = 0,\ \mathbb{V}\{\theta_i\} = \sigma_i^2,\ \text{and } \mathrm{Cov}\{\theta_i;\theta_j\} = \rho_{ij}\sigma_i\sigma_j \text{ with } \rho_{ij} \in [-1;1]. \tag{3}$$

These shocks capture the net effect of stochastic factors that may influence emissions and their associated benefits, e.g. business cycles and technology shocks, jurisdiction-specific events, changes in the price of factors of production, weather fluctuations, etc. For instance, $\theta_i > 0$ can be a favorable productivity shock that increases jurisdiction i's benefits from emissions, and as a consequence, emissions relative to baseline. We assume that $\theta_i > -b_1$ for every jurisdiction and shock realization. This is innocuous and guarantees that $\tilde{q}_i$ is always positive. In sum, jurisdictions are identical up to size and shock.

Emissions caps. The emissions cap profile $(\omega_i)_{i\in\mathcal{I}}$ is exogenous. By implication, jurisdictional caps are independent of the decision to link. That is, domestic caps are fixed once and for all, upheld in all linkage scenarios, and do not constitute a part of the linkage negotiation process. This anchors the aggregate level of emissions at $\Omega_{\mathcal{I}} = \sum_{i\in\mathcal{I}}\omega_i$ and thereby rules out spillovers attributable to linkage. While this assumption is restrictive, it allows us to (*i*) have well-defined (i.e., stable) autarky outcomes; (*ii*) isolate the economic gains directly due to linkage; and (*iii*) compare these gains across multilateral linkages in a meaningful way.

[8] An alternative and observationally-equivalent interpretation of the ratio b_2/ψ_i is that b_2 is the measure of technology in the reference jurisdiction with $\psi_{\text{ref}} = 1$ and that a jurisdiction with $\psi_i > 1$ has access to cheaper abatement opportunities at the margin.

In order to focus our analysis on the benefits of linkage arising from shocks, we further restrict jurisdictional caps to be proportional to jurisdictional size by a common factor of proportionality, that is

$$\omega_i = A \cdot \psi_i \text{ for all } i \in \mathcal{I}, \tag{4}$$

where $A \in (0; b_1/b_2)$. This implies caps are equally stringent relative to baseline. Notice the negative relationship between A and the level of ambition implicitly embedded in domestic caps, specifically as $A \to b_1/b_2$, $\omega_i \to \bar{q}_i$. Moreover, our assumption that A is common to all jurisdictions implies that *expected* autarky permit prices are equal across jurisdictions. This sets the linkage gains associated with the equalization of *expected* marginal benefits across partnering jurisdictions to zero, and attributes the benefits of linking only to those arising from shocks. In Appendix B we show that letting ambition levels differ across jurisdictions does not affect our results. In this appendix we also discuss the implications of alternative cap selection mechanisms and strategic manipulation of caps in anticipation of future linkage.

Autarky equilibria. Under autarky, each jurisdiction complies with its domestic cap. We assume that $\theta_i > (b_2\ \omega_i)/\psi_i - b_1$ for all $i \in \mathcal{I}$ to focus exclusively on interior equilibria. That is, there are weak restrictions on shocks such that all domestic caps are binding and autarky permit prices are positive

$$\bar{p}_i = \bar{p} + \theta_i > 0 \text{ for all } i \in \mathcal{I}, \tag{5}$$

where $\bar{p} = b_1 - b_2 A$ is the expected autarky permit price, and as noted above, is equal across jurisdictions.[9] Therefore, jurisdictions with positive (resp. negative) shock realizations have autarky prices higher (resp. lower) than $\bar{p}$. When autarky prices differ, the aggregate abatement effort is not efficiently allocated among jurisdictions. In particular, cost-efficiency could be improved by shifting abatement away from relatively high- to low-shock jurisdictions until the autarky price differentials disappear. This is the function linkage performs.

3.2 A primer on bilateral linkages

Consider a bilateral link between two jurisdictions i and j and call it $\{i,j\}$-linkage. An interior $\{i,j\}$-linkage equilibrium consists of the triple $(p_{\{i,j\}}, q_{\{i,j\},i}, q_{\{i,j\},j})$ where $p_{\{i,j\}}$ is the

[9] When jurisdiction i's cap is slack, its autarky price is zero and it emits up to its laissez-faire emissions. Linking it to a positive-price jurisdiction would then increase aggregate emissions and could reduce the benefits from linking. Our focus on interior equilibria allows analytical simplification (i) in computing expected gains from linkage as aggregate damages are constant and thus cancel out; (ii) in determining the linking price uniquely. See Goodkind & Coggins (2015) for an extension allowing for corner solutions.

equilibrium permit price in the linked market $\{i,j\}$ and $q_{\{i,j\},i}$ (resp. $q_{\{i,j\},j}$) is the equilibrium emissions level in jurisdiction i (resp. j). The $\{i,j\}$-linkage equilibrium price

$$p_{\{i,j\}} = \bar{p} + \hat{\Theta}_{\{i,j\}} = \frac{\psi_i \bar{p}_i + \psi_j \bar{p}_j}{\psi_i + \psi_j}, \text{ where } \hat{\Theta}_{\{i,j\}} = \frac{\psi_i \theta_i + \psi_j \theta_j}{\psi_i + \psi_j} \tag{6}$$

is the size-averaged shock in the linked system $\{i,j\}$. Because the linking price is also the size-weighted average of autarky prices, it will be closer to that of the larger jurisdiction. Abatement reallocation under $\{i,j\}$-linkage is such that jurisdictional marginal benefits are equalized and the aggregate constraint on emissions $\Omega_{\{i,j\}} = \omega_i + \omega_j$ is satisfied. In particular, jurisdiction i's net demand for permits under $\{i,j\}$-linkage is

$$q_{\{i,j\},i} - \omega_i = \frac{\psi_i}{b_2}(\bar{p}_i - p_{\{i,j\}}) = \frac{\psi_i \psi_j}{b_2\,(\psi_i + \psi_j)}(\theta_i - \theta_j). \tag{7}$$

We note that j's net demand can be obtained from Equation (7) by replacing each occurrence of i with j, and j with i.That is, linkage eliminates the post-shock wedge in realized autarky prices, the magnitude of which is measured by $|\theta_i - \theta_j|$. For given shock realizations, the high-shock (i.e., high-price) jurisdiction will 'import' permits from the low-shock (i.e., low-price) jurisdiction because it values them more. In essence, bilateral linkage increases the effective cap in the high-shock jurisdiction and reduces that of the low-shock jurisdiction by the same amount, thereby leaving the aggregate emissions cap $\Omega_{\{i,j\}}$ unchanged.

Because aggregate emissions are constant, the difference between each jurisdiction's benefits under $\{i,j\}$-linkage and autarky corresponds to its jurisdictional gains from the bilateral link. In i's case, we denote this quantity $\delta_{\{i,j\},i}$ and show in Appendix A.1 that

$$\delta_{\{i,j\},i} = \frac{b_2}{2\psi_i}(q_{\{i,j\},i} - \omega_i)^2. \tag{8}$$

This is non-negative. Following the same steps we obtain $\delta_{\{i,j\},j}$ and find that $\{i,j\}$-linkage is always mutually beneficial. Plugging Equation (7) into Equation (8) then yields

$$\delta_{\{i,j\},i} = \frac{\psi_i}{2b_2}(\bar{p}_i - p_{\{i,j\}})^2 = \frac{\psi_i \psi_j^2}{2b_2(\psi_i + \psi_j)^2}(\theta_i - \theta_j)^2. \tag{9}$$

Next, we define the aggregate gains from $\{i,j\}$-linkage as

$$\Delta_{\{i,j\}} \doteq \delta_{\{i,j\},i} + \delta_{\{i,j\},j} = \frac{\psi_i \psi_j}{2b_2(\psi_i + \psi_j)}(\theta_i - \theta_j)^2. \tag{10}$$

Taking expectations then yields

$$\mathbb{E}\{\Delta_{\{i,j\}}\} = \underbrace{\frac{\psi_i \psi_j}{2b_2(\psi_i + \psi_j)}}_{PSE_{\{i,j\}}} \Big(\underbrace{\sigma_i^2 + \sigma_j^2}_{VE_{\{i,j\}}} \underbrace{-2\rho_{ij}\sigma_i\sigma_j}_{DE_{\{i,j\}}} \Big) \geq 0, \tag{11}$$

where we adapt the terminology in Doda & Taschini (2017) to indicate the pair size effect ($PSE_{\{i,j\}}$), volatility effect ($VE_{\{i,j\}}$) and dependence effect ($DE_{\{i,j\}}$) specific to $\{i,j\}$-linkage. We observe that the influence of jurisdictional sizes (resp. shocks) is confined to PSE (resp. VE and DE). We also observe that the aggregate expected gain is (i) positive as long as jurisdictional shocks are imperfectly correlated and jurisdictional volatility levels differ, for otherwise the two jurisdictions are identical in terms of shock characteristics and there is no gain from linkage,[10] (ii) increasing in both jurisdictional volatilities and sizes, (iii) is higher the more weakly correlated jurisdictional shocks are, and (iv) for a given aggregate size, maximal when jurisdictions have equal sizes.

In addition, we note that the aggregate gain is apportioned between jurisdictions in inverse proportion to size.[11] Formally, $\mathbb{E}\{\delta_{\{i,j\},i}\}/\mathbb{E}\{\delta_{\{i,j\},j}\} = \psi_j/\psi_i$. This is so because, for a given volume of permit trade, the *distance* between the autarky and linkage prices is greater in the smaller jurisdiction. We return to this crucial point below. For future reference, we write the gains from $\{i,j\}$-linkage accruing to jurisdiction i as

$$\mathbb{E}\{\delta_{\{i,j\},i}\} = PSE_{\{i,j\},i} \times \Big(VE_{\{i,j\}} + DE_{\{i,j\}} \Big), \tag{12}$$

where $PSE_{\{i,j\},i} = \psi_j PSE_{\{i,j\}}/(\psi_i + \psi_j)$ and note that $VE_{\{i,j\}} = VE_{\{i,j\},i} = VE_{\{i,j\},j}$ as well as $DE_{\{i,j\}} = DE_{\{i,j\},i} = DE_{\{i,j\},j}$. In sum, a jurisdiction prefers a link with a relatively larger jurisdiction whose permit demand is volatile and weakly correlated to its own.

Analytical example: Jurisdictions i and j

We use a simple analytical example to illustrate the source of the gains in a bilateral setting. Assume jurisdictional shocks in i and j take on two values only which occur with equal probability. Using the conventional notation for lotteries and given Equation (3), we have $\theta_i = (+\sigma_i, .5; -\sigma_i, .5)$ and $\theta_j = (+\sigma_j, .5; -\sigma_j, .5)$. We will consider two cases.

[10] Even when the jurisdictional shocks are perfectly correlated, differences in shock volatilities can generate gains from linkage. Then, the larger the difference in volatility levels, the larger the gains from the link.

[11] With the alternative interpretation that ψ measures jurisdictional abatement technology level, the jurisdiction with a higher-cost abatement technology gains relatively more from the link.

Case 1: $\boldsymbol{\psi_i = 2\psi_j = 2\psi}$, $\boldsymbol{\sigma_i = \sigma_j = \sigma}$ **with arbitrary** $\boldsymbol{\rho_{ij}}$**.**
In this case, the shock affecting the joint system $\{i, j\}$ satisfies

$$\hat{\Theta}_{\{i,j\}} = \Big(+\sigma, (1+\rho_{ij})/4; \sigma/3, (1-\rho_{ij})/4; -\sigma/3, (1-\rho_{ij})/4; -\sigma, (1+\rho_{ij})/4 \Big). \tag{13}$$

Assume the positive shock $+\sigma$ occurs in i. It also occurs in j with probability $(1+\rho_{ij})/2$, in which case autarky prices are equal and there is no gain from linkage. The negative shock $-\sigma$ occurs in j with probability $(1-\rho_{ij})/2$, in which case $\bar{p}_i - \bar{p}_j = 2\sigma \neq 0$ and there are positive gains from linkage. Note that the linking price settles at $p_{\{i,j\}} = (2\bar{p}_i + \bar{p}_j)/3 = \bar{p} + \sigma/3$ because i is twice as large as j. The case of the negative shock occurring in i is symmetric. Jurisdictional price wedges between autarky and $\{i, j\}$-linkage thus read

$$|\bar{p}_i - p_{\{i,j\}}| = \Big(+2\sigma/3, (1-\rho_{ij})/2; 0, (1+\rho_{ij})/2 \Big), \tag{14a}$$

$$|\bar{p}_j - p_{\{i,j\}}| = \Big(+4\sigma/3, (1-\rho_{ij})/2; 0, (1+\rho_{ij})/2 \Big). \tag{14b}$$

Combining these autarky-link price wedges with our results above for $\delta_{\{i,j\},i}$ and $\delta_{\{i,j\},j}$ imply that j benefits more from $\{i, j\}$-linkage than i does because the expected autarky-link price wedge is wider for j than for i. Intuitively, this is because the linking price settles closer to the autarky price of the larger jurisdiction. We will see below that this result will carry over to multilateral linkage – cf. Proposition 1.

Also note that correlation solely influences the probabilities of realization of possible price wedges, but not their magnitude. All else equal, a link between two negatively-correlated jurisdictions increases the chances of non-nil price wedges as compared to a link between two positively-correlated jurisdictions. In particular, when $\rho_{ij} = 0$, jurisdictional gains from $\{i, j\}$-linkage amount to $\mathbb{E}\{\delta_{\{i,j\},i}\} = 2\psi\sigma^2/(9b_2)$ and $\mathbb{E}\{\delta_{\{i,j\},j}\} = 4\psi\sigma^2/(9b_2)$.

Case 2: $\boldsymbol{\psi_i = \psi_j = \psi}$, $\boldsymbol{\sigma_i = 2\sigma_j = 2\sigma}$ **with arbitrary** $\boldsymbol{\rho_{ij}}$**.**
In this case, the shock affecting the joint system $\{i, j\}$ satisfies

$$\hat{\Theta}_{\{i,j\}} = \Big(+3\sigma/2, (1+\rho_{ij})/4; \sigma/2, (1-\rho_{ij})/4; -\sigma/2, (1-\rho_{ij})/4; -3\sigma/2, (1+\rho_{ij})/4 \Big). \tag{15}$$

Assume the positive shock $+2\sigma$ occurs in i. When the positive shock $+\sigma$ also occurs in j (with probability $(1+\rho_{ij})/2$) a price wedge exists because jurisdictional volatility levels differ

($\bar{p}_i - \bar{p}_j = \sigma$) with linking price $p_{\{i,j\}} = \bar{p} + 3\sigma/2$. When the negative shock $-\sigma$ occurs in j (with probability $(1-\rho_{ij})/2$) the price wedge is wider ($\bar{p}_i - \bar{p}_j = 3\sigma$) with linking price $p_{\{i,j\}} = \bar{p} + \sigma/2$. Again, the case of the negative shock occurring in i is symmetric. For all realizations of shock pairs, the linking price is equidistant from the two autarky prices because jurisdictions have the same size. Jurisdictional price wedges thus coincide

$$|\bar{p}_i - p_{\{i,j\}}| = |\bar{p}_j - p_{\{i,j\}}| = \Big(+ 3\sigma/2, (1-\rho_{ij})/2; +\sigma/2, (1+\rho_{ij})/2 \Big). \tag{16}$$

In turn, this means that expected jurisdictional gains are equal. In other words, two jurisdictions of equal size benefit equally from a higher VE. Therefore, for given aggregate gains from a bilateral link, only relative jurisdictional sizes matter in determining how they are apportioned between jurisdiction. In particular, when $\rho_{ij} = 0$, jurisdictional gains from $\{i,j\}$-linkage amount to $\mathbb{E}\{\delta_{\{i,j\},i}\} = \mathbb{E}\{\delta_{\{i,j\},j}\} = 5\psi\sigma^2/(8b_2)$.

3.3 Linkages in a three-jurisdiction world

Before presenting our results for the general case with n jurisdictions, we highlight a number of issues that arise when one leaves the world of bilateral linkages. We do so by analyzing the possible linkages in a three-jurisdiction world $\mathcal{I} = \{i,j,k\}$. We take jurisdiction i's perspective and compare its options of linking with either jurisdiction j or k graphically. We rule out the possibility of a trilateral linkage for the moment and return to it shortly.

In this setting i may prefer a link with j to a link with k, vice versa, or is indifferent between the two depending on jurisdictional characteristics. Figure 1 plots the different linkage indifference frontiers for i. The axes measure the sizes of j and k relative to the size of i and the different panels correspond to various combinations of volatility and correlation parameters. Note that above (resp. below) the frontier i prefers to link with k (resp. j).[12]

In Figure 1a inter-jurisdictional correlations and volatility levels are equal so that jurisdictions only differ by size. That is, $VE_{\{i,j\}} = VE_{\{i,k\}}$ and $DE_{\{i,j\}} = DE_{\{i,k\}}$ and we single out PSE. Not suprisingly, the indifference frontier coincides with the 45-degree line and the central dot represents the point where jurisdictions are identical. All else equal, this shows that i is better off linking with the larger jurisdiction because when $\psi_j \geq \psi_k$, $PSE_{\{i,j\},i} \geq PSE_{\{i,k\},i}$.

Next, Figure 1b isolates VE by considering that shocks are independent and that k is twice

[12] Given our results in the previous section for bilateral linkages, Appendix C derives the analytical expressions for the indifference frontiers depicted in Figures 1 to 3.

as volatile as i and j. This distorts the 45-degree line in favour of $\{i,k\}$ because $VE_{\{i,k\}} > VE_{\{i,j\}}$. There is a similar distortion of the frontier when we isolate DE. For instance, with equal volatility levels, Figure 1c considers that shocks in i and j are negatively correlated while those in i and k are independent, i.e. $DE_{\{i,j\}} > DE_{\{i,k\}} = 0$. Finally, Figure 1d is indicative of the trade-off between VE and DE. In this case, VE 'dominates' DE because the volatility in k is sufficiently high for $VE_{\{i,k\}} + DE_{\{i,k\}} > VE_{\{i,j\}} + DE_{\{i,j\}}$ to hold.

Next we allow for the formation of the trilateral link in addition to the bilateral links just discussed. We delay the characterization of the full $\mathcal{I}$-linkage equilibrium until the next section and focus here on how jurisdiction i fares under the trilateral link $\mathcal{I}$ as compared to the two bilateral links $\{i,j\}$ and $\{i,k\}$. By an extension of notation and taking the expectation of Equation (9), the gains from $\mathcal{I}$-linkage relative to autarky accruing to i amount to

$$\mathbb{E}\{\delta_{\mathcal{I},i}\} = \frac{\psi_i}{2b_2}\mathbb{E}\{(\theta_i - \hat{\Theta}_{\mathcal{I}})^2\}, \text{ where } \hat{\Theta}_{\mathcal{I}} = \frac{\psi_i\theta_i + \psi_j\theta_j + \psi_k\theta_k}{\psi_i + \psi_j + \psi_k}. \tag{17}$$

Using the definition of size-averaged shocks, this can be rewritten as

$$\mathbb{E}\{\delta_{\mathcal{I},i}\} = \frac{\psi_i(\psi_j + \psi_k)^2}{2b_2(\psi_i + \psi_j + \psi_k)^2}\mathbb{E}\{(\theta_i - \hat{\Theta}_{\{j,k\}})^2\}. \tag{18}$$

Therefore, insofar as i is concerned, the trilateral link $\mathcal{I}$ is equivalent to a bilateral link with the joint system $\{j,k\}$. By this artefact and from a jurisdictional perspective, we can apply our analysis of bilateral linkage to that of trilateral linkages. In other words, we can compare PSE, VE and DE across bilateral and trilateral linkages in a meaningful way. Direct computation of $\mathbb{V}\{\hat{\Theta}_{\{j,k\}}\}$ and $\mathrm{Cov}\{\theta_i; \hat{\Theta}_{\{j,k\}}\}$ then yields

$$\mathbb{E}\{\delta_{\mathcal{I},i}\} = \underbrace{\frac{\psi_i(\psi_j + \psi_k)^2}{2b_2(\psi_i + \psi_j + \psi_k)^2}}_{PSE_{\mathcal{I},i}}\Bigg(\underbrace{\sigma_i^2 + \frac{\psi_j^2\sigma_j^2 + \psi_k^2\sigma_k^2 + 2\rho_{jk}\psi_j\psi_k\sigma_j\sigma_k}{(\psi_j + \psi_k)^2}}_{VE_{\mathcal{I},i}}\underbrace{-2\sigma_i\frac{\rho_{ij}\psi_j\sigma_j + \rho_{ik}\psi_k\sigma_k}{\psi_j + \psi_k}}_{DE_{\mathcal{I},i}}\Bigg) \tag{19}$$

As compared to a bilateral link between two jurisdictions, the first thing to note is that the influence of jurisdictional sizes is no longer limited to PSE because the sizes of j and k now affect VE and DE. In a trilateral link, i is better off if it is negatively correlated with both j and k. This is similar to bilateral linkage in that it increases the contribution of DE to i's gains. Here the novelty is that i prefers j and k to be more positively correlated with one another. This amplifies the demand volatility in the joint system $\{j,k\}$ increasing its VE.

Figure 2 illustrates i's linkage preferences by plotting i's linkage indifference frontiers. The reference case is Figure 2a where the jurisdictions are equally volatile and shocks are independent. The dot represents the point where i is indifferent between the three possible links. All else equal, this shows that i prefers the trilateral link when it is of similar or larger size than the others. Otherwise, it prefers a bilateral link with the largest jurisdiction.

Figure 2b considers two alternative situations where i is twice (resp. half) as volatile as both j and k where the indifference point is B (resp. C). This shows i prefers the trilateral link when it is similarly or more volatile than the others. Otherwise, it prefers a bilateral link with the most volatile jurisdiction. There is an observationally equivalent interpretation of Figure 2b where, relative to the case of independent shocks (A) the indifference point becomes B (resp. C) when j and k are positively (resp. negatively) correlated as this increases (resp. decreases) $VE_{\mathcal{I},i}$.

As compared to Figure 2a, Figure 2c depicts the effects of doubling σ_k. All else equal, this makes $\{i,k\}$ more attractive than $\{i,j\}$ and, interestingly, also reduces the region where $\mathcal{I}$ is preferred. Moreover, in the extreme case where $\sigma_j = 0$, Figure 2d shows that $\{i,j\}$ is never the most preferred link for jurisdiction i in the considered size ranges. Note that i's preference between $\{i,k\}$ and $\mathcal{I}$ does not vary much with j's size – in particular, $\{i,k\}$ will always be the preferred link provided that $\psi_k \geq 2\psi_i$.

Next, we analyze the impact of correlation. In Figure 2e, i and k are positively correlated which shrinks the regions for both $\{i,k\}$ and $\mathcal{I}$ relative to $\{i,j\}$. In Figure 2f, conversely, when i and k are negatively correlated, $\{i,j\}$ becomes less attractive than both $\{i,k\}$ and $\mathcal{I}$.

Finally, it is also informative to characterize j and k's linkage preferences. Figure 3 superimposes the linkage indifference frontiers for the three jurisdictions when they are equally volatile and have independent shocks. First, the dark grey area at the center represents the zone where $\mathcal{I}$-linkage is simultaneously preferred by all three jurisdictions and should thus endogenously emerge. This is the case when heterogeneity in size is not too pronounced. Second, the light grey areas at the top and in the south-west corner represent the zones where i and k prefer $\{i,k\}$-linkage the most, respectively. These zones do not overlap, which means that $\{i,k\}$-linkage cannot form endogenously without transfers. In fact, no bilateral linkage can simultaneously be the most preferred link for the two jurisdictions involved. The next section shows that the non-alignment of jurisdictional linkage preferences generalizes to multilateral linkage as a consequence of superadditivity.

Analytical example (cont.): Enter jurisdiction k

Why does the largest and/or most volatile jurisdiction, say i, prefer the trilateral linkage over bilateral linkages? Because the trilateral-link price is 'less driven' by i's autarky price, i.e. the 'distance' (or variability) between i's autarky price and the linking price is greater. We now illustrate this by extending the analytical example in Section 3.2. Without loss of generality, we also assume all shocks are independent.

Case 1: $\boldsymbol{\psi_i = 2\psi_j = 2\psi_k = 2\psi}$, $\boldsymbol{\sigma_i = \sigma_j = \sigma_k = \sigma}$ **with** $\boldsymbol{\rho_{ij} = \rho_{ik} = \rho_{jk} = 0}$**.**
Assume the positive shock $+\sigma$ occurs in i. Then, positive shocks $+\sigma$ occur in both j and k with probability 1/4 and there is no gain from linkage. Conversely, negative shocks $-\sigma$ occur in both j and k with same probability which drive a wedge in autarky prices $\bar{p}_i - \bar{p}_j = \bar{p}_i - \bar{p}_k = 2\sigma$. By symmetry, and with complementary probability 1/2, opposite shocks occur in j and k and linking price reads $p_{\mathcal{I}} = (2\bar{p}_i + \bar{p}_j + \bar{p}_k)/4 = \bar{p} + \sigma/2$. By symmetry, jurisdictional price wedges between autarky and $\mathcal{I}$-linkage read

$$|\bar{p}_i - p_{\mathcal{I}}| = \Big(\sigma, 1/4; \sigma/2, 1/2; 0, 1/4\Big), \tag{20a}$$

$$|\bar{p}_j - p_{\mathcal{I}}| = |\bar{p}_k - p_{\mathcal{I}}| = \Big(3\sigma/2, 1/4; \sigma, 1/4; \sigma/2, 1/4; 0, 1/4\Big), \tag{20b}$$

and gains from $\mathcal{I}$-linkage are $\mathbb{E}\{\delta_{\mathcal{I},i}\} = 3\psi\sigma^2/(8b_2)$ and $\mathbb{E}\{\delta_{\mathcal{I},j}\} = \mathbb{E}\{\delta_{\mathcal{I},k}\} = 7\psi\sigma^2/(16b_2)$. Comparing with Case 1 in Section 3.2 we see that the large jurisdiction i prefers the trilateral link while the small jurisdictions j and k prefer to form a bilateral link with i.

Case 2: $\boldsymbol{\psi_i = \psi_j = \psi_k = \psi}$, $\boldsymbol{\sigma_i = 2\sigma_j = 2\sigma_k = 2\sigma}$ **with** $\boldsymbol{\rho_{ij} = \rho_{ik} = \rho_{jk} = 0}$**.**
Assume the positive shock $+2\sigma$ occurs in i. Then, positive shocks $+\sigma$ occur in both j and k with probability 1/4 and autarky price wedges are such that $\bar{p}_i - \bar{p}_j = \bar{p}_i - \bar{p}_k = \sigma$. Negative shocks $-\sigma$ occur in both j and k with same probability but higher autarky price wedges $\bar{p}_i - \bar{p}_j = \bar{p}_i - \bar{p}_k = 3\sigma$. By symmetry and with complementary probability 1/2, opposite shocks occur in j and k and linking price reads $p_{\mathcal{I}} = (\bar{p}_i + \bar{p}_j + \bar{p}_k)/3 = \bar{p} + 2\sigma/3$. By symmetry, jurisdictional price wedges between autarky and $\mathcal{I}$-linkage read

$$|\bar{p}_i - p_{\mathcal{I}}| = \Big(2\sigma, 1/4; 4\sigma/3, 1/2; 2\sigma/3, 1/4\Big), \tag{21a}$$

$$|\bar{p}_j - p_{\mathcal{I}}| = |\bar{p}_k - p_{\mathcal{I}}| = \Big(5\sigma/3, 1/4; \sigma, 1/4; \sigma/3, 1/2\Big), \tag{21b}$$

and gains from $\mathcal{I}$-linkage are $\mathbb{E}\{\delta_{\mathcal{I},i}\} = \psi\sigma^2/b_2$ and $\mathbb{E}\{\delta_{\mathcal{I},j}\} = \mathbb{E}\{\delta_{\mathcal{I},k}\} = \psi\sigma^2/(2b_2)$. Comparing with Case 1 in Section 3.2, the most volatile jurisdiction i now prefers the trilateral link while the less volatile jurisdictions j and k prefer to form a bilateral link with i.

In the above cases, while $\mathcal{I}$-linkage increases the autarky-linking price variability for i relative to bilateral linkages, it does just the opposite for j and k. These examples illustrate that jurisdictional linkage preferences are not aligned.

3.4 General theory of multilateral linkages

We now generalize the model to the case of multilateral linkage under uncertainty. To do so, we first introduce the definitions and terminology we require. Then we present our analytical results in the form of four propositions.

3.4.1 Definitions and terminology

Let $\mathbf{C} \doteq \{\mathcal{C} : \mathcal{C} \subseteq \mathcal{I}, \mathcal{C} \neq \varnothing\}$ be the set of non-empty coalitions in $\mathcal{I}$ with generic element $\mathcal{C}$ and cardinality $|\mathbf{C}| = 2^n - 1$. Let also $\mathbf{C}_\star$ denote the set of non-trivial coalitions, i.e. $\mathbf{C}_\star \doteq \{\mathcal{C} : \mathcal{C} \in \mathbf{C}, |\mathcal{C}| \geq 2\}$ with cardinality $|\mathbf{C}_\star| = |\mathbf{C}| - n$, whose generic element $\mathcal{C} \in \mathbf{C}_\star$ we call *linkage coalition*. Denote by $\mathbf{S}$ the set of coalition structures, where a coalition structure corresponds to a partition of $\mathcal{I}$.[13] Formally, $\mathcal{S}$ is a coalition structure i.f.f. $\varnothing \notin \mathcal{S}$, $\bigcup_{\mathcal{C}\in\mathcal{S}} \mathcal{C} = \mathcal{I}$, and $\forall(\mathcal{C}, \mathcal{C}') \in \mathcal{S} \times \mathcal{S}_{-\mathcal{C}}, \mathcal{C} \cap \mathcal{C}' = \varnothing$. For instance, among a group of three jurisdictions $\{i, j, k\}$, there exist five coalition structures, namely

$$\underbrace{\{\{i\},\{j\},\{k\}\}}_{\text{complete autarky}}, \quad \underbrace{\{\{i,j,k\}\}}_{\text{global market}}, \quad \underbrace{\{\{i,j\},\{k\}\},\ \{\{i,k\},\{j\}\}, \text{ and } \{\{j,k\},\{i\}\}}_{\text{3 incomplete linkages}}.$$

The first and second coalition structures are the complete autarky and the global market, respectively. Coalition structures in which there are singletons, i.e. some jurisdictions remain in autarky, are referred to as incomplete linkage, e.g. $\{\{i, k\}, \{j\}\}$. Among a group of four jurisdictions $\{i, j, k, l\}$, richer variation in coalition structures emerges consisting of multiple linkage coalitions, e.g. $\{\{i, j\}, \{k, l\}\}$. Coalition structures in which linkage coalitions coexist are referred to as *polycentric* structures. Note that polycentric structures may also contain singletons and therefore exhibit incomplete linkage.

[13]For the sake of expositional clarity and consistently with the language of cooperative game theory, coalition structures can only comprise disjoint coalitions. This is without loss of generality and our machinery can characterize situations where jurisdictions belong to several coalitions. In other words, this could represent an *indirect linkage* as defined in Jaffe et al. (2009) and Tuerk et al. (2009).

Further, let $\mathbf{S}_i$ denote the set of coalition structures containing exactly $i \in [\![1; n]\!]$ coalitions, whose cardinality is given by the Stirling number of the second kind $\left\{ {n \atop i} \right\}$. The cardinality of $\mathbf{S}$ is thus given by the n^{th} Bell number given n agents, that is

$$|\mathbf{S}| \doteq \sum_{i=1}^{n} |\mathbf{S}_i| = \sum_{i=1}^{n} \left\{ {n \atop i} \right\} = \sum_{i=1}^{n} \frac{1}{i!} \sum_{j=0}^{i} (-1)^{i-j} \binom{i}{j} j^n. \tag{22}$$

Table 1 shows that the difference in the number of possible linkage coalitions and coalition structures grows exponentially with the number of jurisdictions.

The concept of *coarsening*, i.e. a refinement coalition structure, is helpful in comparing different coalition structures. To define it formally, we first introduce and define *unitary linkage* between two disjoint coalitions.

Definition 1. ***(Unitary linkage)*** *Let $\mathcal{S} \in \mathbf{S}$ such that $|\mathcal{S}| \geq 2$. A unitary linkage is a mapping*

$$\begin{cases} \mathbf{S} \longrightarrow \mathbf{S} \\ \mathcal{S} \longmapsto \mathcal{S}' = \{\mathcal{C}' \cup \mathcal{C}''\} \cup \mathcal{S} \backslash \{\mathcal{C}', \mathcal{C}''\}, \end{cases}$$

for some $(\mathcal{C}', \mathcal{C}'') \in \mathcal{S} \times \mathcal{S} \backslash \{\mathcal{C}'\}$. That is, $\mathcal{S}'$ obtains from $\mathcal{S}$ by merging exactly two disjoint coalitions in $\mathcal{S}$ and $|\mathcal{S}| - |\mathcal{S}'| = 1$.

Observe that a bilateral linkage is a unitary linkage between two singletons. Next, we define a structure coarsening as a sequence of unitary linkages.

Definition 2. ***(Coarsening)*** *Let $\mathcal{S}$ and $\mathcal{S}'$ in $\mathbf{S}^2$ such that $|\mathcal{S}| \geq 2$ and $d = |\mathcal{S}| - |\mathcal{S}'| \geq 1$. $\mathcal{S}'$ is coarser than $\mathcal{S}$ if there exists $(\mathcal{S}_i)_{i \in [\![0; d]\!]} \in \mathbf{S}^{d+1}$ such that $\mathcal{S}_0 = \mathcal{S}'$, $\mathcal{S}_d = \mathcal{S}$ and for all $i \in [\![1; d]\!]$, $\mathcal{S}_{i-1}$ obtains from $\mathcal{S}_i$ via unitary linkage. That is, for all $i \in [\![1; d]\!]$, there exist $(\mathcal{C}_i', \mathcal{C}_i'')$ in $\mathcal{S}_i \times \mathcal{S}_i \backslash \{\mathcal{C}_i'\}$ such that $\mathcal{S}_{i-1} = \{\mathcal{C}_i' \cup \mathcal{C}_i''\} \cup \mathcal{S}_i \backslash \{\mathcal{C}_i', \mathcal{C}_i''\}$.*

In this sense, linkage can be interpreted as a refinement of the underlying coalition structure.[14] When a coalition structure $\mathcal{S}'$ obtains from $\mathcal{S}$ through linkage, the set of newly formed linkage coalitions is $\mathcal{S}' \backslash \{\mathcal{S}' \cap \mathcal{S}\}$ and has cardinality $|\mathcal{S}| - |\mathcal{S}'|$ at most. Note also that the number of coalition structures that are strictly finer than $\mathcal{S}$ is $2^{|\mathcal{S}|} - |\mathcal{S}| - 1$.

[14] Contrast this with concentration. Formally, $\mathcal{S}'$ is a concentration of $\mathcal{S}$ if it obtains from $\mathcal{S}$ by moving one jurisdiction at a time from a coalition in $\mathcal{S}$ to another coalition of equal or larger cardinality. The relation 'coarser than' implies 'is a concentration of' while the opposite is not true because concentration allows for a gradual dissolution of coalitions.

3.4.2 Analytical Results

For all $\mathcal{C}$ in $\mathbf{C}_\star$, we call $\mathcal{C}$-linkage the formation of a linked market for permits between all jurisdictions in $\mathcal{C}$. By extension, $\mathcal{I}$-linkage corresponds to the global market. An interior $\mathcal{C}$-linkage equilibrium consists of the $(|\mathcal{C}|+1)$-tuple $(p_\mathcal{C}, (q_{\mathcal{C},i})_{i\in\mathcal{C}})$, where $p_\mathcal{C}$ is the equilibrium price in the linked market and $q_{\mathcal{C},i}$ denotes jurisdiction i's equilibrium emissions level. The equilibrium is fully characterized by the equalization of marginal benefits across partnering jurisdictions (to the $\mathcal{C}$-linkage equilibrium price) and the linked market clearing condition, that is

$$b_1 + \theta_i - \frac{b_2}{\psi_i} q_{\mathcal{C},i} = p_\mathcal{C} \text{ for all } i \text{ in } \mathcal{C}, \text{ and } \sum_{i\in\mathcal{C}} q_{\mathcal{C},i} = \Omega_\mathcal{C}, \tag{23}$$

where $\Psi_\mathcal{C} \doteq \sum_{i\in\mathcal{C}} \psi_i$ and $\Omega_\mathcal{C} \doteq \sum_{i\in\mathcal{C}} \omega_i = A \cdot \Psi_\mathcal{C}$. After rearranging, the $\mathcal{C}$-linkage equilibrium price can be expressed as the size-weighted average of jurisdictional autarky prices, that is

$$p_\mathcal{C} = \bar{p} + \hat{\Theta}_\mathcal{C} = \Psi_\mathcal{C}^{-1} \sum_{i\in\mathcal{C}} \psi_i \bar{p}_i, \text{ with } \hat{\Theta}_\mathcal{C} \doteq \Psi_\mathcal{C}^{-1} \sum_{i\in\mathcal{C}} \psi_i \theta_i. \tag{24}$$

Jurisdictional net demands for permits are proportional to both jurisdictional size and the difference between the jurisdictional autarky price and the prevailing linkage price, e.g. for jurisdiction $i \in \mathcal{C}$

$$q_{\mathcal{C},i} - \omega_i = \psi_i (\bar{p}_i - p_\mathcal{C})/b_2. \tag{25}$$

Individual gains in multilateral linkages

Ex post, jurisdiction i imports permits under $\mathcal{C}$-linkage i.f.f. $\bar{p}_i > p_\mathcal{C}$, i.e. the linking price happens to be lower than its autarky price. All else equal, this is equivalent to an increase in jurisdiction i's effective cap. Relative to autarky, the gains from $\mathcal{C}$-linkage accruing to jurisdiction $i \in \mathcal{C}$ are

$$\delta_{\mathcal{C},i} = \frac{b_2}{2\psi_i} (q_{\mathcal{C},i} - \omega_i)^2 = \frac{\psi_i}{2b_2} (\bar{p}_i - p_\mathcal{C})^2. \tag{26}$$

We summarize the above in the following proposition.

Proposition 1. *Under $\mathcal{C}$-linkage, the expected economic gains of jurisdiction $i \in \mathcal{C}$ are*

$$\mathbb{E}\{\delta_{\mathcal{C},i}\} = \frac{\psi_i}{2b_2} \mathbb{E}\{(\bar{p}_i - p_\mathcal{C})^2\} \geq 0. \tag{27}$$

Proof. Relegated to Appendix A.1. □

Jurisdiction i's expected gains from $\mathcal{C}$-linkage are always non-negative – and positive provided that the i's autarky price differs from the $\mathcal{C}$-linkage price almost surely.[15] That is, every partnering jurisdiction in any multilateral linkage is always at least as well off as compared to autarky. Note that jurisdictional expected gains are proportional to both jurisdictional size and the expectation of the square of the difference in autarky and $\mathcal{C}$-linkage prices.

Price variability

Loosely speaking, the more 'variable' the linking price relative to autarky price, the more a jurisdiction benefits from the link.[16,17] For instance, controlling for size, a jurisdiction will prefer to be part of linkage coalitions in which the linking price happens to be high when its domestic price happens to be low, and vice versa.

Additionally, Equation (24) implies that the linking price is primarily driven by the autarky prices of the larger jurisdictions. Similarly, for jurisdictions of equal sizes, the linking price is in large part determined by those jurisdictions whose permit demand is highly variable. In other words, large and volatile jurisdictions will prefer to link with many jurisdictions in a bid to augment their autarky-link price distances. Conversely, small jurisdictions may prefer to link exclusively with one relatively large jurisdiction, for otherwise the influence of that large jurisdiction on the link outcome is likely to be mitigated.

Importantly, Equation (27) can be decomposed into two non-negative components, namely

$$\mathbb{E}\{\delta_{\mathcal{C},i}\} = \frac{\psi_i}{2b_2}\Big((\mathbb{E}\{\bar{p}_i\} - \mathbb{E}\{p_{\mathcal{C}}\})^2 + \mathbb{V}\{\bar{p}_i - p_{\mathcal{C}}\}\Big). \tag{28}$$

The first component relates to the difference in expected autarky-link prices, i.e. in jurisdictional cap stringencies or ambition levels.[18] Intuitively, the larger this difference, the larger the benefits associated with the equalization of marginal benefits on average. The second component relates to the variance of the difference in autarly-link prices. That is, linking ETSs induces a positive *additional gain* relative to the case without uncertainty as soon as shocks are different across linked jurisdictions. This is the source of the Pareto-improvement due to the absorption of shocks. In this paper, we neutralize the first component of linkage gains by setting expected autarky prices equal across jurisdictions through Equation (4) and only focus on those gains that arise due to uncertainty.

[15] This result is the analog of the expected gains from merging ETSs obtained by Caillaud & Demange (2017). Note also that summing $\delta_{\mathcal{C},i} = b_2\,(q_{\mathcal{C},i} - \omega_i)^2/(2\psi_i)$ over $i \in \mathcal{C}$ would yield the comparative advantage of decentralization w.r.t. centralization for uniformly-mixed pollutants in Yates (2002).

[16] In other economic contexts, Waugh (1944) and Oi (1961) observed that variability could be beneficial.

[17] Formally, the appropriate term is 'distance'. Indeed, if $\mathcal{L}^2 = \{f|\mathbb{E}\{f^2\} < \infty\}$ then $(\mathcal{L}^2, \langle\cdot\rangle)$ is a Hilbert space with inner product $\langle f, g\rangle = \mathbb{E}\{fg\}$ and $(f,g) \mapsto (\langle f-g, f-g\rangle)^{1/2}$ is the distance induced by $\langle\cdot\rangle$.

[18] More details can be found in Appendix B.1 – see Equation (B.5) in particular.

We summarize the properties of the $\mathcal{C}$-linkage price in the following proposition.

Proposition 2. *Under $\mathcal{C}$-linkage, the permit price volatility is bounded from above*

$$\mathbb{V}\left\{p_{\mathcal{C}}\right\} \leq \Psi_{\mathcal{C}}^{-1} \sum_{i \in \mathcal{C}} \psi_i \mathbb{V}\left\{\bar{p}_i\right\}, \text{ with } \mathbb{E}\{p_{\mathcal{C}}\} = \bar{p}.$$

Only when jurisdictional shocks are independent does it hold that $\text{p-lim}_{|\mathcal{C}| \to +\infty} p_{\mathcal{C}} = \bar{p}$. *Linkage always lowers price volatility in higher volatility jurisdictions but may increase it in lower volatility jurisdictions, especially when the latter are relatively small.*

Proof. Relegated to Appendix A.2. □

The first statement places an upper bound on the linking price volatility. Moreover, the inequality holds strictly provided that at least two jurisdictions in $\mathcal{C}$ are not perfectly positively correlated and/or have different volatility levels. This illustrates the shock absorption mechanism associated with linkage and suggests that overall permit price volatility is reduced as a result of the link. The second statement, however, clarifies this point and shows that as the linkage coalition expands, the linking price converges in probability towards its expected value $\bar{p}$ only when jurisdictional shocks are independent. In other words, in the general case, there is no reason that the linking price volatility should gradually diminish and converge to zero as the number of linked jurisdictions increases.

Proposition 2 also clarifies the effects of linking on price volatility from a jurisdictional perspective. Volatile jurisdictions experience reduced price volatility as domestic shocks are spread over a thicker market and thus better cushioned. Conversely, as links create exposure to shocks abroad, stable jurisdictions may face higher price volatility relative to autarky. All else equal, this is more likely in small jurisdictions because the influence of larger jurisdictions on the link outcomes is greater. However, we stress that linkage is always preferred to autarky despite that it might lead to higher price volatility. This is so because jurisdictions that 'import' some volatility as a result of the link are well compensated for doing so.

Bilateral decomposition of multilateral linkages

Jurisdictional expected gains in Equation (27) are in a compact form that provides an intuitive interpretation in terms of autarky-link price distance. However, this does not directly relate to jurisdictional characteristics so we unpack it. Then, as per our definition of emissions caps in Equation (4) the autarky-linking price wedge solely relates to shocks, that is

$$\bar{p}_i - p_{\mathcal{C}} = \theta_i - \hat{\Theta}_{\mathcal{C}}. \tag{29}$$

Plugging Equation (29) into Equation (26) and using the definition of $\hat{\Theta}_{\mathcal{C}}$, we obtain

$$\delta_{\mathcal{C},i} = \frac{\psi_i}{2b_2\Psi_{\mathcal{C}}^2}\left(\sum_{j\in\mathcal{C}_{-i}} \psi_j\left(\theta_i - \theta_j\right)\right)^2. \tag{30}$$

Expanding the above and taking expectations then yields

$$\begin{aligned}\mathbb{E}\{\delta_{\mathcal{C},i}\} = \frac{\psi_i}{2b_2\Psi_{\mathcal{C}}^2}\Bigg(&\sum_{j\in\mathcal{C}_{-i}} \psi_j^2\left(\sigma_i^2 + \sigma_j^2 - 2\rho_{ij}\sigma_i\sigma_j\right)\\ &+ \sum_{(j,k)\in\mathcal{C}_{-i}\times\mathcal{C}_{-i}} \psi_j\psi_k\left(\sigma_i^2 + \rho_{jk}\sigma_j\sigma_k - \rho_{ik}\sigma_i\sigma_k - \rho_{ij}\sigma_i\sigma_j\right)\Bigg).\end{aligned} \tag{31}$$

The above expression, however, is cumbersome and does not lend itself to easy interpretation. We could pursue a similar approach as in Section 3.3 to write $\mathbb{E}\{\delta_{\mathcal{C},i}\}$ as the expected gains from a bilateral link between i and $\mathcal{C}_{-i}$, but the nature of the entity $\mathcal{C}_{-i}$ is already hard to grasp for quadrilateral links. In general, when it comes to a multilateral linkage, it will be more convenient to express the associated quantities as a function of its internal bilateral linkage quantities. By an argument of symmetry and with the convention that for all $i \in \mathcal{I}$, $\Delta_{\{i,i\}} = 0$, Appendix A.3 shows that $\mathcal{C}$-linkage gains accruing to jurisdiction $i \in \mathcal{C}$ write

$$\delta_{\mathcal{C},i} = \Psi_{\mathcal{C}}^{-2}\sum_{j\in\mathcal{C}_{-i}}\left\{\Psi_{\mathcal{C}_{-i}}(\psi_i+\psi_j)\Delta_{\{i,j\}} - \frac{\psi_i}{2}\sum_{k\in\mathcal{C}_{-i}}(\psi_j+\psi_k)\Delta_{\{j,k\}}\right\}. \tag{32}$$

Therefore, jurisdiction i is better off linking with groups of jurisdictions such that on the one hand, the aggregate gains in bilateral links between i and each jurisdiction in these groups are high, and on the other hand, the aggregate gains in bilateral links internal to these groups are low. Then, summing over all $i \in \mathcal{C}$ yields the following result.

Proposition 3. *Any $\mathcal{C}$-linkage can be decomposed into its internal bilateral linkages, that is*

$$\Delta_{\mathcal{C}} \doteq \sum_{i\in\mathcal{C}}\delta_{\mathcal{C},i} = (2\Psi_{\mathcal{C}})^{-1}\sum_{(i,j)\in\mathcal{C}^2}(\psi_i+\psi_j)\Delta_{\{i,j\}}. \tag{33}$$

The number of such internal bilateral links is triangular and equals $\binom{|\mathcal{C}|+1}{2}$.

Proof. Relegated to Appendix A.3. Appendix B.1 shows that Proposition 3 continues to hold for jurisdictional cap profiles that do not satisfy Equation (4). □

In words, the aggregate gain from $\mathcal{C}$-linkage writes as a size-weighted function of all gains from bilateral links between jurisdictions belonging to the linkage coalition $\mathcal{C}$. This shortens equations and provides a convenient way to compute gains associated with large coalitions. However, it is not clear from Equation (33) what are the implications (e.g., in terms of aggregate expected gains) of enlarging a linkage coalition. This is what we analyze next.

Linkage between linkage coalitions and superadditivity

We define the aggregate gains generated by any coalition structure $\mathcal{S}$ in $\mathbf{S}$ by $\Delta_{\mathcal{S}} \doteq \sum_{\mathcal{C}\in\mathcal{S}} \Delta_{\mathcal{C}}$ and adopt the convention that $\Delta_{\mathcal{C}} = 0$ whenever $\mathcal{C} \in \mathbf{C}\backslash\mathbf{C}_\star$. Now let $(\mathcal{C}, \mathcal{C}') \in \mathbf{C}_\star \times \mathbf{C}$ such that $\mathcal{C}' \subset \mathcal{C}$ and denote by $\mathcal{C}''$ the complement of $\mathcal{C}'$ in $\mathcal{C}$, i.e. $\mathcal{C} = \mathcal{C}' \cup \mathcal{C}''$ and $\mathcal{C}' \cap \mathcal{C}'' = \varnothing$. Then, we can express the aggregate gains in $\mathcal{C}$ as a function of those in $\mathcal{C}'$ and $\mathcal{C}''$ by unpacking Equation (33), that is

$$\Delta_{\mathcal{C}} = \Psi_{\mathcal{C}}^{-1}\Bigg(\Psi_{\mathcal{C}'}\Delta_{\mathcal{C}'} + \Psi_{\mathcal{C}''}\Delta_{\mathcal{C}''} + \sum_{(i,j)\in\mathcal{C}'\times\mathcal{C}''}(\psi_i+\psi_j)\Delta_{\{i,j\}}\Bigg). \tag{34}$$

Note that the third term in the parenthesis captures the interaction among jurisdictions in $\mathcal{C}'$ and $\mathcal{C}''$, which is what we want to isolate. To do so, we denote the aggregate gains of linking coalitions $\mathcal{C}'$ and $\mathcal{C}''$ by $\Delta_{\{\mathcal{C}',\mathcal{C}''\}}$ and define them such that

$$\Delta_{\{\mathcal{C}',\mathcal{C}''\}} \doteq \Delta_{\mathcal{C}} - \Delta_{\mathcal{C}'} - \Delta_{\mathcal{C}''}. \tag{35}$$

With this definition, we can establish the following result.

Proposition 4. *Let $\mathcal{S}$ and $\mathcal{S}'$ be as in Definition 2 where $\mathcal{S}'$ is coarser than $\mathcal{S}$ and $d = |\mathcal{S}| - |\mathcal{S}'| \geq 1$. Linkage is a superadditive mechanism, that is*

$$\mathbb{E}\{\Delta_{\mathcal{S}'}\} - \mathbb{E}\{\Delta_{\mathcal{S}}\} = \sum_{i=1}^{d}\Big\{\mathbb{E}\{\Delta_{\mathcal{S}_i}\} - \mathbb{E}\{\Delta_{\mathcal{S}_{i-1}}\}\Big\} = \sum_{i=1}^{d}\mathbb{E}\{\Delta_{\{\mathcal{C}_i',\mathcal{C}_i''\}}\} \geq 0, \tag{36}$$

where in particular, for all $i \in [\![1;d]\!]$,

$$\mathbb{E}\{\Delta_{\{\mathcal{C}_i',\mathcal{C}_i''\}}\} = \Psi_{\{\mathcal{C}_i'\cup\mathcal{C}_i''\}}^{-1}\Bigg(\sum_{(j,k)\in\mathcal{C}_i'\times\mathcal{C}_i''}(\psi_j+\psi_k)\mathbb{E}\{\Delta_{\{j,k\}}\} - \Psi_{\mathcal{C}_i''}\mathbb{E}\{\Delta_{\mathcal{C}_i'}\} - \Psi_{\mathcal{C}_i'}\mathbb{E}\{\Delta_{\mathcal{C}_i''}\}\Bigg) \geq 0. \tag{37}$$

Proof. Relegated to Appendix A.4. Appendix B.1 shows that Proposition 4 continues to hold for jurisdictional cap profiles that do not satisfy Equation (4). $\square$

In words, the aggregate expected gain from the union of disjoint coalitions is no less than the sum of the separate coalitions' aggregate expected gains. The proof for the non-negativity of

$\mathbb{E}\{\Delta_{\{\mathcal{C}_i',\mathcal{C}_i''\}}\}$ in Equation (37) intuitively follows from the definition of bilateral linkage and the fact that it is almost surely mutually beneficial – here generally considered between two coalitions in lieu of two jurisdictional markets (i.e., singletons).

We now illustrate the implications of superadditivity. In particular, because singletons have zero value, linkage also satisfies monotonicity, that is

$$\forall(\mathcal{C},\mathcal{C}') \in \mathbf{C}^2,\ \mathcal{C}' \subseteq \mathcal{C} \Rightarrow \mathbb{E}\{\Delta_{\mathcal{C}'}\} \leq \mathbb{E}\{\Delta_{\mathcal{C}}\}. \tag{38}$$

Therefore, $\mathcal{I}$-linkage is the linkage coalition that is the most advantageous in aggregate expected terms. Superadditivity, in fact, provides the stronger result that linkage satisfies cohesiveness, that is

$$\forall \mathcal{S} \in \mathbf{S},\ \mathbb{E}\{\Delta_{\mathcal{I}}\} \geq \mathbb{E}\{\Delta_{\mathcal{S}}\}. \tag{39}$$

Therefore, $\mathcal{I}$-linkage is the socially optimal linkage coalition structure in that it is conducive to the highest aggregate gross cost savings in meeting the aggregate emissions cap $\Omega_{\mathcal{I}}$.[19] In words, from a global perspective, a single linkage coalition consisting of all jurisdictions linked together outperforms any possible grouping of disjoint linkage coalitions. In addition, superadditivity allows us to generalize the observations made for the three-jurisdiction world in Section 3.3 regarding jurisdictional linkage preferences, absent inter-jurisdictional transfers.

Corollary 1. *Assume inter-jurisdictional transfers away. Then, jurisdictional linkage preferences are not aligned in the sense that*
(i) $\mathcal{I}$-linkage may not be the most preferred linkage coalition for all jurisdictions in $\mathcal{I}$;
(ii) any $\mathcal{C} \in \mathbf{C}_\star \backslash \mathcal{I}$ cannot be the most preferred linkage coalition for all jurisdictions in $\mathcal{C}$.

Proof. Relegated to Appendix A.5. □

Statement (i) can be reformulated as follows: There exists a set of jurisdictional characteristics such that $\mathcal{I}$-linkage is the most preferred link for all jurisdictions. We can conjecture based on the results in Section 3.3 that this set contains jurisdictions which are homogeneous enough in terms of size and volatility. This is far from the case in practice. Therefore, although $\mathcal{I}$-linkage is the most efficient outcome from an aggregate perspective, it is unlikely that it will be the most preferred outcome jurisdictionally speaking. In other words, absent

[19] Formally, cohesiveness requires the aggregate gains from the grand coalition (i.e., $\mathcal{I}$-linkage) to be larger than under no agreement (i.e., complete autarky) or any partial agreement (i.e., incomplete linkage). Superadditivity is a stronger property as it requires that this holds for all intermediary linkage coalition structures as well. We also note that the particular functional forms that are assumed in the IEA literature generally imply cohesivess but not necessarily superadditivity.

inter-jurisdictional transfers, the global market is unlikely to emerge endogenously as some jurisdictions will oppose it and prefer to form smaller linkage coalitions.[20]

Such smaller coalitions can form provided that jurisdictional linkage preferences happen to tally with one another. However, statement (*ii*) indicates that one jurisdiction's most preferred linkage coalition cannot simultaneously be the favourite coalition for every jurisdiction thereof. In a world where monetary transfers can run into significant political-economy obstacles and thereby prove unwieldy, this non-alignment result can in part explain why linkage negotiations fall short of leading to large linkage coalitions in the short run.[21]

Finally, our analysis naturally extends to linkages between more than two linkage coalitions by rewriting Equation (32). That is, for any linkage coalition $\mathcal{C} = \bigcup_i \mathcal{C}_i$ where for all $i \neq j$, $\mathcal{C}_i \cap \mathcal{C}_j = \varnothing$, the gain accruing to coalition $\mathcal{C}_i$ in forming $\mathcal{C}$ reads

$$\Delta_{\mathcal{C},\mathcal{C}_i} = \Psi_{\mathcal{C}}^{-2} \sum_{\mathcal{C}' \in \mathcal{C}_{-\mathcal{C}_i}} \left\{ \Psi_{\mathcal{C}_{-\mathcal{C}_i}} \Big(\Psi_{\mathcal{C}_i} + \Psi_{\mathcal{C}'} \Big) \Delta_{\{\mathcal{C}_i, \mathcal{C}'\}} - \frac{\Psi_{\mathcal{C}_i}}{2} \sum_{\mathcal{C}'' \in \mathcal{C}_{-\mathcal{C}_i}} \Big(\Psi_{\mathcal{C}'} + \Psi_{\mathcal{C}''} \Big) \Delta_{\{\mathcal{C}', \mathcal{C}''\}} \right\}. \tag{40}$$

We deliberately abstain from characterizing how this gain is apportioned within linkage coalitions. As discussed in Appendix B.2, this would require additional assumptions on how gains are shared among the members of an existing coalition, i.e. the degree of consolidation.

4 Quantitative illustration

In this section we illustrate the quantitative implications of our theory using historical emissions data to discipline the selection of model parameters for five real-world jurisdictions: China (CHN), the United States (USA), the block of European countries currently participating in the EUETS (EUR), Korea (KOR) and Egypt (EGY). We assume that there is a hypothetical ETS which covers all carbon emissions in each jurisdiction.

[20] If inter-jurisdictional transfers were feasible, cohesiveness ensures that it would always be possible to find a transfer scheme that satisfies 'grand' coalition rationality, i.e. no subcoalition is better off deviating from the global market. In other words, there exists (at least) one allocation of the gains from the global market that lies in the core of the coalitional game, i.e. the global market can be sustained. Note that, utilizing the solution concept of Partial Agreement Nash Equilibrium, Chander & Tulkens (1995, 1997) prove non-emptiness of the (γ)-core precisely by pointing out a specific stabilizing transfer scheme in the standard coalitional game with transboundary externalities. Helm (2001) generalizes this result by showing that such a game is 'balanced' provided that cohesivess and standard convexity assumptions about the payoff functions hold. Invoking the Bondareva-Shapley theorem, the core is non-empty.

[21] Inter-jurisdictional transfers could stabilize linkage coalitions in the sense of both internal and external stability as defined in Cartel games (D'Aspremont et al., 1983). In fact, all linkage coalitions are 'potentially internally stable' in the sense of Carraro et al. (2006).

Our calibration strategy is similar to Doda & Taschini (2017) and described in more detail in Appendix D. We calibrate jurisdictions' sizes and shock properties using the level and fluctuation of jurisdictionsal historical emissions. The results are reported in Tables 2 and 3. Specifically, Table 2 provides jurisdictions' sizes, where China's size is normalized to 100, and shock volatilities. Table 3 lists the pairwise jurisdictional shock correlations.

We consider only five jurisdictions for clarity of exposition and emphasize that our sample selection is deliberate. One of the jurisdictions, CHN, is very large relative to the rest. Two other jurisdictions are large and approximately of equal size, USA and EUR. The remaining two, KOR and EGY, are relatively small and substantially more volatile than the larger jurisdictions. Finally, EGY is negatively correlated with all other jurisdictions to varying degrees. To a large extent, our sample spans the diversity present in the data.

4.1 Linkage with several real-world jurisdictions

We adopt a combinatorial approach to evaluating the gains from, or equivalently the *value* of, every possible linkage arrangement to illustrate the mechanisms that govern multilateral linkage. At this level of abstraction, value is measured in arbitrary units but its magnitude is comparable across jurisdictions, linkage coalitions, and linkage coalition structures. This ensures that we can compare multilateral linkages in a consistent way and thus, that we can characterize jurisdictional linkage preferences.

Our theoretical results indicate that the global market always generates the largest aggregate value. However, by Corollary 1, it may not be the most preferred link for each jurisdiction. Indeed, in our sample the global market is not each jurisdiction's best option. This is shown in Table 4 which illustrates the jurisdictions' linkage preferences by listing their most and second most preferred linkage coalitions.

In particular, Table 4 shows that size is a key factor determining the most preferred coalition. The size of CHN is so dominant that only when all others join CHN in a global market, the value CHN receives is the largest. This is in line with Doda & Taschini (2017) who show that a jurisdiction prefers a larger partner, all else equal. For the remaining jurisdictions, however, a bilateral link with CHN is preferred to all other coalitions. This is true despite the fact that adding other jurisdictions to the bilateral link with CHN increases the overall size of the market. This demonstrates why it may be misleading to apply the results and intuition derived in a bilateral setting to a multilateral context.

Although Table 4 reveals much about the complex interactions that determine jurisdictional linkage preferences, this is only part of a much richer story. To illustrate this, Figure 4 provides more detailed information by illustrating the gains from every linkage coalition for CHN, USA and EGY. The gains CHN derives from being a member of a linkage coalition is increasing in the total size of the remaining members of the coalition of a given cardinality, which is intuitive. Moreover, CHN always prefers to expand the present linkage coalition by including the largest partner among those available.

For USA and EGY, the effects of coalitional cardinality are more subtle. First, for these jurisdictions the global market is far from being a desirable coalition. Second, starting from their most preferred coalition, a bilateral link with CHN, entry by a new jurisdiction in the coalition, say at the insistence of CHN, implies marginal losses which are increasing in the new member's size. Third and conversely, starting from their second most preferred bilateral coalition under the assumption that CHN is not available to link, entry by a new jurisdiction can actually improve the gains USA and EGY obtain. Fourth, the gain improvements highlighted in the previous sentence, which is realized when EGY joins the USA-EUR link and KOR joins the EGY-USA link respectively, do not require the entry of the largest jurisdiction that is available to link. Taken together these four observations reinforce the message of Table 4 that the general model of multilateral linking presented above is essential for ranking alternative coalitions from a jurisdiction's perspective.

Finally, we highlight the existence of preference clusters in Figure 4. There are three such clusters for CHN. An upper cluster where it partners with the two other large jurisdictions (USA and EUR), a middle cluster where CHN is linked with either USA or EUR but not both, and a lower cluster where CHN is only linked to small jurisdictions (KOR and EGY). In each of these clusters, CHN prefers linkage coalitions with the largest members available. For USA, two such clusters exist. An upper cluster where CHN is in the linkage coalition and a lower cluster where it is not. Note again the effect of size: in the lower cluster where USA is the largest jurisdiction, it prefers links with the largest jurisdictions available while in the upper cluster where it is not the largest jurisdiction, it prefers links with the smallest jurisdictions available. For EGY, the smallest jurisdiction in our sample, such clusters are more circumscribed. We observe that the dispersion of jurisdictional gains across linkage coalitions is modest, suggesting that a clear ranking of links for small jurisdictions can be more arduous. In this case, a combination of sizes and shock characteristics govern the ranking of the possible multilateral linkages, which illustrates that without a theory it is difficult to determine their net effect.

4.2 Policy application: multilateral linkage with costs

Linkage coalitions that generate high gains involve large partners or many partners, or both. In practice, however, these could be relatively costly to form. In particular, in the presence of costs associated with the formation of linkage coalitions, hereafter linkage costs, it might well be that a coalition structure different from the global market yields the highest aggregate payoff, net of costs. In this section we look at two aspects of this question. First, given such linkage costs, we explore the nature of globally efficient coalition structures, or GECS for short. Second, we compare the ability of various inter-jurisdictional cost-sharing arrangements in making GECS Pareto-improving with respect to autarky.[22]

We model the linkage costs as having two distinct variable components: (*i*) a linkage implementation part capturing that the larger the potential jurisdictions involved, the larger are the implementation-related administrative costs, e.g. the costs of harmonizing the rules of the previously independent systems; and (*ii*) a linkage negotiation part reflecting that costs in forging and establishing climate policy linkage agreements are increasing in the number of participating jurisdictions. Fixed per-link sunk costs are not considered as they are blind to both the composition of coalitions and the architecture of coalition structures, thereby unable to discriminate between them. These considerations are captured by the following cost structure:

$$\kappa(\mathcal{C};\varepsilon_0,\varepsilon_1) \doteq \varepsilon_0 \cdot \Psi_{\mathcal{C}} + \varepsilon_1 \cdot |\mathcal{C}|^2, \text{ for all } \mathcal{C} \in \mathbf{C}_\star, \tag{41}$$

where $(\varepsilon_0, \varepsilon_1) \in \mathbb{R}^2_+$ are scaling parameters for the implementation and negotiation costs, respectively. Given these costs, we define the efficient structure as follows.

Definition 3. ***(Globally efficient coalition structure, GECS)*** *Given cost parameters $(\varepsilon_0, \varepsilon_1) \in \mathbb{R}^2_+$ net aggregate economic gains from any $\mathcal{S} \in \mathbf{S}$ write*

$$\tilde{\Delta}_{\mathcal{S}}(\varepsilon_0,\varepsilon_1) \doteq \Delta_{\mathcal{S}} - \sum_{\mathcal{C}\in\mathcal{S}} \kappa(\mathcal{C};\varepsilon_0,\varepsilon_1). \tag{42}$$

Then GECS, denoted $\mathcal{S}^*$, is unique and satisfies

$$\mathcal{S}^*(\varepsilon_0,\varepsilon_1) \doteq \arg\max_{\mathcal{S}\in\mathbf{S}} \left\langle \mathbb{E}\{\tilde{\Delta}_{\mathcal{S}}(\varepsilon_0,\varepsilon_1)\}\right\rangle. \tag{43}$$

Although our definition of linkage costs in Equation (41) is exogenously imposed, we emphasize that costs associated with the formation of coalition structures are endogenous to

[22] This analysis could also be applied to any other 'desirable' coalition structures.

the optimization program in Equation (43). On the one hand, for a pair of cost parameters low enough, linkage may remain superadditive and GECS correspond to the global market. In particular, $\mathcal{S}^*(0,0) = \mathcal{I}$. On the other hand, for a pair of cost parameters high enough, linkage may become subadditive and GECS correspond to complete autarky. For cost parameters such that linkage is neither superadditive nor subadditive, below we numerically explore the nature of GECS in terms of polycentricity and incompleteness of linkage. To that end, we first introduce alternative inter-jurisdictional cost-sharing arrangements and adopt a criterion to discriminate between alternative arrangements.

Definition 4. ***(Cost-sharing arrangements)*** *Given $\mathcal{C} \in \mathbf{C}_\star$, a cost-sharing arrangement is a collection of non-negative weights $(\phi_{\mathcal{C},i})_{i\in\mathcal{C}}$ such that $\sum_{i\in\mathcal{C}} \phi_{\mathcal{C},i} = 1$ where $\phi_{\mathcal{C},i}$ is the share of the aggregate cost of forming coalition $\mathcal{C}$ incurred by jurisdiction $i \in \mathcal{C}$.*

For any cost-sharing arrangement, net gains from forming any $\mathcal{C}$ in $\mathbf{C}_\star$ accruing to i in $\mathcal{C}$ thus write

$$\tilde{\delta}_{\mathcal{C},i}(\varepsilon_0, \varepsilon_1) \doteq \delta_{\mathcal{C},i} - \phi_{\mathcal{C},i} \cdot \kappa(\mathcal{C}; \varepsilon_0, \varepsilon_1). \tag{44}$$

We consider seven cost-sharing arrangements that are listed and described in Table 5. These rules apportion costs equally, based on size, on cost type, on the gains that a jurisdiction obtains, etc. They are not meant to be exhaustive but simply illustrative. Notice that cost-sharing arrangements can be assimilated to inter-jurisdictional transfer schemes.

We adopt a weak notion of incentive-compatibility to discriminate between alternative outcomes and require that jurisdictions be at least as well off as under autarky, i.e. jurisdictional expected gains net of linkage costs must be non-negative. Formally, for given $(\varepsilon_0, \varepsilon_1) \in \mathbb{R}^2_+$, GECS is said to be Pareto-improving with respect to autarky if it holds that, for all $\mathcal{C}$ in $\mathcal{S}^*(\varepsilon_0, \varepsilon_1)$ and all i in $\mathcal{C}$

$$\mathbb{E}\{\tilde{\delta}_{\mathcal{C},i}(\varepsilon_0, \varepsilon_1)\} \geq 0. \tag{45}$$

Next we compare the seven cost-sharing arrangements in their ability to implement GECS, that is to make GECS Pareto-improving w.r.t. autarky.

Introducing linkage costs requires us to parametrize the cost function in Equation (41). This is difficult even at this level of abstraction because there is very little empirical guidance to select the pair $(\varepsilon_0, \varepsilon_1)$. To discipline the parametrization, we report three sets of results which are comparable in the sense that the most costly coalition structure, i.e. that where jurisdictions negotiate a global market, generates costs equal to 75% of the gross benefits it delivers. Note that this does not identify $(\varepsilon_0, \varepsilon_1)$ individually. To pin down a unique pair, we further assume that a share z of the linkage costs are implementation costs, and report results for $z \in \{0, 0.5, 1\}$. In particular, the aggregate gross gain from the global market is 0.0473 with coalition cardinality 5 and aggregate size 202.738. With $z = 1$ there are only implementation costs and $(\varepsilon_0, \varepsilon_1)$ is uniquely determined $(1.75 \cdot 10^{-4}, 0)$. Conversely, with $z = 0$, there are only negotiation costs and we have the parameter pair $(0, 1.42 \cdot 10^{-3})$. Finally, with $z = 0.5$ the parameter pair is $(8.75 \cdot 10^{-5}, 7.01 \cdot 10^{-4})$.

Table 6 presents the results of combining these cost assumptions with the seven cost-sharing arrangements. In particular, we report GECS ($\mathcal{S}^*$) and the associated expected aggregate net gains $\mathbb{E}\{\tilde{\Delta}_{\mathcal{S}^*}\}$. We say that $\mathcal{S}^*$ is blocked by a jurisdiction i under a rule $R\#$, if i receives negative net benefits, i.e. it is worse off under $\mathcal{S}^*$ than autarky. Table 6 also reports the blocking jurisdictions, if any, under a given rule.

First, when only linkage negotiation costs are involved ($z = 0$), GECS corresponds to a linked system among the three largest jurisdictions on the one hand, and another system consisting of the two smallest jurisdictions on the other, i.e. GECS is a polycentric complete linkage. If we were to increase cost parameters further, a GECS where some jurisdictions remain in autarky, i.e. an incomplete linkage, would emerge. However, only cost-sharing rules $R3$ and $R5$ are consistent with no jurisdiction blocking this efficient structure. If any other rule were adopted ex ante, KOR would block its linkage coalition with EGY thereby precluding the implementation of GECS. Thus, cost-sharing rules are critical for whether GECS constitutes a Pareto-improvement w.r.t. autarky, i.e. whether GECS is implementable or not.

Second, we observe that when $z = 0.5$, GECS is unchanged. Although the net gains from GECS are half those that obtain with $z = 0$, GECS is now feasible under cost-sharing rules $R1$, $R4$, $R5$ and $R7$. This shows that high aggregate gains from GECS do not necessarily make incentive-compatibility easier to achieve. Additionally, KOR is no longer the sole blocking jurisdiction, as EUR might also oppose GECS under certain cost-sharing rules.

Third, when only linkage implementation costs are involved ($z = 1$), GECS corresponds to the global market. However, it is only achievable under $R5$ and all jurisdictions but EUR may block it depending on the rule considered. Although GECS with $z = 1$ corresponds to the global market, it brings about half the aggregate gains as GECS with $z = 0$, which differs from the global market. In addition, it seems less likely to obtain because there are more potential blocking jurisdictions and fewer cost-sharing rules that can implement it.

The above suggests that polycentric GECSs are likely when the share of negotiation costs is larger, while the level of implementation costs determines whether system integration is complete or incomplete. Among the cost specifications and cost-sharing rules considered in Table 6, $R5$ always renders GECS viable. That is, splitting linkage costs in proportion to jurisdictional linkage gains might facilitate the implementation of efficient structures. In practice, however, the determination of the associated cost shares might not be as straightforward as it is for simpler rules, e.g. egalitarian or per size.

5 Conclusion

Linkages between ETSs will have an important role to play in the successful, cost-effective implementation of the Paris Agreement. While the theory of bilateral linkages is well established, we know relatively little about multilaterally linked systems. In this paper we advance the frontier of research on this topic by proposing a general theory to describe and analyze multilateral linkages between ETSs. In our theory, the magnitude of individual gains in linkages and the variance of prices in linkage equilibria are well defined objects and we study their analytical properties. In particular, we provide a formula for the gains from trade in multilaterally linked ETSs as a function of coalitional sizes and shock characteristics. Importantly, we decompose any multilateral linkage into its internal bilateral linkages. We use this decomposition to characterize aggregate and jurisdictional gains from any linkage coalition as a size-weighted function of aggregate gains from all bilateral links that can be formed among its constituents. Finally, this decomposition result enables us to analytically characterize the aggregate expected gains from the union of disjoint linkage coalitions, which we show to be no less than the sum of separate coalitions' expected gains, i.e. linkage is superadditive.

A direct consequence of superadditivity is that the global market is the efficient coalition structure from a social perspective. Absent inter-jurisdictional transfer agreements, however, the global market is not necessarily the most preferred linkage coalition when viewed from the perspective of a single jurisdiction. Therefore, even without linkage costs it is not a forgone conclusion that the globally linked market will endogenously emerge. In a world where inter-jurisdictional transfers are politically unpalatable, this may be an important reason why multilateral linkages are uncommon.

A quantitative illustration loosely calibrated to five real-world jurisdictions provides evidence on the potential practical relevance of our theoretical findings. For example, the global market is *not* the most preferred coalition of every jurisdiction in our numerical analysis. When we additionally introduce reasonably parametrized linkage costs, coalition structures different from the global market may be efficient but not necessarily implementable depending on how linkage costs are shared. This is clearly an area where additional academic and policy work would be useful.

Acknowledgments

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Tables

Table 1: Number of linkage coalitions and coalition structures

Number of jurisdictions	3	4	5	10	15
Number of linkage coalitions	4	11	26	1,013	32,752
Number of coalition structures	5	15	52	115,975	1,382,958,545

Table 2: Calibration results: Size and volatility (ψ_i and σ_i)

	CHN	USA	EUR	KOR	EGY
ψ_i	100	55.038	38.699	6.645	2.356
σ_i	0.028	0.019	0.017	0.034	0.050

Table 3: Calibration results: Pairwise correlation coefficients (ρ_{ij})

	CHN	USA	EUR	KOR	EGY
CHN	1.000				
USA	0.525	1.000			
EUR	0.460	0.652	1.000		
KOR	0.247	0.419	0.277	1.000	
EGY	-0.395	-0.186	-0.101	-0.397	1.000

Table 4: Jurisdictional rankings of linkage coalitions

	Most preferred coalition	Second most preferred coalition
CHN	{CHN,USA,EUR,KOR,EGY}	{CHN,USA,EUR,KOR}
USA	{CHN,USA}	{CHN,USA,EGY}
EUR	{CHN,EUR}	{CHN,EUR,KOR,EGY}
KOR	{CHN,KOR}	{CHN,KOR,EGY}
EGY	{CHN,EGY}	{CHN,KOR,EGY}

Table 5: Alternative inter-jurisdictional cost-sharing arrangements

Rule number	Share of total costs incurred by jurisdiction i ($\phi_{\mathcal{C},i}$)†
$R1$	$\lvert\mathcal{C}\rvert^{-1}$
$R2$	$\psi_i \cdot \Psi_{\mathcal{C}}^{-1}$
$R3$	$\psi_i^{-1} \cdot \left(\sum_{i \in \mathcal{C}} \psi_i^{-1} \right)^{-1}$
$R4$	$\varepsilon_0 \cdot \psi_i + \varepsilon_1 \cdot \lvert\mathcal{C}\rvert$
$R5$	$\mathbb{E}\{\delta_{\mathcal{C},i}\} \cdot \mathbb{E}\{\Delta_{\mathcal{C}}\}^{-1}$
$R6$	$\Big(\mathbb{E}\{\Delta_{\mathcal{C}}\} - \mathbb{E}\{\Delta_{\mathcal{C}_{-i}}\}\Big)^{-1} \cdot \Big(\sum_{j \in \mathcal{C}} \Big(\mathbb{E}\{\Delta_{\mathcal{C}}\} - \mathbb{E}\{\Delta_{\mathcal{C}_{-j}}\}\Big)^{-1} \Big)^{-1}$
$R7$	$\Big(\mathbb{E}\{\Delta_{\mathcal{C}}\} - \mathbb{E}\{\Delta_{\mathcal{C}_{-i}}\}\Big) \cdot \Big(\lvert\mathcal{C}\rvert \cdot \mathbb{E}\{\Delta_{\mathcal{C}}\} - \sum_{j \in \mathcal{C}} \mathbb{E}\{\Delta_{\mathcal{C}_{-j}}\}\Big)^{-1}$

Note: †: except for $R4$ where the total linkage costs incurring to jurisdiction i are indicated.

Legend: $R1$ is an egalitarian rule: all jurisdictions incur the same costs. With $R2$ and $R3$ linkage costs are shared in proportion to size or inverse of size, respectively. $R4$ is a mixed rule where implementation costs are shared in proportion to size and negotiation costs are evenly shared among jurisdictions. Under $R5$ jurisdictions incur costs in proportion to what they gain from the coalition. $R6$ (resp. $R7$) considers that the more desirable one jurisdiction is, the less (resp. more) it contributes to linkage cost payment.

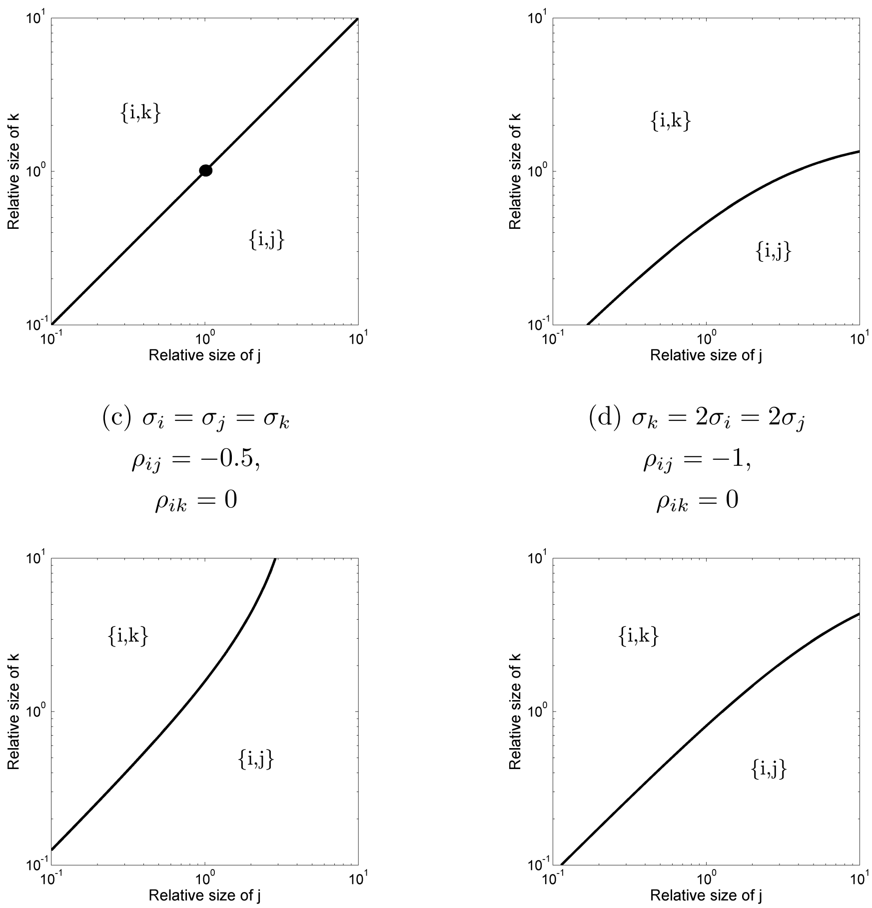
Table 6: Multilateral linkage with costs and alternative cost-sharing rules ($x = 0.75$)

	$\mathcal{S}^*$	$\mathbb{E}\{\tilde{\Delta}_{\mathcal{S}^*}\}$	Set of blocking jurisdiction under R#
$z = 0$	{{CHN,USA,EUR},{KOR,EGY}}	0.0221	$R3$ and $R5$: $\emptyset$ $R1$, $R2$, $R4$, $R6$ and $R7$: {KOR}
$z = 0.5$	{{CHN,USA,EUR},{KOR,EGY}}	0.0137	$R1$, $R4$, $R5$ and $R7$: $\emptyset$ $R2$: {KOR}; $R3$: {EUR} $R6$: {EUR,KOR}
$z = 1$	{{CHN,USA,EUR,KOR,EGY}}	0.0118	$R5$: $\emptyset$; $R1$: {KOR}; $R7$: {CHN} $R2$ and $R4$: {CHN,USA} $R3$ and $R6$: {KOR,EGY}

Note: Cost parameters are set such that linkage costs (i) amount to a share x of the aggregate gains from the global market; (ii) are composed of a share z (resp. $1 - z$) of implementation (resp. negotiation) costs.

Figures

Figure 1: Bilateral linkage preferences for jurisdiction i



(a) $\sigma_i = \sigma_j = \sigma_k$
$\rho_{ij} = \rho_{ik}$

(b) $\sigma_k = 2\sigma_i = 2\sigma_j$
$\rho_{ij} = \rho_{ik} = 0$

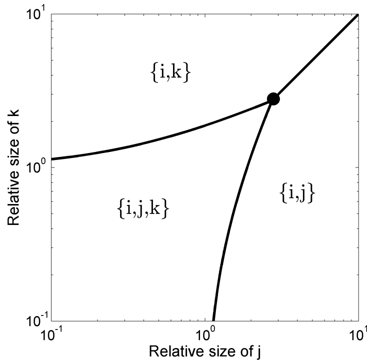
(c) $\sigma_i = \sigma_j = \sigma_k$
$\rho_{ij} = -0.5,$
$\rho_{ik} = 0$

(d) $\sigma_k = 2\sigma_i = 2\sigma_j$
$\rho_{ij} = -1,$
$\rho_{ik} = 0$

Figure 2: Bi- and trilateral linkage preferences for jurisdiction i

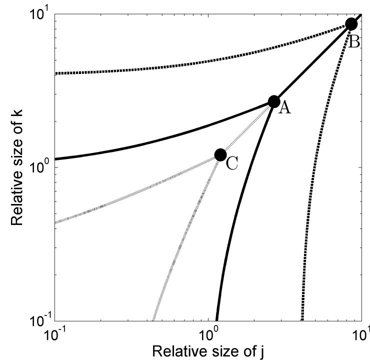
(a) $\sigma_i = \sigma_j = \sigma_k$

$\rho_{ij} = \rho_{ik} = \rho_{jk} = 0$



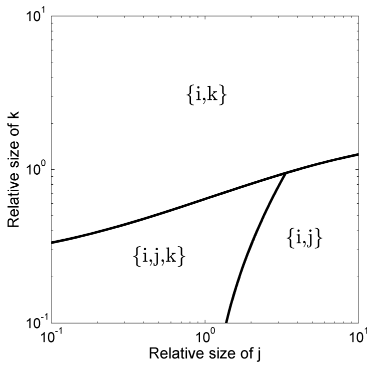
(b) B: $\sigma_i = 2\sigma_j = 2\sigma_k$

C: $2\sigma_i = \sigma_j = \sigma_k$



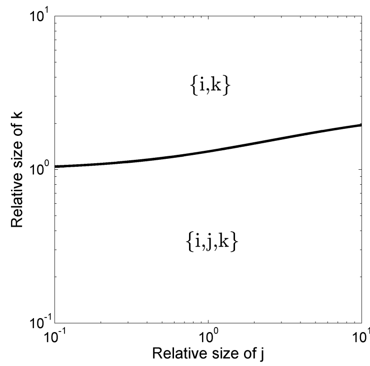
(c) $\sigma_k = 2\sigma_i = 2\sigma_j$

$\rho_{ij} = \rho_{ik} = \rho_{jk} = 0$



(d) $\sigma_i = \sigma_k$, $\sigma_j = 0$

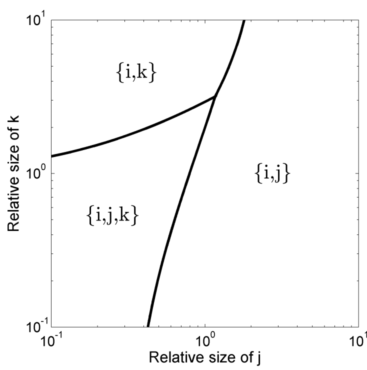
$\rho_{ij} = \rho_{ik} = \rho_{jk} = 0$



(e) $\sigma_i = \sigma_j = \sigma_k$,

$\rho_{ij} = \rho_{jk} = 0$

$\rho_{ik} = .5$



(f) $\sigma_i = \sigma_j = \sigma_k$,

$\rho_{ij} = \rho_{jk} = 0$

$\rho_{ik} = -.5$

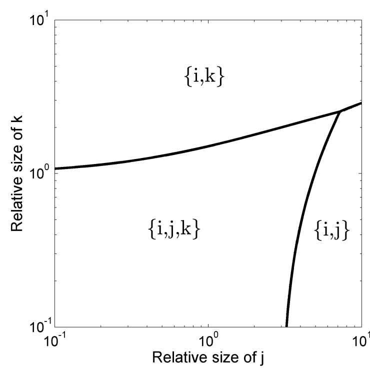


Figure 3: Linkage preferences for the three jurisdictions

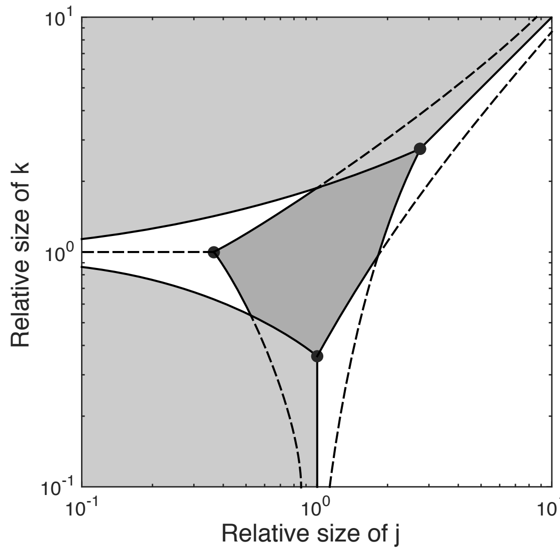
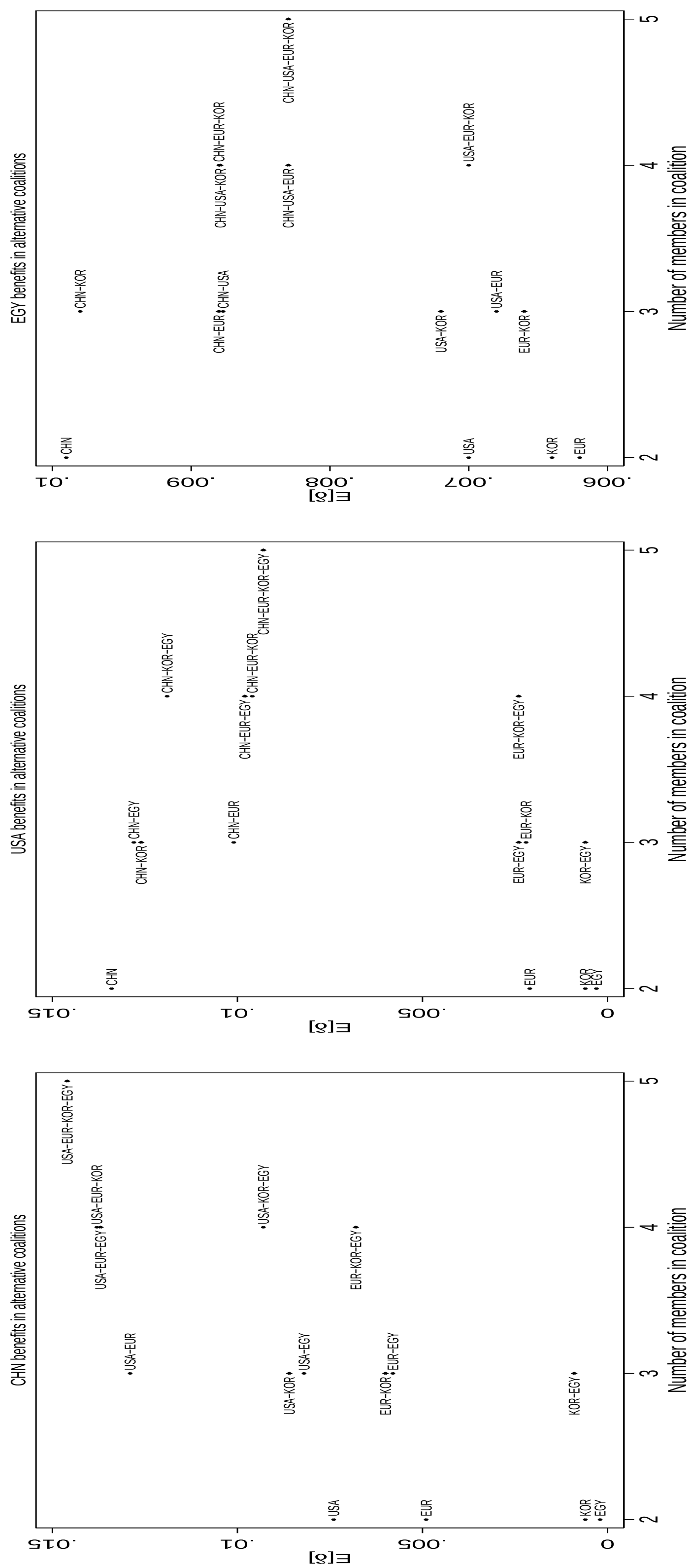


Figure 4: Jurisdictional preferences in terms of linkage coalition



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Appendices & Supplemental Material

A Collected proofs

Throughout Appendix A we fix $\mathcal{C} \in \mathbf{C}_\star$ and we assume without loss of generality that $\mathcal{C} = \{1, 2, \ldots, m\}$ with some $m \in [\![3; n]\!]$.

A.1 Proof of Proposition 1 (compact form of linkage gains)

Recalling that aggregate emissions are constant and do not vary with the linkage coalition structure, the gross economic gains of jurisdiction $i \in \mathcal{C}$ correspond to the difference between its benefits under $\mathcal{C}$-linkage and autarky, respectively, that is

$$\begin{aligned}
\delta_{\mathcal{C},i} &= (b_1 + \theta_i - p_{\mathcal{C}})(q_{\mathcal{C},i} - \omega_i) - \frac{b_2}{2\psi_i}(q_{\mathcal{C},i}^2 - \omega_i^2) \\
&= \frac{b_2}{\psi_i} q_{\mathcal{C},i}(q_{\mathcal{C},i} - \omega_i) - \frac{b_2}{2\psi_i}(q_{\mathcal{C},i}^2 - \omega_i^2) \\
&= \frac{b_2}{2\psi_i}(q_{\mathcal{C},i} - \omega_i)^2 = \frac{\psi_i}{2b_2}(\bar{p}_i - p_{\mathcal{C}})^2,
\end{aligned} \tag{A.1}$$

where the second and fourth equalities obtain via the necessary first-order condition in Equation (23) and the expression for the net demand for permits in Equation (25), respectively. This delivers Equation (26) and taking expectations proves Proposition 1.

A.2 Proof of Proposition 2 (linking price properties)

Since the variance is a symmetric bilinear form, it jointly holds that

$$\mathbb{V}\{p_{\mathcal{C}}\} = \mathbb{V}\{\hat{\Theta}_{\mathcal{C}}\} = \Psi_{\mathcal{C}}^{-2}\left(\sum_{i=1}^{m} \psi_i^2 \sigma_i^2 + 2 \sum_{1 \le i < j \le m} \psi_i \psi_j \rho_{ij} \sigma_i \sigma_j\right), \text{ and} \tag{A.2a}$$

$$\Psi_{\mathcal{C}} \sum_{j=1}^{m} \psi_j \mathbb{V}\{\bar{p}_j\} = \sum_{i=1}^{m}\sum_{j=1}^{m} \psi_i \psi_j \sigma_j^2 = \sum_{i=1}^{m} \psi_i^2 \sigma_i^2 + \sum_{1 \le i < j \le m} \psi_i \psi_j (\sigma_i^2 + \sigma_j^2). \tag{A.2b}$$

Then, $\mathbb{V}\{p_{\mathcal{C}}\} \le \Psi_{\mathcal{C}}^{-1} \sum_{i \in \mathcal{C}} \psi_i \mathbb{V}\{\bar{p}_i\}$ follows since $\sigma_i^2 + \sigma_j^2 \ge 2\rho_{ij}\sigma_i\sigma_j$. The inequality holds strictly when $\exists (i,j) \in \mathcal{C}^2$ such that $\rho_{ij} < 1$ and/or $\sigma_i \neq \sigma_j$.

Next, it is sufficient to verify the statement on jurisdictional price variability as a result of linkage for bilateral links; the argument naturally extends to multilateral links. Then, by applying Equation (A.2a) to $\{i,j\}$-linkage it holds that

$$\mathbb{V}\{p_{\{i,j\}}\} = (\psi_i + \psi_j)^{-2}\Big(\psi_i^2\mathbb{V}\{\bar{p}_i\} + \psi_j^2\mathbb{V}\{\bar{p}_j\} + 2\rho_{ij}\psi_i\psi_j(\mathbb{V}\{\bar{p}_i\}\mathbb{V}\{\bar{p}_j\})^{1/2}\Big). \tag{A.3}$$

Assume w.l.o.g. that jurisdiction i is the less volatile jurisdiction, i.e. $\sigma_j \geq \sigma_i$. Then, $\{i,j\}$-linkage reduces price volatility in the high-volatility jurisdiction i.f.f. $\mathbb{V}\{\bar{p}_j\} \geq \mathbb{V}\{p_{\{i,j\}}\}$, that is i.f.f.

$$\psi_i(\sigma_j^2 - \sigma_i^2) + 2\psi_j\sigma_j(\sigma_j - \rho_{ij}\sigma_i) \geq 0, \tag{A.4}$$

and unconditionally holds, i.e. for all ψ_i, ψ_j, $\sigma_j \geq \sigma_i$ and $\rho_{ij} \in [-1;1]$. For the low-volatility jurisdiction, however, $\mathbb{V}\{\bar{p}_i\} \geq \mathbb{V}\{p_{\{i,j\}}\}$ holds if and only if

$$\psi_j(\sigma_i^2 - \sigma_j^2) + 2\psi_i\sigma_i(\sigma_i - \rho_{ij}\sigma_j) \geq 0 \;\Leftrightarrow\; \frac{\psi_j}{\psi_i} \leq \frac{2\sigma_i(\sigma_i - \rho_{ij}\sigma_j)}{\sigma_j^2 - \sigma_i^2}. \tag{A.5}$$

For a given triple $(\sigma_i, \sigma_j, \rho_{ij})$, $\{i,j\}$-linkage effectively reduces volatility in the low-volatility jurisdiction provided that the high-volatility jurisdiction is not too large in comparison.

Finally, we establish the statement on price convergence in probability. Assume $\mathcal{C}$ is ordered such that $\psi_1 \leq \cdots \leq \psi_m$, let $\bar{\sigma} = \max_{i\in\mathcal{C}} \sigma_i$ and fix $\varepsilon > 0$. Then, it holds that

$$\begin{aligned}
\mathbb{P}\Big(|\hat{\Theta}_{\mathcal{C}} - \mathbb{E}\{\hat{\Theta}_{\mathcal{C}}\}| > \varepsilon\Big) &\leq \varepsilon^{-2}\mathbb{E}\Big\{(\hat{\Theta}_{\mathcal{C}} - \mathbb{E}\{\hat{\Theta}_{\mathcal{C}}\})^2\Big\} = \varepsilon^{-2}\mathbb{V}\{\hat{\Theta}_{\mathcal{C}}\} \\
&= \varepsilon^{-2}\psi_{\mathcal{C}}^{-2}\sum_{i=1}^{m}\left\{\psi_i^2\sigma_i^2 + \sum_{j=1}^{m}\rho_{ij}\psi_i\psi_i\sigma_i\sigma_j\right\} \\
&\leq \left(\frac{\psi_m\bar{\sigma}}{\psi_1\varepsilon}\right)^2\left[\frac{1}{m} + 1\right],
\end{aligned} \tag{A.6}$$

where the first inequality is Chebyshev's inequality and the second follows by construction. Since ψ_m and $\bar{\sigma}$ are finite, only when the second term in the above bracket is nil (i.e., shocks are independent) we have that $p_{\mathcal{C}}$ converges in probability towards $\bar{p}$ as $|\mathcal{C}|$ tends to infinity, that is $\lim_{m\to+\infty}\mathbb{P}\Big(|\hat{\Theta}_{\mathcal{C}} - \mathbb{E}\{\hat{\Theta}_{\mathcal{C}}\}| > \varepsilon\Big) = 0$, i.e. $\lim_{m\to+\infty}\mathbb{P}\Big(|\hat{\Theta}_{\mathcal{C}} - \mathbb{E}\{\hat{\Theta}_{\mathcal{C}}\}| \leq \varepsilon\Big) = 1$.

A.3 Proof of Proposition 3 (bilateral decomposition)

We first establish Equation (32). Note that by plugging $\bar{p}_i - p_{\mathcal{C}} = \theta_i - \hat{\Theta}_{\mathcal{C}}$ into Equation (26) and applying the definition of $\hat{\Theta}_{\mathcal{C}}$, we obtain Equation (30). Expanding further yields

$$\delta_{\mathcal{C},i} = \frac{\psi_i}{2b_2\Psi_{\mathcal{C}}^2}\sum_{j=1,j\neq i}^{m}\psi_j\left\{\psi_j(\theta_i - \theta_j)^2 + 2\sum_{k>j,k\neq i}^{m}\psi_k(\theta_i - \theta_j)(\theta_i - \theta_k)\right\}. \tag{A.7}$$

It is useful to note that the two following identities hold true

$$\begin{aligned} 2(\theta_i-\theta_j)(\theta_i-\theta_k) &= (\theta_i-\theta_k+\theta_k-\theta_j)(\theta_i-\theta_k)+(\theta_i-\theta_j)(\theta_i-\theta_j+\theta_j-\theta_k) \\ &= (\theta_i-\theta_j)^2+(\theta_i-\theta_k)^2-(\theta_j-\theta_k)^2, \end{aligned} \tag{A.8}$$

$$\sum_{j=1,j\neq i}^{m}\sum_{k>j,k\neq i}^{m}\psi_j\psi_k\Big\{(\theta_i-\theta_j)^2+(\theta_i-\theta_k)^2\Big\}=\sum_{j=1,j\neq i}^{m}\sum_{k=1,k\neq i,j}^{m}\psi_j\psi_k(\theta_i-\theta_j)^2. \tag{A.9}$$

Using these identities and rearranging the sums in Equation (A.7), we obtain that

$$\delta_{\mathcal{C},i}=\frac{\psi_i}{2b_2\Psi_{\mathcal{C}}^2}\sum_{j=1,j\neq i}^{m}\psi_j\left\{(\Psi_{\mathcal{C}}-\psi_i)(\theta_i-\theta_j)^2-\sum_{k>j,k\neq i}^{m}\psi_k(\theta_j-\theta_k)^2\right\}. \tag{A.10}$$

Recall that the aggregate gross economic gains from $\{i,j\}$-linkage read

$$\Delta_{\{i,j\}}=\frac{\psi_i\psi_j}{2b_2(\psi_i+\psi_j)}(\theta_i-\theta_j)^2. \tag{A.11}$$

Finally noting that $\Psi_{\mathcal{C}_{-i}}=\Psi_{\mathcal{C}}-\psi_i$, Equation (A.10) coincides with Equation (32).

We can now prove the proposition. Summing Equation (32) over all $i\in[\![1;m]\!]$ gives

$$\Delta_{\mathcal{C}}\doteq\sum_{i=1}^{m}\delta_{\mathcal{C},i}=\Psi_{\mathcal{C}}^{-2}\sum_{i=1}^{m}\left\{\sum_{j=1,j\neq i}^{m}\left\{\Psi_{\mathcal{C}_{-i}}(\psi_i+\psi_j)\Delta_{\{i,j\}}-\psi_i\sum_{k>j,k\neq i}^{m}(\psi_j+\psi_k)\Delta_{\{j,k\}}\right\}\right\}. \tag{A.12}$$

Regrouping terms by bilateral linkages, Equation (A.12) rewrites

$$\begin{aligned} \Delta_{\mathcal{C}} &= \Psi_{\mathcal{C}}^{-2}\sum_{1\leq i<j\leq m}\left\{\Big(\Psi_{\mathcal{C}_{-i}}+\Psi_{\mathcal{C}_{-j}}\Big)(\psi_i+\psi_j)\Delta_{\{i,j\}}-\sum_{k=1,k\neq i,j}^{m}\psi_k(\psi_i+\psi_j)\Delta_{\{i,j\}}\right\} \\ &= \Psi_{\mathcal{C}}^{-2}\sum_{1\leq i<j\leq m}\Big\{\Big(\Psi_{\mathcal{C}_{-i}}+\Psi_{\mathcal{C}_{-j}}-\Psi_{\mathcal{C}_{-\{i,j\}}}\Big)(\psi_i+\psi_j)\Delta_{\{i,j\}}\Big\} \\ &= \Psi_{\mathcal{C}}^{-1}\sum_{1\leq i<j\leq m}(\psi_i+\psi_j)\Delta_{\{i,j\}}. \end{aligned} \tag{A.13}$$

By symmetry, i.e. $\Delta_{\{i,j\}}=\Delta_{\{j,i\}}$, Equation (A.13) coincides with Equation (33). Recalling that expectation and variance are linear and symmetric bilinear operators, respectively,

$$\mathbb{E}\{\Delta_{\mathcal{C}}\}=(2\Psi_{\mathcal{C}})^{-1}\sum_{(i,j)\in\mathcal{C}\times\mathcal{C}}\Psi_{\{i,j\}}\mathbb{E}\{\Delta_{\{i,j\}}\}, \tag{A.14a}$$

$$\mathbb{V}\{\Delta_{\mathcal{C}}\}=(2\Psi_{\mathcal{C}})^{-2}\sum_{(i,j)\in\mathcal{C}\times\mathcal{C}}\Psi_{\{i,j\}}\sum_{(k,l)\in\mathcal{C}\times\mathcal{C}}\Psi_{\{k,l\}}\mathrm{Cov}\{\Delta_{\{i,j\}};\Delta_{\{k,l\}}\}. \tag{A.14b}$$

The following will establish that $\mathcal{I} = \arg\max_{\mathcal{C}\in\mathbf{C}_\star} \mathbb{E}\{\Delta_\mathcal{C}\}$ but there is no reason that forming larger coalitions reduces volatility of gains and a fortiori that $\mathcal{I} = \arg\min_{\mathcal{C}\in\mathbf{C}_\star} \mathbb{V}\{\Delta_\mathcal{C}\}$.

A.4 Proof of Proposition 4 (superadditivity)

Given $\mathcal{S}$ and $\mathcal{S}'$ in Definition 2, Equation (36) obtains by telescoping the sequence $(\mathcal{S}_i)_{i\in[\![0;d]\!]} \in \mathbf{S}^{d+1}$ with $\mathcal{S}_0 = \mathcal{S}'$ and $\mathcal{S}_d = \mathcal{S}$. It is thus sufficient to prove Equation (37) for any $i \in [\![1;d]\!]$. Fix $\mathcal{C}$ and $\mathcal{C}'$ in $\mathbf{C}_\star$ with $\mathcal{C}' \subset \mathcal{C}$ and $\mathcal{C}''$ the complement of $\mathcal{C}'$ in $\mathcal{C}$, i.e. $\mathcal{C} = \mathcal{C}' \cup \mathcal{C}''$ and $\mathcal{C}' \cap \mathcal{C}'' = \varnothing$. Expanding Equation (33) then gives

$$\begin{aligned}\Delta_\mathcal{C} &= (2\Psi_\mathcal{C})^{-1}\left(\sum_{(i,j)\in\mathcal{C}'\times\mathcal{C}'}\Psi_{\{i,j\}}\Delta_{\{i,j\}} + \sum_{(i,j)\in\mathcal{C}''\times\mathcal{C}''}\Psi_{\{i,j\}}\Delta_{\{i,j\}} + 2\sum_{(i,j)\in\mathcal{C}'\times\mathcal{C}''}\Psi_{\{i,j\}}\Delta_{\{i,j\}}\right)\\ &= \Psi_\mathcal{C}^{-1}\left(\Psi_{\mathcal{C}'}\Delta_{\mathcal{C}'} + \Psi_{\mathcal{C}''}\Delta_{\mathcal{C}''} + \sum_{(i,j)\in\mathcal{C}'\times\mathcal{C}''}\Psi_{\{i,j\}}\Delta_{\{i,j\}}\right).\end{aligned} \tag{A.15}$$

The aggregate gain from merging $\mathcal{C}'$ and $\mathcal{C}''$ is $\Delta_{\{\mathcal{C}',\mathcal{C}''\}} \doteq \Delta_\mathcal{C} - \Delta_{\mathcal{C}'} - \Delta_{\mathcal{C}''}$ so that

$$\begin{aligned}\Delta_{\{\mathcal{C}',\mathcal{C}''\}} &= \Psi_\mathcal{C}^{-1}\left(\sum_{(i,j)\in\mathcal{C}'\times\mathcal{C}''}\Psi_{\{i,j\}}\Delta_{\{i,j\}} + \Big(\Psi_{\mathcal{C}'} - \Psi_\mathcal{C}\Big)\Delta_{\mathcal{C}'} + \Big(\Psi_{\mathcal{C}''} - \Psi_\mathcal{C}\Big)\Delta_{\mathcal{C}''}\right)\\ &= \Psi_\mathcal{C}^{-1}\left(\sum_{(i,j)\in\mathcal{C}'\times\mathcal{C}''}\Psi_{\{i,j\}}\Delta_{\{i,j\}} - \Psi_{\mathcal{C}''}\Delta_{\mathcal{C}'} - \Psi_{\mathcal{C}'}\Delta_{\mathcal{C}''}\right).\end{aligned} \tag{A.16}$$

By transposition of the definition of the expected gains in a bilateral link between two singletons to two linkage coalitions, we obtain

$$\mathbb{E}\{\Delta_{\{\mathcal{C}',\mathcal{C}''\}}\} = \frac{\Psi_{\mathcal{C}'}\Psi_{\mathcal{C}''}}{2b_2\Psi_\mathcal{C}}\Big(\mathbb{V}\{p_{\mathcal{C}'}\} + \mathbb{V}\{p_{\mathcal{C}''}\} - 2\mathrm{Cov}\{p_{\mathcal{C}'}; p_{\mathcal{C}''}\}\Big) \geq 0, \tag{A.17}$$

which by definition is non negative and proves superadditivity.

A.5 Proof of Corollary 1 (non alignment of preferences)

Fix $\mathcal{C}' \in \mathbf{C}_\star\backslash\mathcal{I}$. Let $\mathcal{C} \supset \mathcal{C}'$ be a proper superset of $\mathcal{C}'$ and denote by $\mathcal{C}'' = \mathcal{C} \cap \mathcal{C}'$ the complement of $\mathcal{C}'$ in $\mathcal{C}$. By way of contradiction, assume that $\mathbb{E}\{\delta_{\mathcal{C}',i}\} \geq \mathbb{E}\{\delta_{\mathcal{C},i}\}$ holds for all $i \in \mathcal{C}'$, with at least one inequality holding strictly. By summation over $i \in \mathcal{C}'$

$$\sum_{i\in\mathcal{C}'}\mathbb{E}\{\delta_{\mathcal{C}',i}\} = \mathbb{E}\{\Delta_{\mathcal{C}'}\} > \sum_{i\in\mathcal{C}'}\mathbb{E}\{\delta_{\mathcal{C},i}\} = \mathbb{E}\{\Delta_\mathcal{C}\} - \sum_{i\in\mathcal{C}''}\mathbb{E}\{\delta_{\mathcal{C},i}\} \tag{A.18}$$

Recalling the definition of the gains in a link between $\mathcal{C}'$ and $\mathcal{C}''$ in Equation (35), Equation (A.18) imposes

$$\mathbb{E}\{\Delta_{\mathcal{C}''}\} + \mathbb{E}\{\Delta_{\{\mathcal{C}',\mathcal{C}''\}}\} - \sum_{i\in\mathcal{C}''} \mathbb{E}\{\delta_{\mathcal{C},i}\} < 0, \tag{A.19}$$

and contradicts superadditivity, which requires the above expression to be non-negative. That is, $\mathcal{C}'$ cannot be the most weakly preferred linkage coalition for all jurisdictions thereof.

B Model generalization and extensions

B.1 The two components of linkage gains

We consider the general case where jurisdictions in $\mathcal{C}$ have different domestic ambition levels, i.e. the exogenous cap profile $(\omega_i)_{i\in\mathcal{C}}$ does not satisfy Equation (4) and expected autarky prices differ across jurisdictions in $\mathcal{C}$. In this case, the $\mathcal{C}$-linkage equilibrium permit price reads

$$p_{\mathcal{C}} = b_1 - b_2\Omega_{\mathcal{C}}\Psi_{\mathcal{C}}^{-1} + \hat{\Theta}_{\mathcal{C}}. \tag{B.1}$$

Considering the difference between the $\mathcal{C}$-linkage price and the autarky price in jurisdiction $i \in \mathcal{C}$, we obtain

$$\bar{p}_i - p_{\mathcal{C}} = \theta_i - \hat{\Theta}_{\mathcal{C}} - b_2\Big(\Omega_{\mathcal{C}}\Psi_{\mathcal{C}}^{-1} - \omega_i\psi_i^{-1}\Big). \tag{B.2}$$

Note that Equation (B.2) reduces to Equation (29) when $\Omega_{\mathcal{C}}\Psi_{\mathcal{C}}^{-1} = \omega_i\psi_i^{-1}$. This occurs when Equation (4) holds, i.e. when $\exists A > 0$ such that $\omega_i = A \cdot \psi_i$ for all $i \in \mathcal{C}$.

Before showing how our results in the main text generalize, we illustrate how they hold irrespective of the common stringency parameter A To this end, we compare $\mathcal{C}$-linkage equilibria with two ambition parameters A and A' such that A is more stringent than A', i.e. $A' > A$. This implies that both autarky and $\mathcal{C}$-linkage prices under A are higher than under A', that is

$$p_{\mathcal{C}}^{A} = b_1 - b_2 A + \hat{\Theta}_{\mathcal{C}} > p_{\mathcal{C}}^{A'} = b_1 - b_2 A' + \hat{\Theta}_{\mathcal{C}}. \tag{B.3}$$

However, jurisdictional net permit demands, and consequently the linkage gains, are unaltered. In fact,

$$q_{\mathcal{C},i}^{A} - \omega_i^{A} = q_{\mathcal{C},i}^{A'} - \omega_i^{A'} = \psi_i(\theta_i - \hat{\Theta}_{\mathcal{C}})/b_2, \tag{B.4}$$

holds for all $i \in \mathcal{C}$ and is independent of the coalition-wide stringency parameter.

In the general case, plugging Equation (B.2) into Equation (26) and taking expectations gives

$$\mathbb{E}\{\delta_{\mathcal{C},i}\} = \underbrace{\frac{b_2\psi_i}{2}\Big(\Omega_{\mathcal{C}}\Psi_{\mathcal{C}}^{-1} - \omega_i\psi_i^{-1}\Big)^2}_{\text{stringency-dependent only}} + \underbrace{\frac{\psi_i}{2b_2}\mathbb{E}\{(\theta_i - \hat{\Theta}_{\mathcal{C}})^2\}}_{\text{shock-dependent only}}. \tag{B.5}$$

Similar to Equation (28), the expected gains from $\mathcal{C}$-linkage accruing to jurisdiction $i \in \mathcal{C}$ can be decomposed into two components. The first source of linkage gains directly relates to the within-coalition differences in jurisdiction-specific ambition levels (i.e., wedges in expected autarky prices) and is *independent* of the shocks. The second source of linkage gains, which is our primary focus, directly relates to jurisdiction-specific uncertainty and is *independent* of the ambition levels.

Because both the first and second components of the linkage gains are non-negative, it is straightforward to show that Proposition 4 (superadditivity) continues to hold. However, a formal proof is required to show that Proposition 3 (bilateral decomposition) is maintained. For this purpose, consider that shocks are absent. In this case, aggregate gains from $\mathcal{C}$-linkage read

$$\Delta_{\mathcal{C}} = \frac{b_2}{2}\sum_{i=1}^{m}\psi_i\Big(\Omega_{\mathcal{C}}\Psi_{\mathcal{C}}^{-1} - \omega_i\psi_i^{-1}\Big)^2 = \frac{b_2}{2}\Big(\sum_{i=1}^{m}\omega_i^2\psi_i^{-1} - \Omega_{\mathcal{C}}^2\Psi_{\mathcal{C}}^{-1}\Big). \tag{B.6}$$

Applying Equation (B.6) for bilateral linkages, it follows that

$$\begin{aligned}
\sum_{1\le i<j\le m}\Psi_{\{i,j\}}\Delta_{\{i,j\}} &= \frac{b_2}{2}\sum_{1\le i<j\le m}\Big\{\Psi_{\{i,j\}}(\omega_i^2\psi_i^{-1} + \omega_j^2\psi_j^{-1}) - \Omega_{\{i,j\}}^2\Big\} \\
&= \frac{b_2}{2}\Big(\sum_{i=1}^{m}\sum_{j=1,j\neq i}^{m}\Psi_{\{i,j\}}\omega_i^2\psi_i^{-1} - \sum_{1\le i<j\le m}\Omega_{\{i,j\}}^2\Big) \\
&= \frac{b_2}{2}\sum_{i=1}^{m}\Big\{\Big(\Psi_{\mathcal{C}} + (m-2)\psi_i\Big)\omega_i^2\psi_i^{-1} - \sum_{j=1,j\neq i}^{m}\Big\{\omega_i^2 + \omega_i\omega_j\Big\}\Big\} \\
&= \frac{b_2}{2}\Big(\Psi_{\mathcal{C}}\sum_{i=1}^{m}\omega_i^2\psi_i^{-1} - \sum_{i=1}^{m}\Big\{\omega_i^2 + 2\sum_{j>i}\omega_i\omega_j\Big\}\Big) \\
&= \frac{b_2}{2}\Big(\Psi_{\mathcal{C}}\sum_{i=1}^{m}\omega_i^2\psi_i^{-1} - \Omega_{\mathcal{C}}^2\Big).
\end{aligned} \tag{B.7}$$

Multiplying both sides by $\Psi_{\mathcal{C}}^{-1}$ shows that Equations (33) and (B.6) coincide.

B.2 Distribution of gains between and within linkage coalitions

In order to proceed with the analysis of how linkage gains are apportioned within linkage coalitions we must first assume that previously formed linkage coalitions have evolved into an integrated system for otherwise the entity 'linkage coalition' would be ill-defined. In practice, we assume that linkage coalitions consolidate and transmute into one new, single entity (Caparrós & Péreau, 2017). Because there is no time in our model, consolidation can be envisaged as commitment in the sense of Carraro & Siniscalco (1993).

As in the main text, consider the situation where two disjoint linkage coalitions $\mathcal{C}'$ and $\mathcal{C}''$ link. Let $\mathcal{C} = \mathcal{C}' \cup \mathcal{C}''$ and assume that $\mathcal{C}'$ and $\mathcal{C}''$ have consolidated prior to linking. By definition of bilateral linkage the aggregate gross gains $\mathbb{E}\{\Delta_{\{\mathcal{C}',\mathcal{C}''\}}\}$ are shared between $\mathcal{C}'$ and $\mathcal{C}''$ in inverse proportion to linkage coalition size. To understand how these gains are then shared within each linkage coalition, first note that the aggregate abatement effort required of, say, $\mathcal{C}'$ must be apportioned among its internal jurisdictions according to some optimality criterion, in particular in proportion to jurisdictional abatement opportunities.

In our model, the ratio ψ_i/b_2 measures abatement opportunities at the margin: the larger this ratio the cheaper are the abatement opportunities in jurisdiction $i \in \mathcal{C}'$. By summation, the aggregate abatement opportunities of the consolidated linkage coalition $\mathcal{C}'$ is $\Psi_{\mathcal{C}'}/b_2$. Optimality requires that within-$\mathcal{C}'$ jurisdiction i's net permit demand under $\{\mathcal{C}',\mathcal{C}''\}$-linkage satisfies the equality $q_{\{\mathcal{C}',\mathcal{C}''\},i} - \omega_i = (\psi_i/\Psi_{\mathcal{C}'})(q_{\{\mathcal{C}',\mathcal{C}''\},\mathcal{C}'} - \Omega_{\mathcal{C}'})$. In turn, the gains in $\mathcal{C}'$ are apportioned among internal jurisdictions in proportion to size as well. For instance, the gains accruing to jurisdiction $i \in \mathcal{C}'$ amount to $(\psi_i/\Psi_{\mathcal{C}'})(\Psi_{\mathcal{C}''}/\Psi_{\mathcal{C}})\mathbb{E}\{\Delta_{\{\mathcal{C}',\mathcal{C}''\}}\}$.

Special case: Unitary accretion. It is also of interest to characterize the special case where a linkage coalition is linked to an individual jurisdiction (i.e., singleton). This clarifies how overall gross gains from the link are distributed between jurisdictions. Fix $\mathcal{C} \in \mathbf{C}$ and $i \in \mathcal{I}_{-\mathcal{C}}$ and let $\mathcal{C}' = \mathcal{C}$ and $\mathcal{C}'' = \{i\}$ in Equation (37), then

$$\mathbb{E}\{\Delta_{\{\mathcal{C},\{i\}\}}\} = \mathbb{E}\{\Delta_{\mathcal{C}\cup\{i\}}\} - \mathbb{E}\{\Delta_{\mathcal{C}}\} = \Psi_{\mathcal{C}\cup\{i\}}\Psi_{\mathcal{C}}^{-1}\mathbb{E}\{\delta_{\mathcal{C}\cup\{i\},i}\} = (1 + \psi_i\Psi_{\mathcal{C}}^{-1})\mathbb{E}\{\delta_{\mathcal{C}\cup\{i\},i}\}. \quad \text{(B.8)}$$

In words, linking jurisdiction $i \notin \mathcal{C}$ to the linkage coalition $\mathcal{C}$ generates an overall gross gain equal to $\mathbb{E}\{\delta_{\mathcal{C}\cup\{i\},i}\} + \psi_i\mathbb{E}\{\delta_{\mathcal{C}\cup\{i\},i}\}/\Psi_{\mathcal{C}}$ where the first term accrues to jurisdiction i and the second one accrues to the linkage coalition $\mathcal{C}$. Put differently, jurisdictions in $\mathcal{C}$ get a portion $\psi_i/\Psi_{\mathcal{C}\cup\{i\}}$ of the overall gross gain $\mathbb{E}\{\Delta_{\{\mathcal{C},\{i\}\}}\}$ that they share in proportion to size.

We provide an alternative proof of Equation (B.8). Fix w.l.o.g. $i = m$ such that $\mathcal{C}_{-i} = \{1, 2, \ldots, m-1\}$. By subtracting Equation (33) for coalitions $\mathcal{C}$ and $\mathcal{C}_{-i}$, we obtain

$$\begin{aligned}
\Delta_{\mathcal{C}} - \Delta_{\mathcal{C}_{-i}} &= \Psi_{\mathcal{C}}^{-1} \sum_{1 \le j < k \le i} (\psi_j + \psi_k)\Delta_{\{j,k\}} - \Psi_{\mathcal{C}_{-i}}^{-1} \sum_{1 \le j < k \le i-1} (\psi_j + \psi_k)\Delta_{\{j,k\}} \\
&= \Psi_{\mathcal{C}}^{-1} \sum_{j=1}^{i-1} (\psi_j + \psi_i)\Delta_{\{j,i\}} - \sum_{1 \le j < k \le i-1} (\Psi_{\mathcal{C}_{-i}}^{-1} - \Psi_{\mathcal{C}}^{-1})(\psi_j + \psi_k)\Delta_{\{j,k\}} \\
&= \Psi_{\mathcal{C}}^{-1}\Psi_{\mathcal{C}_{-i}}^{-1} \left(\sum_{j=1}^{i-1} \Psi_{\mathcal{C}_{-i}} (\psi_j + \psi_i)\Delta_{\{j,i\}} - \psi_i \sum_{1 \le j < k \le i-1} (\psi_j + \psi_k)\Delta_{\{j,k\}} \right) \\
&= \Psi_{\mathcal{C}}\Psi_{\mathcal{C}_{-i}}^{-1} \delta_{\mathcal{C},i},
\end{aligned} \tag{B.9}$$

where the last line follows from Equation (32).

B.3 Alternative domestic cap selection mechanisms

This appendix considers alternative cap selection mechanisms. For example, jurisdictional caps on emissions could be set non-cooperatively. That is, jurisdiction $i \in \mathcal{I}$ maximizes the difference between expected benefits of pollution and damages from pollution by operating its own market for permit in autarky, taking other jurisdictions' cap levels $(\omega_j)_{j \in \mathcal{I}_{-i}}$ as given. In the case of a uniformly-mixed stock pollutant, damages exclusively depend on aggregate emissions $Q_{\mathcal{I}} = \sum_{i \in \mathcal{I}} q_i$. For the time being, assume every jurisdiction faces the same damages from pollution given by

$$D(Q_{\mathcal{I}}) = d_1 Q_{\mathcal{I}} + d_2 (Q_{\mathcal{I}})^2/2, \tag{B.10}$$

where d_1, d_2 are positive parameters. Now, with $\Omega_{-i} = \sum_{j \in \mathcal{I}_{-i}} \omega_j$, these Cournot-Nash caps satisfy

$$\omega_i \doteq \arg\max_{\omega \ge 0} \mathbb{E}\Big\{ B_i(\omega; \theta_i) - D\left(\omega + \Omega_{-i}\right) \Big\} \text{ for all } i \in \mathcal{I}. \tag{B.11}$$

In this case, jurisdictional caps are proportional to jurisdictional size

$$\omega_i = A_1 \cdot \psi_i \text{ for all } i \in \mathcal{I}, \text{ where } A_1 = \frac{b_1 - d_1}{b_2 + d_2 \Psi_{\mathcal{I}}} > 0 \tag{B.12}$$

measures the non-cooperative abatement effort that is common to all jurisdictions. Here we assume $b_1 > d_1$. As long as the damage functions are identical across jurisdictions and there is no anticipation of linkage, similar results obtain under various cooperation levels and alternative conjectural variations.

Let $\mathcal{C} \in \mathbf{C}$ be a coalition on cap selection, i.e. jurisdictions in $\mathcal{C}$ set their caps cooperatively.

Denote by $\bar{\mathcal{C}}$ the complement of $\mathcal{C}$ in $\mathbf{C}$ and assume members of $\bar{\mathcal{C}}$ behave as singletons w.r.t. cap selection. We assume Stackelberg conjectural variations where $\mathcal{C}$ behaves as the leader. Note that our results would slightly differ under alternative conjectural variations, see e.g. MacKenzie (2011) and Gelves & McGinty (2016). For instance, with Cournot conjectural variations we would solve for the coalitional Nash equilibrium in cap selection, see e.g. Bloch (2003). The aggregate reaction function of singletons to the emissions cap $\Omega_{\mathcal{C}}$ selected by $\mathcal{C}$ reads

$$\Omega^r_{\bar{\mathcal{C}}}(\Omega_{\mathcal{C}}) = \frac{(b_1 - d_1 - d_2\Omega_{\mathcal{C}})}{b_2 + d_2\Psi_{\bar{\mathcal{C}}}} \cdot \Psi_{\bar{\mathcal{C}}}. \tag{B.13}$$

Coalition $\mathcal{C}$ recognizes $\Omega^r_{\bar{\mathcal{C}}}$ when jointly deciding upon $\Omega_{\mathcal{C}}$, that is

$$\max_{(\omega_i)_{i\in\mathcal{C}}} \left\{ \sum_{i\in\mathcal{C}} B_i(\omega_i;\theta_i) - |\mathcal{C}| D\Big(\Omega_{\mathcal{C}} + \Omega^r_{\bar{\mathcal{C}}}(\Omega_{\mathcal{C}})\Big)\right\}. \tag{B.14}$$

Solving Equation (B.14) and summing over i in $\mathcal{C}$ gives the $\mathcal{C}$-coalition aggregate cap

$$\Omega_{\mathcal{C}} = A_{\mathcal{C}} \cdot \Psi_{\mathcal{C}}, \text{ with } A_{\mathcal{C}} \doteq \frac{b_1(b_2 + d_2\Psi_{\bar{\mathcal{C}}})^2 - b_2|\mathcal{C}|\Big(d_1(b_2 + d_2\Psi_{\bar{\mathcal{C}}}) + d_2(b_1 - d_1)\Psi_{\mathcal{C}}\Big)}{b_2\Big((b_2 + d_2\Psi_{\bar{\mathcal{C}}})^2 + b_2 d_2|\mathcal{C}|\Psi_{\mathcal{C}}\Big)}. \tag{B.15}$$

Substituting the above in Equation (B.13) gives the $\bar{\mathcal{C}}$-aggregate cap

$$\Omega_{\bar{\mathcal{C}}} = A_{\bar{\mathcal{C}}} \cdot \Psi_{\bar{\mathcal{C}}}, \text{ with } A_{\bar{\mathcal{C}}} \doteq \frac{b_1 - d_1 - d_2 A_{\mathcal{C}} \cdot \Psi_{\mathcal{C}}}{b_2 + d_2\Psi_{\bar{\mathcal{C}}}}. \tag{B.16}$$

Differentiating the abatement effort coefficients above w.r.t. the cardinality of $\mathcal{C}$ gives

$$\frac{\partial A_{\mathcal{C}}}{\partial|\mathcal{C}|} < 0, \text{ and } \frac{\partial A_{\bar{\mathcal{C}}}}{\partial|\mathcal{C}|} = -\frac{d_2\Psi_{\mathcal{C}}}{b_2 + d_2\Psi_{\bar{\mathcal{C}}}}\frac{\partial A_{\mathcal{C}}}{\partial|\mathcal{C}|} > 0. \tag{B.17}$$

The first inequality tells us that the higher the number of cooperating jurisdictions, the the larger is the proportion of pollution externalities that are internalized, and consequently the larger the partnering jurisdictions' individual abatement efforts. The second inequality reflects the standard free-rider problem and the crowding-out effect of domestic abatement efforts. Indeed, domestic abatement efforts are strategic substitutes.[23] That is, in response to higher abatement efforts from jurisdictions in $\mathcal{C}$, jurisdictions in $\bar{\mathcal{C}}$ will lower their own. In particular, $\mathcal{C} = \mathcal{I}$ corresponds to full cooperation where the common abatement effort is

[23] This will always be the case in a pure emissions game. In an international market for permits, note that Holtsmark & Midttømme (2015) are able to transform domestic abatement efforts into strategic complements by tying the dynamic emissions game to the dynamics of (investments in) renewables.

$A_n = \frac{b_1 - nd_1}{b_2 + nd_2\Psi_{\mathcal{I}}} > 0$ (we assume $b_1 > nd_1$). Symmetrically, $\bar{\mathcal{C}} = \mathcal{I}$ coincides with the Cournot-Nash solution in Equation (B.11) with $A_1 = \frac{b_1 - d_1}{b_2 + d_2\Psi_{\mathcal{I}}} > A_n$ as jurisdictions do not internalize the negative externality generated by their pollution on the other $n-1$ jurisdictions.

B.4 Cap selection in anticipation of linkage

First note that differentiating Equation (B.5) w.r.t. ω_i gives

$$\frac{\partial \mathbb{E}\{\delta_{\mathcal{C},i}\}}{\partial \omega_i} = b_2\psi_i\Big(\Omega_{\mathcal{C}}\Psi_{\mathcal{C}}^{-1} - \omega_i\psi_i^{-1}\Big)\Big(\Psi_{\mathcal{C}}^{-1} - \psi_i^{-1}\Big) \geq 0 \Leftrightarrow \omega_i\psi_i^{-1} \geq \Omega_{\mathcal{C}}\Psi_{\mathcal{C}}^{-1}. \tag{B.18}$$

Irrespective of the shock structure, jurisdictions with size-adjusted cap stringency lower than that of $\mathcal{C}$ (i.e., $\omega_i\psi_i^{-1} \geq \Omega_{\mathcal{C}}\Psi_{\mathcal{C}}^{-1}$) in expectations are potential permit sellers (i.e., $\mathbb{E}\{\bar{p}_i\} \leq \Psi_{\mathcal{C}}^{-1}\sum_{j\in\mathcal{C}}\psi_j\mathbb{E}\{\bar{p}_j\}$). One would expect that these jurisdictions have an incentive to inflate their domestic caps to increase permit sales and thus economic gains from linkage (Helm, 2003). That being said, such an incentive is mitigated by the contrasting downward pressure exerted by the extra supply of permits on the linked permit price. Conversely, jurisdictions whose ambition levels are above the $\mathcal{C}$-average ambition level are potential permit buyers in expectations and have the incentive to strengthen ambition.

As an illustration we consider the situation where jurisdictions anticipate $\mathcal{C}$-linkage when selecting their domestic cap. This corresponds to a two-stage game where jurisdictions determine their caps at stage one and permit trading on the linked market occurs at stage 2. We solve the game using backward induction and focus on subgame perfect Nash equilibria.

Stage 2: Permit trading and jurisdictional emissions choices.

The linked market equilibrium obtains by equalization of marginal benefits across jurisdictions and linked market closure. Given cap and realized shock profiles $(\omega_i)_{i\in\mathcal{C}}$ and $(\theta_i)_{i\in\mathcal{C}}$, respectively, we denote by $q^*_{\mathcal{C},i}$ and $p^*_{\mathcal{C}}$ the equilibrium emission level in i and linking price

$$q^*_{\mathcal{C},i} \equiv q^*_{\mathcal{C},i}(\Omega_{\mathcal{C}};(\theta_i)_{i\in\mathcal{C}}) = \psi_i(\theta_i - \hat{\Theta}_{\mathcal{C}})/b_2 + \psi_i\Omega_{\mathcal{C}}\Psi_{\mathcal{C}}^{-1}, \tag{B.19a}$$

$$p^*_{\mathcal{C}} \equiv p^*_{\mathcal{C}}(\Omega_{\mathcal{C}};(\theta_i)_{i\in\mathcal{C}}) = b_1 + \hat{\Theta}_{\mathcal{C}} - b_2\Omega_{\mathcal{C}}\Psi_{\mathcal{C}}^{-1}. \tag{B.19b}$$

As is standard, we note that $\partial p^*_{\mathcal{C}}/\partial_{\Omega_{\mathcal{C}}} = -b_2\Psi_{\mathcal{C}}^{-1} < 0$ and $\partial q^*_{\mathcal{C},i}/\partial_{\Omega_{\mathcal{C}}} = \psi_i\Psi_{\mathcal{C}}^{-1} \in (0;1)$. For simplicity, we assume in the following that $d_2 = 0$, i.e. jurisdictional reaction functions for cap selection are orthogonal, and that jurisdictional damages are proportional to size.

Stage 1: Non-cooperative jurisdictional cap selection with linkage anticipation.

Each jurisdiction recognizes the effects of its domestic cap decision on both the linked permit price and its own market position. We consider Cournot conjectural variations (i.e., caps are announced simultaneously) and each jurisdiction takes other jurisdictional caps as given. Jurisdictional caps with strategic anticipation of linkage $(\hat{\omega}_i)_{i\in\mathcal{C}}$ satisfy, for all i in $\mathcal{C}$,

$$\begin{aligned}\hat{\omega}_i \doteq \arg\max_{\omega} \mathbb{E}\Big\{ B_i\Big(q^*_{\mathcal{C},i}(\omega+\Omega_{\mathcal{C}_{-i}};(\theta_i)_{i\in\mathcal{C}});\theta_i\Big) - \psi_i d_1(\omega+\Omega_{\mathcal{C}_{-i}}) \\ + p^*_{\mathcal{C}}(\Omega_{\mathcal{C}};(\theta_i)_{i\in\mathcal{C}})\Big(\omega - q^*_{\mathcal{C},i}(\omega+\Omega_{\mathcal{C}_{-i}};(\theta_i)_{i\in\mathcal{C}})\Big)\Big\}.\end{aligned} \tag{B.20}$$

Now assume that $\mathcal{C}=\{i,j\}$ where $\psi_j>\psi_i$, i.e. j is the larger and higher-damage jurisdiction. By stage-2 optimality, i.e. $\partial B_i(q^*_{\mathcal{C},i};\theta_i)/\partial q_i = p^*_{\mathcal{C}}$, and taking expectations, the necessary first-order condition associated with Programme (B.20) writes

$$-b_2\Psi^{-2}_{\{i,j\}}(\psi_j\hat{\omega}_i - \psi_i\hat{\omega}_j) + b_1 - b_2(\hat{\omega}_1+\hat{\omega}_2)\Psi^{-1}_{\{i,j\}} - d_1\psi_i = 0. \tag{B.21}$$

Summing over i and j gives $\hat{\omega}_1+\hat{\omega}_2 = \Psi_{\{i,j\}}(2b_1 - d_1\Psi_{\{i,j\}})/(2b_2)$. Plugging this expression into Equation (B.21) then yields

$$\hat{\omega}_i = \psi_i(b_1 - d_1\Psi_{\{i,j\}})/b_2 + d_1\psi_j\Psi_{\{i,j\}}/(2b_2). \tag{B.22}$$

When there is no anticipation of linkage, emissions caps are determined by Equation (B.11), i.e. $\omega_i = \psi_i(b_1 - d_1\psi_i)/b_2$. As in Helm (2003), it holds that $\hat{\omega}_i > \omega_i$ and $\hat{\omega}_j < \omega_j$, i.e. the low-damage (resp. high-damage) jurisdiction increases (resp. decreases) its domestic cap in the perspective of $\{i,j\}$-linkage. In aggregate, anticipation of linkage leads to increased emissions since

$$\hat{\omega}_i + \hat{\omega}_j \geq \omega_i + \omega_j \Leftrightarrow (\psi_i - \psi_j)^2 \geq 0. \tag{B.23}$$

If additional damages associated with this increase in emissions are high enough, linkage (when anticipated) can be suboptimal relative to autarky (Holtsmark & Sommervoll, 2012).

C Linkage indifference frontiers

We define the relative size and volatility parameters by $\psi_j = x\psi_i$, $\psi_k = y\psi_i$, $\sigma_j = \alpha\sigma_j$ and $\sigma_k = \beta\sigma_i$. Then, jurisdiction i prefers $\{i,j\}$ over $\{i,k\}$, $\{i,j\}$ over $\mathcal{I}=\{i,j,k\}$ and $\{i,k\}$

over $\mathcal{I}$ i.f.f. the following inequalities respectively hold

$$y \leq \frac{x\sqrt{1+\alpha^2-2\rho_{ij}\alpha}}{\sqrt{1+\beta^2-2\rho_{ik}\beta}+x\left(\sqrt{1+\beta^2-2\rho_{ik}\beta}-\sqrt{1+\alpha^2-2\rho_{ik}\alpha}\right)} \tag{C.1a}$$

$$y \leq \frac{2x(1+x)\left(x(1+\alpha^2-2\rho_{ij}\alpha)-(1+x)(1-\rho_{ij}\alpha-\rho_{ik}\beta+\rho_{jk}\alpha\beta)\right)}{(1+x)^2(1+\beta^2-2\rho_{ik}\beta)-x^2(1+\alpha^2-2\rho_{ij}\alpha)} \tag{C.1b}$$

$$x \leq \frac{2y(1+y)\left(y(1+\beta^2-2\rho_{ik}\beta)-(1+y)(1-\rho_{ij}\alpha-\rho_{ik}\beta+\rho_{jk}\alpha\beta)\right)}{(1+y)^2(1+\alpha^2-2\rho_{ij}\alpha)-y^2(1+\beta^2-2\rho_{ik}\beta)}, \tag{C.1c}$$

and define the indifference frontiers depicted in Figures 1, 2 and 3. Similarly, we could define indifference frontiers for j and k. Comparative statics results can be obtained by directly differentiating the frontiers in Equation (C.1). However, we prefer to proceed graphically to intuitively explain the movements of the frontiers when we vary jurisdictional characteristics.

In particular, to better discipline our characterization of i's relative preferences for the trilateral link w.r.t. bilateral links it is useful to consider the following ratio

$$\frac{\mathbb{E}\{\delta_{\mathcal{I},i}\}}{\mathbb{E}\{\delta_{\{i,j\},i}\}} = \frac{PSE_{\mathcal{I},i}}{PSE_{\{i,j\},i}} \times \frac{VE_{\mathcal{I},i}+DE_{\mathcal{I},i}}{VE_{\{i,j\},i}+DE_{\{i,j\},i}}. \tag{C.2}$$

First note that the coefficient PSE is always higher for the trilateral link than for an internal bilateral link, that is

$$PSE_{\mathcal{I},i} > PSE_{\{i,j\},i}. \tag{C.3}$$

In addition, the ratio $PSE_{\mathcal{I},i}/PSE_{\{i,j\},i}$ increases with ψ_i and ψ_k but decreases with ψ_j. The implications of this are twofold. First, all else equal, when i is relatively larger than both j and k, the PSE ratios are such that i prefers the trilateral link over the two bilateral links. Second, all else equal, when j (resp. k) is relatively larger than both i and k (resp. j), the PSE ratios steer i's preferences towards the bilateral link $\{i,j\}$ (resp. $\{i,k\}$).

Next, in order to analyze the relative VE, assume that $\rho_{jk}=0$ to start with. In this case

$$VE_{\{i,j\},i} \geq VE_{\mathcal{I},i} \Leftrightarrow \sigma_j\sqrt{\psi_k+2\psi_j} \geq \sigma_k\sqrt{\psi_k}, \tag{C.4}$$

which holds unconditionally when $\sigma_j \sim \sigma_k$ since $\sigma_j^2 > \mathbb{V}\{\hat{\Theta}_{\{j,k\}}\}$ or provided that $\sqrt{3}\sigma_j \geq \sigma_k$ when $\psi_j=\psi_k$. All else equal, note that $VE_{\{i,j\},i}/VE_{\mathcal{I},i} \geq 1$ is more (resp. less) likely to hold when $\rho_{jk}<0$ (resp. $\rho_{jk}>0$), i.e. i prefers to link with both j and k when the latter are positively correlated. In addition, the ratio $VE_{\{i,j\},i}/VE_{\mathcal{I},i}$ increases with σ_j, decreases with σ_k and decreases with σ_i i.f.f. Inequality (C.4) holds. Finally, in terms of relative DE, it holds that

$$DE_{\{i,j\},i} \geq DE_{\mathcal{I},i} \Leftrightarrow \rho_{ij}\sigma_j \leq \rho_{ik}\sigma_k, \tag{C.5}$$

which is always the case when $\rho_{ik} > 0$ and $\rho_{ij} < 0$, and conversely never holds when $\rho_{ik} < 0$ and $\rho_{ij} > 0$. Note also that $DE_{\{i,j\},i}/DE_{\mathcal{I},i} \geq 1$ provided that j is more (resp. less) volatile than k when $\rho_{ij} = \rho_{ik} < 0$ (resp. $\rho_{ij} = \rho_{ik} > 0$). In addition, the ratio $DE_{\{i,j\},i}/DE_{\mathcal{I},i}$ increases in σ_j (resp. σ_k) provided that $\rho_{ij} \cdot \rho_{ik} > 0$ (resp. $\rho_{ij} \cdot \rho_{ik} < 0$).

D Calibration methodology

This appendix describes the steps we take in calibrating jurisdictional characteristics, namely size (ψ_i) shock volatility (σ_i) as well as the pair-specific correlation (ρ_{ij}).

We obtain annual country level carbon dioxide emissions data covering 1950-2012 from the World Resources Institute – observed emissions from jurisdiction i in year t is denoted e_{it}. For China we exclude observations from 1950-1975 because this period features uncharacteristic fluctuations associated with the Great Leap Forward and Cultural Revolution.

Taking the natural logarithm of laissez-faire emissions given in Equation (2) gives

$$\ln(\tilde{q}_i) = \ln(b_2/\psi_i) + \ln(b_1 + \theta_i). \tag{D.1}$$

We associate each component of $\ln(\tilde{q}_i)$ with the trend and cyclical components of emissions obtained using the Hodrick-Prescott (HP) filter with the penalty parameter $\lambda = 6.25$ for annual data – see Hodrick & Prescott (1997) and Ravn & Uhlig (2002) for details. This is in the spirit of Doda (2014) and congruent with our interpretation of variation in marginal benefits of emissions as being driven by business cycles, technology shocks, changes in the prices of factors of production, jurisdiction-specific events, weather fluctuations, etc.

The HP filter decomposes the observed series $\{\ln(e_{it})\}$ into two time series $\{e^t_{it}, e^c_{it}\}$ where $\ln(e_{it}) = e^t_{it} + e^c_{it}$ in each year t. We acknowledge that assuming jurisdictions have identical technology (b_2) lacks realism. However, it reduces the data required to calibrate the model substantially and we consider this assumption to be a reasonable first pass. Since our model is static, we also assume that the final observation of the trend component is related to size of jurisdiction i through

$$\ln(b_2/\psi_i) = e^t_{i,2012}. \tag{D.2}$$

Given our assumptions that jurisdictions are identical up to size and shock, we can normalize

$\psi_{CHN} = 100$ and set $b_2 = 0.5$. These amount to choosing the units in which gains are measured. Consequently, the quantitative results in the main text are comparable across linkage coalitions and jurisdictions. However, the value of the gain from a particular link and how it is shared between jurisdictions, remains sensitive to technology differences and, given those differences, to the calibration of ψ_i. Jurisdictional sizes are listed in Table 2.

To calibrate σ_i and ρ_{ij} we assume that the cyclical components e^c_{it} provide information about the distribution of the underlying jurisdiction-specific shocks θ_i. Then, given our modelling framework, e^c_{it} is related to a draw from the distribution of θ_i so that

$$\ln(b_1 + \theta_i) = e^c_{it}. \tag{D.3}$$

We note that e^c_{it} obtained using the HP filter is a stationary time series. We can thus compute the standard deviation of θ_i consistent with the model using

$$\sigma_i = \sigma(\exp(e^c_{it})). \tag{D.4}$$

Jurisdictional volatilities are given in Table 2. Finally, we calibrate ρ_{ij} using

$$\rho_{ij} = \mathrm{Corr}(\exp(e^c_{it}), \exp(e^c_{jt})). \tag{D.5}$$

and highlight that ρ_{ij} – reported in Table 3 – can be positive, approximately zero, or negative. We note that this large variation in ρ_{ij} is to be expected.

To see why note that emissions of jurisdictions whose economies are tightly interconnected through trade and financial flows will likely move together, especially if jurisdictions' emissions are procyclical. If the economic links between jurisdictions are weak and/or they are geographically distant, one would expect a low level of correlation. Finally, if a jurisdiction's business cycles are negatively correlated with others, also observing negative correlations in emissions fluctuations would not be surprising. These conjectures are consistent with empirical studies such as Calderón et al. (2007) which provides evidence on international business cycle synchronization and trade intensity, and Doda (2014) which analyzes the business cycle properties of emissions. Finally, Burtraw et al. (2013) suggest that demand for permits may be negatively correlated over space due to exogenous weather shocks.

We highlight the following three points regarding our calibration strategy and results. First, we assume that the pair characteristics are not affected by the recent introduction of climate change policies. Some emitters in some of the jurisdictions in our sample are regulated under these policies. We argue that any possible effect would be limited because these policies have not been particularly stringent, affect only a portion of the jurisdiction's emissions, and do so only in the last few years of our sample.

Second, we use the HP filter to decompose the observed emissions series into its trend and cyclical components. Not surprisingly, the calibrated pair characteristics are altered somewhat when we alternatively use the band pass filter recommended by Baxter & King (1999), the random walk band pass filter recommended by Christiano & Fitzgerald (2003) or the simpler log quadratic/cubic detrending procedures. However, their effect on the results are minimal so we restrict our attention to the HP filter.

Third, we take the calibrated ρ_{ij}'s at face value in our computations, rather than setting insignificant correlations to zero, which does not alter the results in a meaningful way.